Noise

1- Effective Spring Constant

The spring constant was defined by k = |F/z|, where F is the force acting on the spring and z the resultant deflection. Another definition of the spring constant k can be derived from the potential energy W of a deformed spring, or lever,

$$W = \frac{1}{2} k z^2 \qquad (1)$$

by taking the second derivative of the energy W in respect to z,

$$k = \frac{\partial^2 W}{\partial z^2} \qquad (2)$$

Equation (2) is convenient for finding the effective spring constant in the presence of a force $F(z)$ that has a derivative in the direction of deflection of the spring. In this case we can expand the force to first order,

$$F(z) = F(z_0) + \frac{\partial F(z_0)}{\partial z} \delta z$$

and get an effective spring constant, k',

$$k' = k - F_1$$

where $F_1 = \partial F/\partial z$. Usually the probe that senses the vibration of the tip is above the tip while the sample is below the tip.

For convenience, we denote a force derivative pointing toward the sample as positive. Therefore, a tip-sample attractive force with a positive derivative decreases the resonance frequency of a lever.

- Fluctuation theory of statistical mechanics - Nyquist´s Theorem

Fluctuations in the entropy of the RC circuit; since there is no free energy in the resistor ($I = 0$) itself, the variation in entropy is the negative of the energy delivered to the capacitor
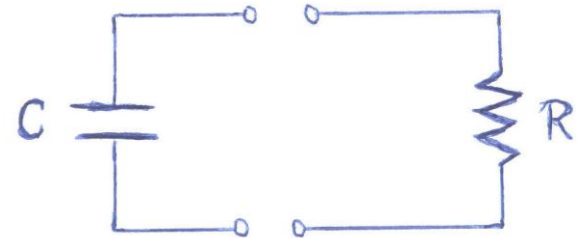
$$\Delta F = T(S - S_0) = -\frac{1}{2} CE^2$$

where E is the thermal noise voltage across C. Since the circuit in thermal equilibrium with the reservoir the probability of finding a value of voltage between E and E + ΔE is

$$P(E)dE = Be^{-\frac{CE^2}{2KT}}$$

$$\int_{-\infty}^{+\infty} P(E)dE = 1 = \int_{-\infty}^{+\infty} Be^{-\frac{CE^2}{2KT}} dE$$

$$B = \sqrt{\frac{2\pi KT}{C}}$$

where P(E) is the probability of finding the voltage between E and E + dE. Therefore, the mean square value of the voltage is

$$\int_{-\infty}^{+\infty} E^2 P(E)de = \int_{-\infty}^{+\infty} \sqrt{\frac{C}{2\pi KT}} E^2 e^{-\frac{CE^2}{2KT}} dE = \frac{KT}{C} \Rightarrow \frac{1}{2} CE^2 = \frac{1}{2} KT$$

2- Sources of Noise

i- Introduction

A typical scanning force microscope consists of a lever and its force sensing tip; a bimorph, and a tip deflection probe, each of which contributes noise to the system. The primary sources of the laser-diode noise (Petermann 1988 and Agrawal and Dutta 1986) include shot intensity, and phase noise, which is converted into amplitude noise in an interferometric configuration.

Initially, we address the thermal noise of the detector and the limiting effect it has on the sensitivity of the measurements. We then discuss the noise at the insulation table and finally at the cantilever.

ii- General Discussion of Noise

A time-dependent optical power, $P(t)$, can be written as a sum of two terms:

$$P(t) = P_0 + \delta P(t)$$

where $P_0 = \langle P \rangle$ is the average power defined as

$$\langle P \rangle = \lim_{t' \to \infty} \frac{1}{t'} \int_{t_0}^{t_0 + t'} P(t)dt$$

and $\delta P(t)$ is the noise term. The spectral density of the power, $W_p(\omega_m)$, where ω_m is the operating frequency can be obtained from $\delta P(t)$ in terms of a Fourier transform by

$$W_p(\omega_m) = \lim_{t' \to \infty} \frac{1}{t'} \left| \int_0^{t'} \delta P(t)\exp(-j\omega_m t)dt \right|^2$$

$$W_p(\omega_m) = \langle |\Delta P(\omega_m)|^2 \rangle$$

and the mean square of the noise, $\langle \delta P^2(t) \rangle$, is given by

$$\langle \delta P^2(t) \rangle = \int_{-\infty}^{\infty} W_p(\omega_m) df = \int_{-\infty}^{\infty} \langle |\Delta P(\omega_m)|^2 \rangle df$$

