Alcance Máximo com ~~atrito~~ Resistência

Sendo $k = \frac{b}{m}$

$$y(x) = \frac{g}{k^2} \cdot \ln\left(1 - \frac{kx}{v_{ox}}\right) + \left(\frac{g}{k} + v_{oy}\right)\frac{x}{v_{ox}}$$

$$y(x_{máx}) = 0 \Rightarrow \frac{g}{k^2} \cdot \ln\left(1 - \frac{kx}{v_{ox}}\right) = -\left(\frac{g}{k} + v_{oy}\right)\frac{x}{v_{ox}}$$

Vamos supor k pequeno $\Rightarrow$

$$\ln\left(1 - \frac{kx}{v_{ox}}\right) \simeq 0 - \frac{x}{v_{ox}} \cdot \frac{1}{1} \cdot k - \frac{1}{2}\frac{x^2}{v_{ox}^2} \cdot k^2 - \frac{2}{6} \cdot \frac{x^3}{v_{ox}^3} \cdot k^3$$

$$\Rightarrow \frac{g}{\cancel{k^2}} \cdot \left(\cancel{-\frac{xk}{v_{ox}}} - \frac{1}{2}\frac{x^2}{v_{ox}^2} \cancel{k^2} - \frac{1}{3}\frac{x^3}{v_{ox}^3}\cancel{k^3}\right) = \cancel{-\frac{g}{k}\frac{x}{v_{ox}}} - \frac{v_{oy}}{v_{ox}} x$$

$$\cancel{x} \cdot \frac{v_{oy}}{\cancel{v_{ox}}} = +\frac{g}{2}\frac{x^{\cancel{2}}}{v_{ox}^{\cancel{2}}} + \frac{1}{3}\frac{x^{\cancel{3}\,2}}{v_{ox}^{\cancel{3}\,2}} \cdot gk \qquad \text{se } x \neq 0$$

$$x^2 \cdot \left(\frac{1}{3} g \frac{k}{v_{ox}^2}\right) + x \cdot \frac{g}{2}\frac{1}{v_{ox}} - v_{oy} = 0$$

$$x = \frac{-\frac{g}{2}\frac{1}{v_{ox}} \pm \sqrt{\frac{g^2}{4} \cdot \frac{1}{v_{ox}^2} + \frac{4 v_{oy} g k}{3 v_{ox}^2}}}{2 \cdot \frac{1}{3} g \frac{k}{v_{ox}^2}}$$

Solução (−) não faz sentido físico ⇒

$$x_{máx} = \frac{-\frac{g}{2}\frac{1}{v_{ox}} + \frac{1}{v_{ox}}\sqrt{\frac{g^2}{4} + \frac{4}{3} v_{oy}\, g k}}{\frac{2}{3}\frac{gk}{v_{ox}^2}}$$

$$x_{máx} \simeq \frac{+\frac{g}{2}\frac{1}{v_{ox}} \cdot \left(-1 + \sqrt{1 + \frac{16}{3} v_{oy}\frac{k}{g}}\right)}{\frac{2}{3}\frac{gk}{v_{ox}^2}}$$

como $k \to 0 \Rightarrow \sqrt{1 + ax} \simeq 1 + \frac{1}{2}ax - \frac{a^2}{8} \cdot x^2$

$$x_{máx} \simeq \frac{\frac{\cancel{g}}{2} \cdot \frac{1}{\cancel{v_{ox}}} \cdot \left(\cancel{-1} \cancel{+1} + \frac{1}{2}\frac{16}{3} v_{oy} \cdot \frac{k}{g} - \left(\frac{16}{3}\right)^2 \cdot \frac{1}{8} \cdot \frac{v_{oy}^2}{g^2} \cdot k^2\right)}{\frac{2}{3}\frac{\cancel{g}k}{v_{ox}^{\cancel{2}}}}$$

$$x_{máx} \simeq \frac{\frac{1}{2} \cdot \left(\frac{\overset{4}{\cancel{8}} v_{oy}}{\cancel{3}}\frac{\cancel{k}}{g} - \frac{\overset{16}{\cancel{32}}}{\cancel{9}\,3} \cdot \frac{v_{oy}^2}{g^2} \cdot k^{\cancel{2}}\right)}{\frac{\cancel{2}}{\cancel{3}}\frac{\cancel{k}}{v_{ox}}} =$$

$$\boxed{x_{máx} \simeq \frac{2 v_{oy} v_{ox}}{g} - \frac{16}{3} \cdot \frac{v_{oy}^2 \cdot v_{ox} \cdot k}{g^2}}$$

