Illuminating pions with photons

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$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

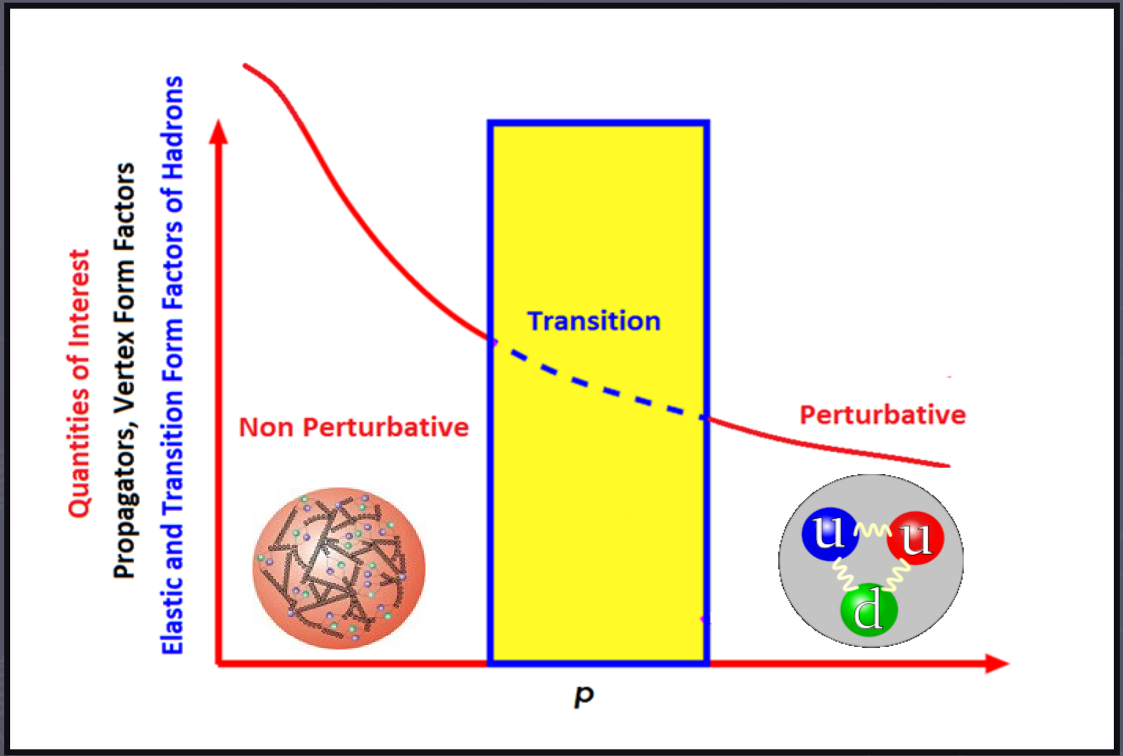
QCD-TNT4 - Unraveling the organization of the QCD tapestry
IhaBela, São Paulo, Brazil - August 31 to September 04, 2015

Introduction

- QCD has two facets: Perturbative & non perturbative. Perturbative -> Asymptotic freedom.
- Non perturbative QCD has emergent phenomena not treatable in perturbation theory.
- Dynamical mass generation for massless quarks; (DCSB). No degeneracy between $J^{P=+}$ and $J^{P=-}$.
- Color degrees of freedom (quarks and gluons) are not observable (confinement).
- Partons of perturbative QCD produced at high energy convert into color singlet hadrons of non perturbative QCD by the time they reach the detector.
- Challenge is to understand hadrons starting from the fundamental degrees of freedom of quarks and gluons.

Introduction

Schwinger-Dyson equations are the fundamental equations of QCD and combine its UV and IR behaviour.