The noise, passing through a filter that has a bandwidth of B and transmission of unity around the center frequency ω_m, is

$$\langle \delta P^2 \rangle = 2B \langle |\Delta P(\omega_m)|^2 \rangle = 2B W_p(\omega_m)$$

A widely used definition of noise is given in terms of the relative intensity noise (RIN), defined by

$$RIN \equiv \frac{\langle \delta P^2 \rangle}{\langle P \rangle^2}$$

iii- Noise in Laser Detection

Since an optical power incident on a photoconductive detector generates a current given by $\quad i = \eta P$

where η is in mA/mW, we can write the RIN as $\quad RIN \equiv \dfrac{\langle \delta i^2 \rangle}{\langle i \rangle^2}$

a- Shot Noise

The RIN arising from statistics of photons incident on a photoconductive detector can be obtained from

$$RIN = \frac{2B\langle|\Delta P(\omega_m)|^2\rangle}{\langle P^2\rangle} = \frac{2BW_p(\omega_m)}{\langle P\rangle^2}$$

From the definition of shot noise,

$$\langle|\Delta i(\omega_m)|^2\rangle = e\langle i\rangle$$

where *e* is the electronic charge, we get the shot-noise-limited RIN,

$$RIN_{shot} = \frac{2Be}{\langle i\rangle} = \frac{3.2\times10^{-16}mA}{\langle i\rangle} \; \frac{B}{Hz}$$

The RIN of any experimental system will therefore obey

$$RIN > RIN_{shot}$$

$$\langle \delta i^2\rangle_{shot}{}^{1/2} = \sqrt{2e\eta P_0 B} \qquad\qquad \langle \delta V^2\rangle_{shot}{}^{1/2} = R\sqrt{2e\eta P_0 B}$$

We find that a shot noise current

$$\langle \delta i^2 \rangle_{shot}{}^{1/2} = \sqrt{2e\eta P_0 B}$$

generates a voltage across a load resistor R given by

$$\langle \delta V^2 \rangle_{shot}{}^{1/2} = R\sqrt{2e\eta P_0 B}$$

b- Resistor Johnson Noise

The agitation of electrons in the load resistor R in the first stage of a photodetector amplifier gives rise to *a* RIN, noise current, and noise voltage given by

$$RIN_J = \frac{4KTB}{R\eta^2 P^2_0} \qquad \langle \delta i^2 \rangle_J{}^{1/2} = \sqrt{\frac{4KTB}{R}}$$

and

$$\langle \delta V^2 \rangle_J{}^{1/2} = \sqrt{4KTBR}$$

Here K is Boltzmann's constant and T is the absolute temperature.

c- Laser Intensity Noise

The dominant source of noise in laser diodes is spontaneous emission. The laser diode, which behaves like a thermal source, is actually an amplifier for this noise. Apart from the spontaneous-emission noise we have to take into account the mode-partition noise, mode-hopping noise, and 1/f noise. The theory of these contributions is lengthy and we will not deal with it here. Practically, we can use the specifications accompanying a laser diode that give the RIN measured in a given bandwidth around a center frequency. A typical value for the laser intensity- related RIN, including the shot noise, is

$$RIN_{int} = 10^{-13} \, B/[Hz]$$

iv- Environmental Noise (1/f)