Agora, assumindo que

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \operatorname{sen}\theta \quad :$$

$$\boxed{x_{máx} = \frac{2 v_0^2 \operatorname{sen}\theta\cos\theta}{g} - \overbrace{\frac{16}{3}\cdot v_0^3 \cdot \frac{\operatorname{sen}^2\theta\cos\theta}{g^2}\cdot k}^{\text{devido à resistência do ar.}}}$$

$$= \frac{v_0^2}{g}\operatorname{sen}2\theta - \frac{8}{3}\frac{v_0^3}{g^2}k \operatorname{sen}\theta\cdot\operatorname{sen}2\theta$$

$$\frac{dx_{máx}}{d\theta} = \frac{2 v_0^2}{g}\cos 2\theta - \frac{4}{3}\frac{v_0^3 k}{g^2}\cdot\left(3\operatorname{sen}3\theta - \operatorname{sen}\theta\right) = 0$$

$$\therefore \quad \cos 2\theta = \frac{2}{3}\frac{v_0}{g}\cdot k\cdot\left(3\operatorname{sen}3\theta - \operatorname{sen}\theta\right)$$

$$\operatorname{sen}3\theta \downarrow 3\operatorname{sen}\theta\cos^2\theta - \operatorname{sen}^3\theta$$

$$\therefore \cancel{2\cdot\operatorname{sen}\theta\cos\theta} \Rightarrow \cos^2\theta - \operatorname{sen}^2\theta = \frac{2}{3}\cdot\frac{v_0 k}{g}\cdot\operatorname{sen}\theta\left(9\cancel{\operatorname{sen}\theta}\cdot\cos^2\theta - 3\operatorname{sen}^{2}\theta - \cancel{\operatorname{sen}\theta}\,1\right)$$

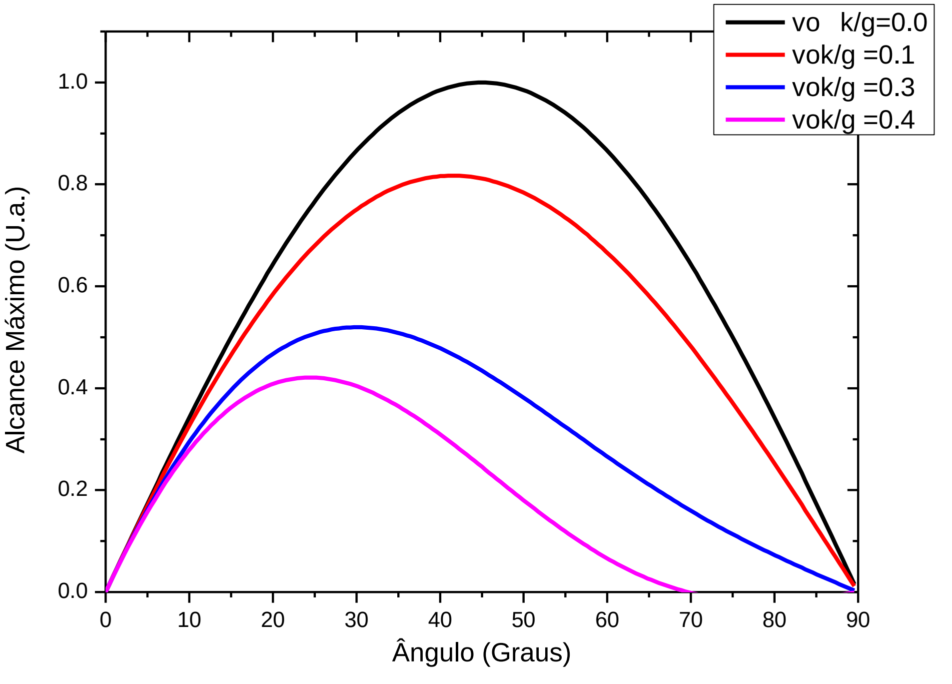
$$\cos^2\theta = 1 - \operatorname{sen}^2\theta \quad \therefore$$

$$\underbrace{1 - 2\operatorname{sen}^2\theta = \frac{2}{3}\frac{v_0 k}{g}\cdot\operatorname{sen}\theta\cdot\left(9 - 9\operatorname{sen}^2\theta - 3\operatorname{sen}^2\theta - 1\right)}$$

muito complicado, mas note que a solução depende de $\frac{v_0 k}{g}$.

fazendo esta conta p/ alguns valores de $\frac{v_0 k}{g}$:

Resolvendo para 3 valores de vok/g vemos que o alcance atinge seu valor máximo para lançamentos com ângulos menores quando a resistência do meio é mais forte.



Tomando a derivada do alcance máximo em relação ao ângulo de lançamento vemos mais facilmente o máximo.