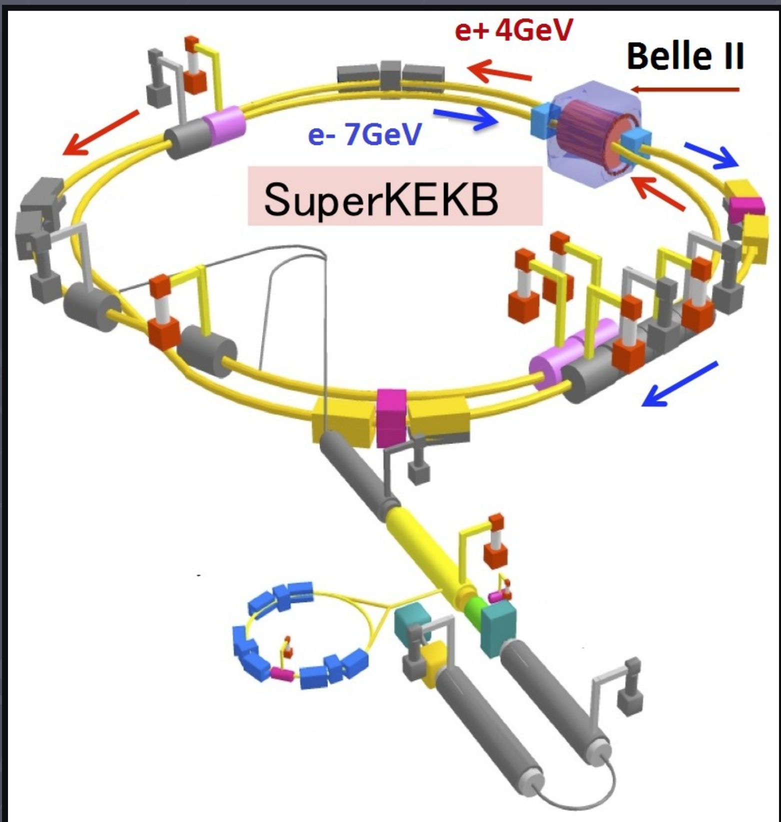


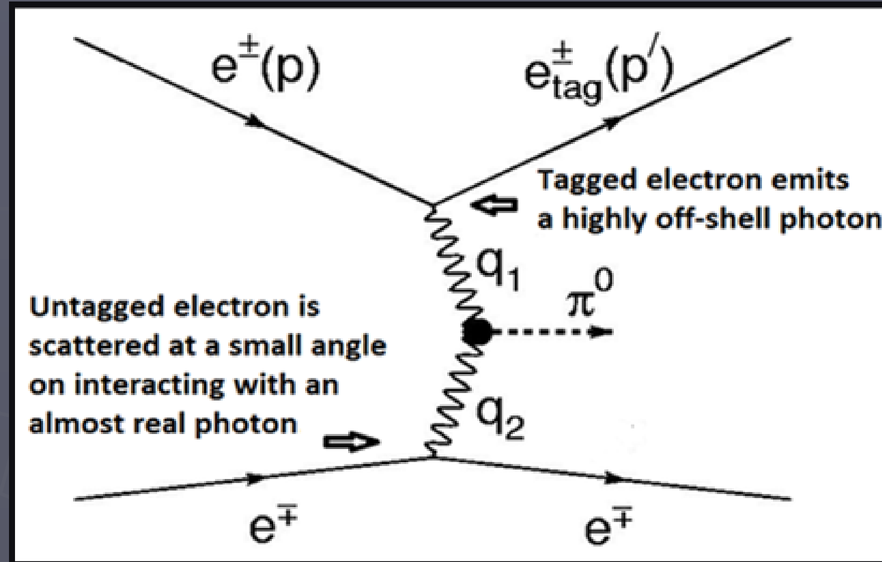
Observing the transition of the pion (form factors, such as $\boxed{\gamma^*\gamma \rightarrow \pi^0}$) from quarks and gluons to one with valence quarks alone can be studied naturally through SDE.

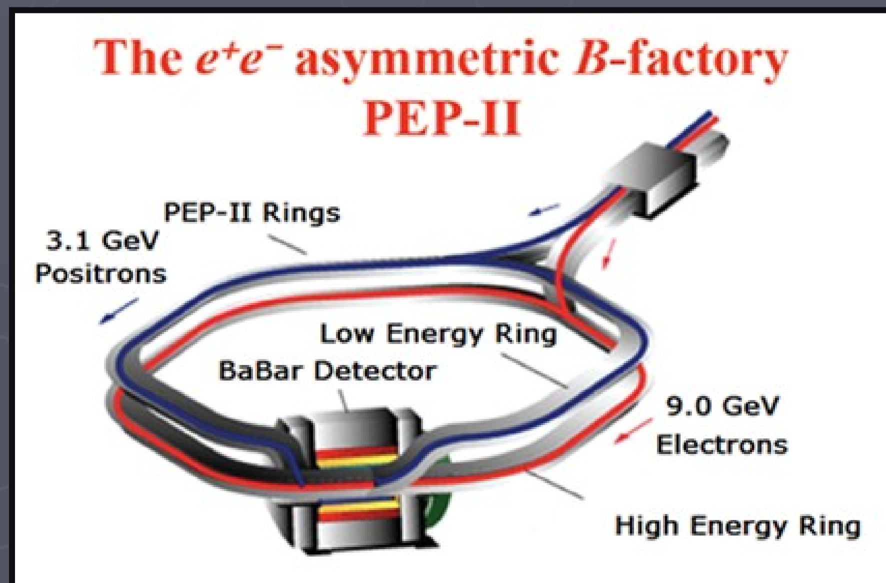
Introduction

The $\boxed{\gamma^*\gamma \to \pi^0}$ transition is studied through process:

$$\boxed{e^+e^- \Rightarrow e^+e^-\pi^0}$$

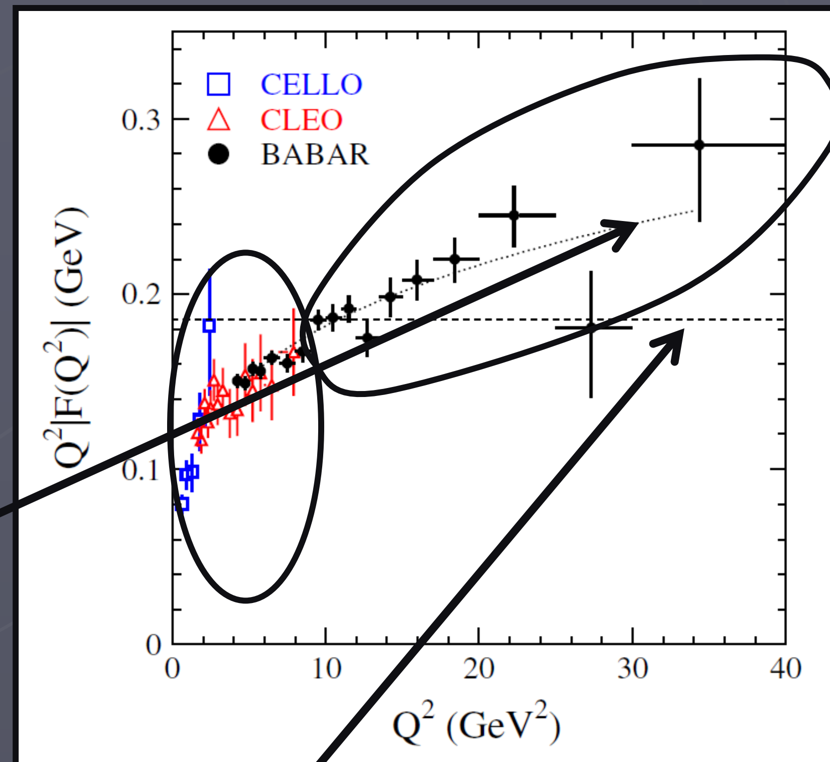






Introduction

The $\boxed{\gamma^*\gamma \to \pi^0}$ transition form factor:



$$Q^2|F(Q^2)| = A\left(\frac{Q^2}{10\ \mathrm{GeV}^2}\right)^{\beta}$$

$$A = 0.182 \pm 0.002\ \mathrm{GeV}$$

$$\beta = 0.25 \pm 0.02$$

$$\text{form factor } (\sim 1/Q^{3/2})$$

CELLO H.J. Behrend et.al., Z. Phys C49 401 (1991). 0.7 – 2.2 GeV^2

The leading twist asymp QDC calculation:

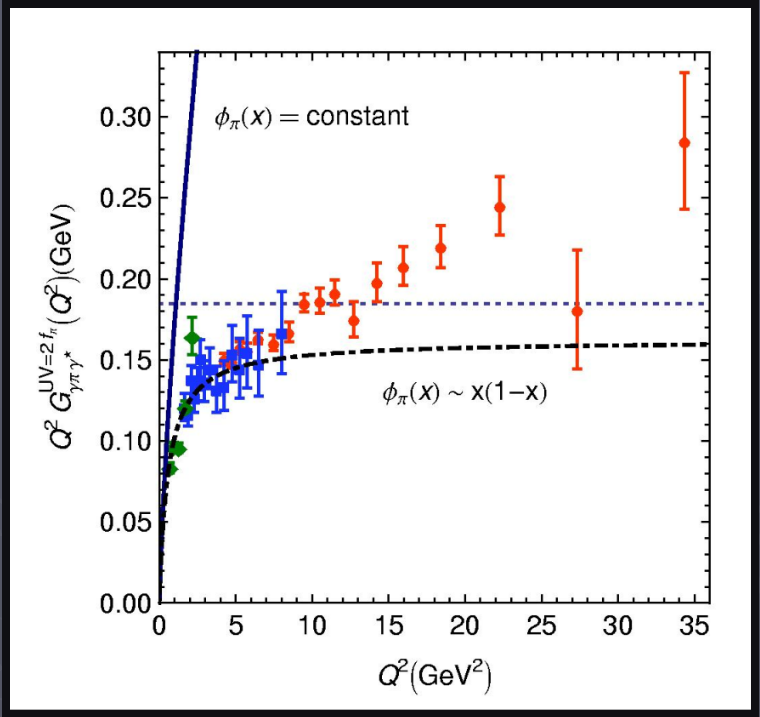
CLEO J. Gronberg et. al., Phys. Rev. D57 33 (1998). 1.7 – 8.0 GeV^2

G.P. Lepage, and S.J. Brodsky, Phys. Rev. D22, 2157 (1980).