Isolation Table

$$m\ddot{x} = F_{apl} - F_K - F_r$$

$$m\frac{dv}{dt} + F_r + F_K = F_{apl}$$
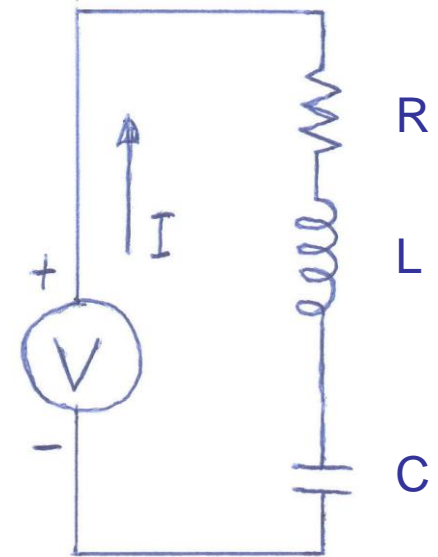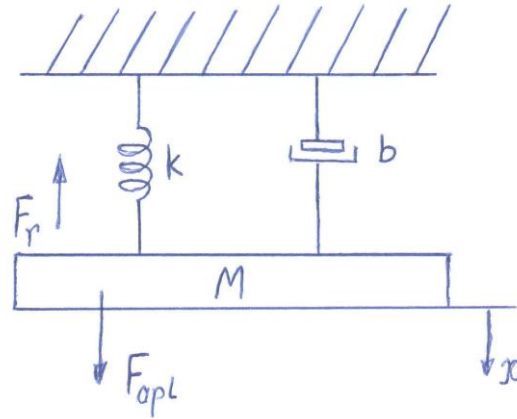
$$F_{apl} = m\frac{dv}{dt} + bv + k\int_0^t v\,dt$$

$$v = v_0 e^{j\omega t}$$

$$F_{apl} = j\omega m v + bv + \frac{k}{j\omega}v$$

$$\frac{F_{apl}}{v} = b + j\omega m - j\frac{k}{\omega} = Z_{mec}$$

↓   ↓

$$\frac{V}{I} = R + j\omega L + \frac{1}{j\omega C} = Z_{elet}$$

v   = velocity
m   = cantilever mass
F_K = elastic restoring force = kx
F_r = dissipative force (friction) = bV
F_{apl} = external force applied to the cantilever oscillation

Equivalence of the above equation with that of an RLC circuit where b = R, m = L and k = 1/C

Roots of the equation are obtained using the Laplace transform (s)

$$F_{apl} = m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx$$

$$x = x_0 e^{st}$$

$$F(s) = ms^2 x + bsx + kx$$

$$\frac{F}{x} = ms^2 + bs + k$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

v- Cantilever Noise

Rewriting in terms of parameter ζ, ω_0 and Q we get

$$\frac{F}{xm} = s^2 + \frac{b}{m}s + \frac{k}{m}$$

$$= s^2 + 2\zeta\omega_0 s + \omega_0^2 \quad\Rightarrow\quad w_0^2 = \frac{k}{m}$$

$$S_{1,2} = \frac{-2\zeta\omega_0 \pm \sqrt{4\zeta^2\omega_0^2 - 4\omega_0^2}}{2}$$

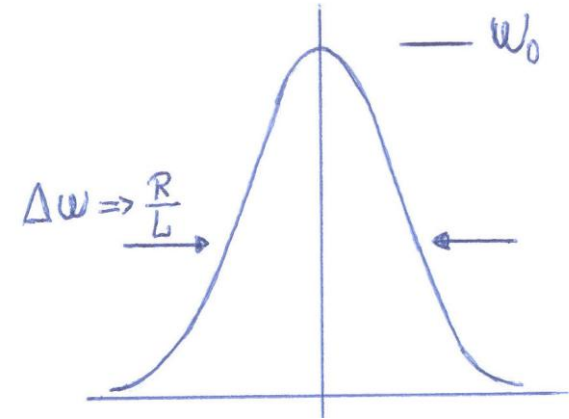$$= \omega_0\zeta \pm \sqrt{\zeta^2\omega_0^2 - \omega_0^2}$$

$$= \omega_0\zeta \pm j\omega_0\sqrt{1-\zeta^2}$$

$$\zeta = \frac{b}{2\omega_0 m}$$

$$2\zeta\omega_0 = \frac{b}{m} \quad\therefore\quad \zeta = \frac{b}{2m}\frac{1}{\sqrt{\dfrac{k}{m}}} \qquad \zeta = \frac{b}{2}\frac{1}{\sqrt{km}}$$

$$2\zeta\omega_0 = \Delta\omega$$

$$\zeta = \frac{1}{2}\frac{\Delta\omega}{\omega_0} = \frac{1}{2}\frac{1}{Q}$$

$$\Delta\omega \Rightarrow \frac{R}{L}$$

$$S_{1,2} = \pm\frac{\omega_0}{2Q} + j\omega_0\sqrt{1-\zeta^2}$$

$$\zeta \ll 1$$

$$S_{1,2} = \pm\frac{\Delta\omega}{2} + j\omega_0$$

Solution

$$m\ddot{x} + b\ddot{x} + kx = F_0 e^{j(\omega t - \phi)}$$

$$m(-1)A_0 e^{j(\omega t - \phi)}\omega^2 - j\omega A_0 e^{j(\omega t + \phi)} + ke^{j(\omega t - \phi)} = F_0 e^{j(\omega t + \phi)}$$

$$(-m\omega^2 + k)A_0 e^{j(\omega t - \phi)} - j\omega A_0 e^{j(\omega t + \phi)} = F_0 e^{j(\omega t + \phi)}$$

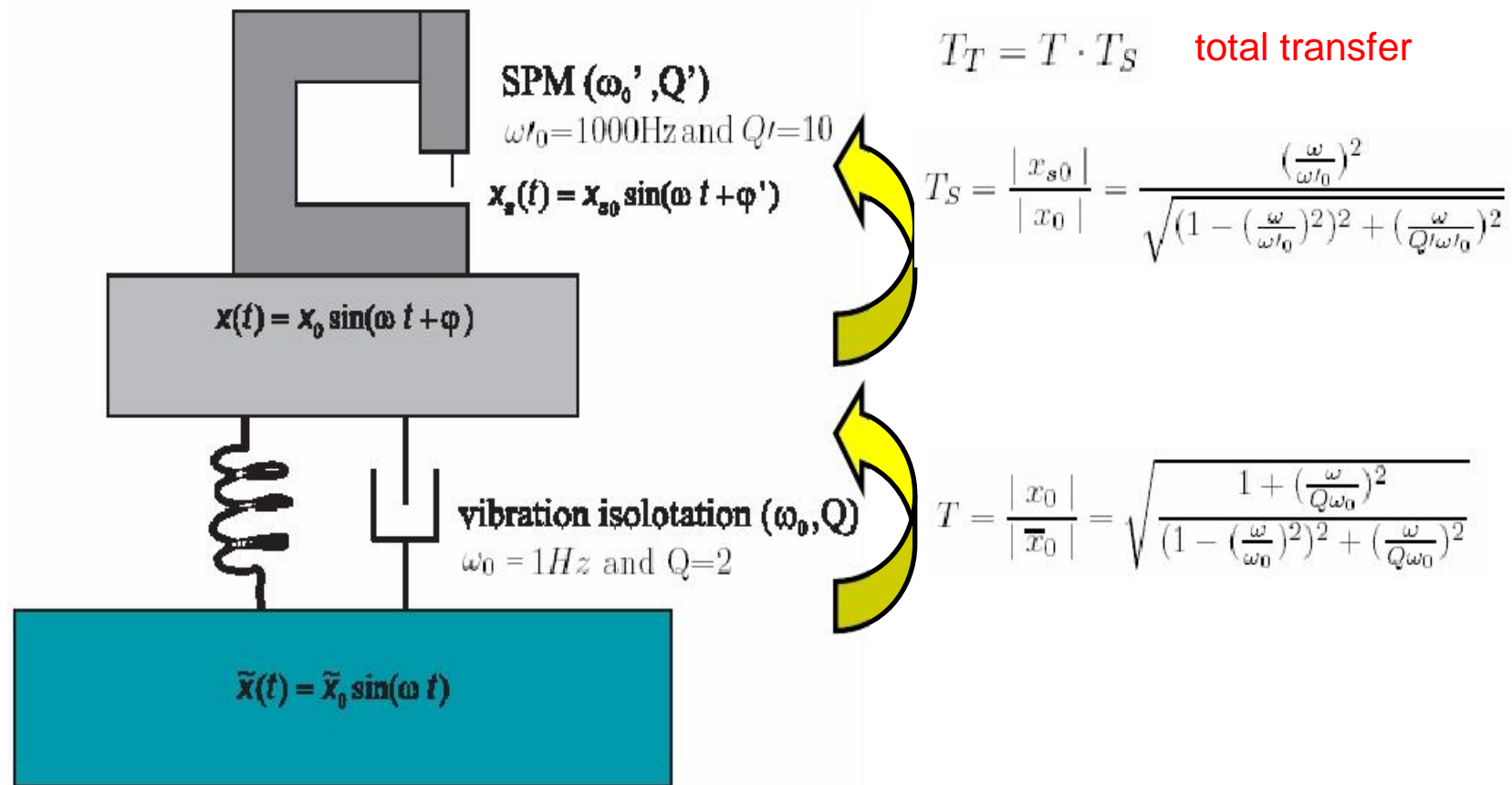$$\left[(\omega_0^2 - \omega^2) - j\omega b\right]x(t) = \frac{F_0}{m}e^{j(\omega t + \phi)}$$

$$x(t) = \frac{F_0 / m}{(\omega_0^2 - \omega^2) - j\omega b}e^{j(\omega t - \phi)}$$

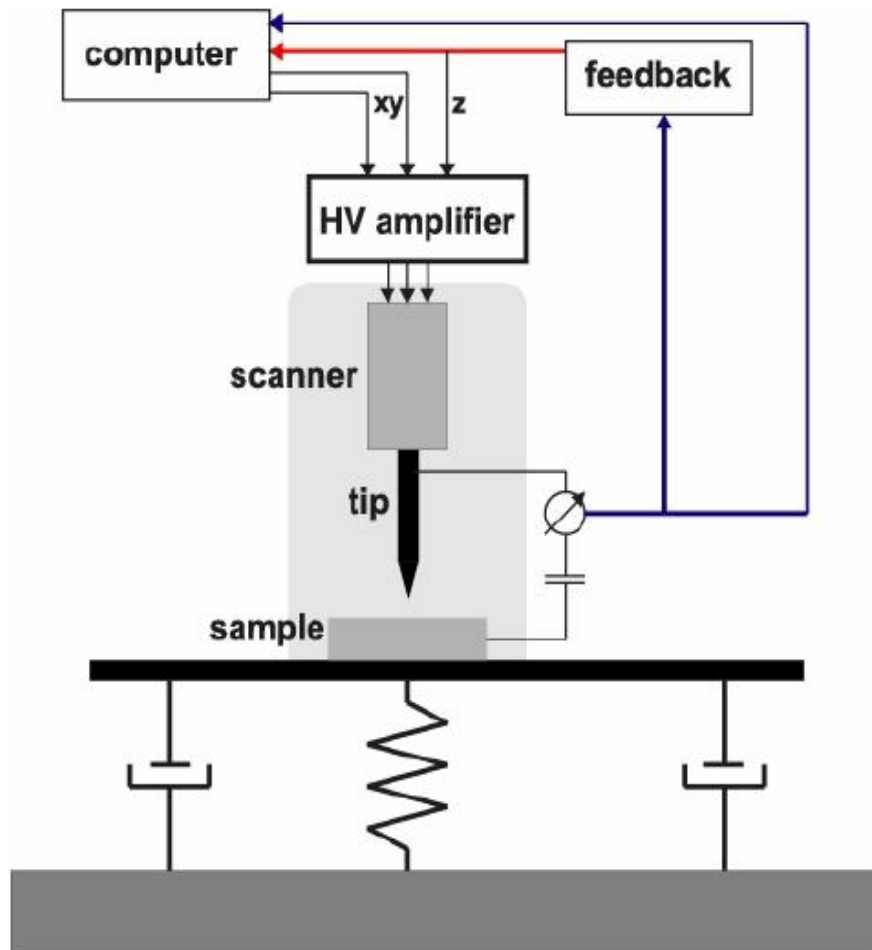$$|x(t)| = \frac{F_0 / m}{\sqrt{(\omega_0^2 - \omega^2)^2 + \omega^2 b^2}}\cos(\omega t - \phi)$$

$$e \qquad b^2 = 4\gamma^2$$

vi- Experimental Values

- Vibration Isolation

SPM (ω_0',Q')

$\omega_0'=1000 \text{Hz}$ and $Q'=10$

$x_s(t) = x_{s0}\sin(\omega t + \varphi')$

$x(t) = x_0 \sin(\omega t + \varphi)$

vibration isolotation (ω_0, Q)

$\omega_0 = 1Hz$ and $Q=2$

$\tilde{x}(t) = \tilde{x}_0 \sin(\omega t)$

$T_T = T \cdot T_S$ total transfer

$$T_S = \frac{|x_{s0}|}{|x_0|} = \frac{(\frac{\omega}{\omega_0'})^2}{\sqrt{(1 - (\frac{\omega}{\omega_0'})^2)^2 + (\frac{\omega}{Q'\omega_0'})^2}}$$

$$T = \frac{|x_0|}{|\overline{x}_0|} = \sqrt{\frac{1 + (\frac{\omega}{Q\omega_0})^2}{(1 - (\frac{\omega}{\omega_0})^2)^2 + (\frac{\omega}{Q\omega_0})^2}}$$

vii- Noise Isolation in AFM

- good vibration isolation of the experiment is prerequisite for high resolution imaging

- the mechanical stability of the experimental setup turns out to be a prerequisite for successful measurements on the atomic scale

- the tip is moved in three dimensions by piezoelectric actuators

- a high voltage amplifier is required to drive the piezoelectric scanner

- the tunneling current is used to control the tip-sample distance z via a feedback circuit that keeps the tunneling current constant

- the distance z is recorded by a computer as function of the scanned coordinates x,y

In an STM typically two signals are acquired, namely the z-coordinate (topography) and the tunneling current (error signal)

viii- Thermally Induced Lever Noise

The lever of a scanning force microscope acts as an isolated one dimensional harmonic oscillator whose energy is

$$W = \frac{1}{2} m \left(\frac{\partial z}{\partial t} \right)^2 + \frac{1}{2} m\omega_0^2 z^2$$

Here each of the two degrees of freedom contributes 1/2 KT to the thermal energy of the lever, giving a total thermal energy of KT. The spectral density of the thermal noise of this free-running lever is

$$W_p(\omega) = \frac{2\gamma KT}{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

where $\gamma = m\omega_0/Q$ and $\omega_0^2 = k/m$. The above equation rewritten as

$$2B\ W_p(\omega) = \frac{4KTBQ}{k\omega_0} \frac{1}{Q^2(1 - \omega^2/\omega_0^2)^2 + \omega^2/\omega_0^2}$$

The RMS noise of the cantilever, obtained from the thermal noise spectral density, is

$$\langle \delta z^2 \rangle^{1/2} = \sqrt{2 B W_p(\omega)}$$

where B is the bandwidth of the spectrum analyzer.

The RMS noise of the cantilever can be obtained by

$$\langle \delta z^2 \rangle^{1/2} = \left[\frac{4kTB}{k\omega_0} \frac{Q}{Q^2(1 - \omega^2/\omega_0^2)^2 + \omega^2/\omega_0^2} \right]^{1/2}$$

ix- Flicker effect

The surface is in a continual process of change due to such causes as evaporation, diffusion, chemical action, structural rearrangements and gas bombardment.

x- Summary

The main results obtained in this chapter are the contributions of the shot noise

$$\langle \delta i^2 \rangle^{1/2}_{shot} = \sqrt{2e\eta P_0 B}$$

the Johnson noise of the load resistor

$$\langle \delta i^2 \rangle^{1/2}_{J} = \sqrt{\frac{4KTB}{R}}$$

the relative intensity noise of the laser

$$\langle \delta i^2 \rangle^{1/2}_{int} = \eta P_0 \sqrt{RIN_{int}}$$

the phase noise of the laser

$$\langle \delta i^2 \rangle^{1/2}_{phase} = \eta P_0 \sqrt{4\pi \Delta\nu B \tau}$$

the thermal noise of the lever on resonance

$$\langle \delta i^2(\omega_0) \rangle^{1/2}_{lever} = \frac{4\pi}{\lambda} \eta P_0 \sqrt{\frac{4KTBQ}{k\omega_0}}$$

and the thermal noise of the lever off resonance

$$\langle \delta i^2(\omega < \omega_0) \rangle^{1/2}_{lever}{}^{1/2} = \frac{4\pi}{\lambda} \eta P_0 \sqrt{\frac{4KTB}{Qk\omega_0}}$$