BaBar R. Aubert et. al., Phys. Rev. D80 052002 (2009). 4.0 – 40.0 GeV^2

Introduction

The $\gamma^*\gamma \to \pi^0$ transition form factor:



H.L.L. Robertes, C.D. Roberts, AB, L.X. Gutiérrez and P.C. Tandy, Phys. Rev. C82, (065202:1-11) 2010.

CELLO H.J. Behrend et.al., Z. Phys C49 401 (1991). 0.7 – 2.2 GeV^2

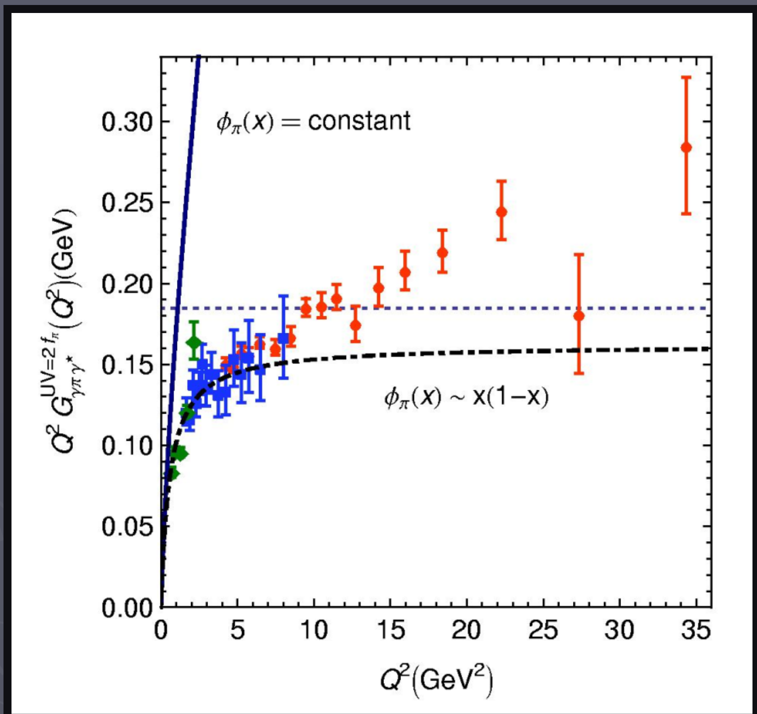
CLEO J. Gronberg et. al., Phys. Rev. D57 33 (1998). 1.7 – 8.0 GeV^2

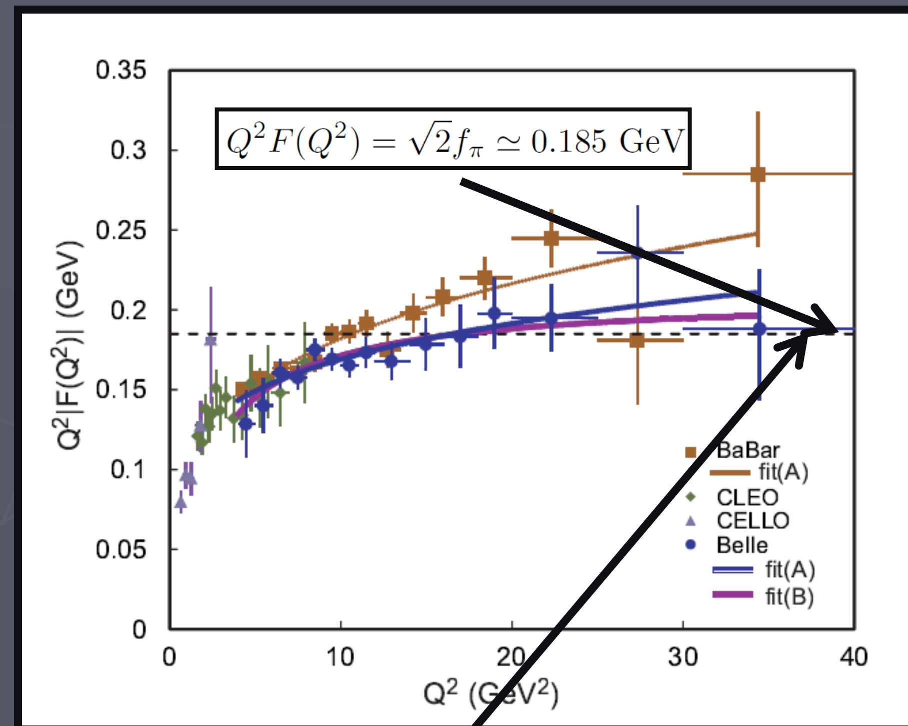
BaBar R. Aubert et. al., Phys. Rev. D80 052002 (2009). 4.0 – 40.0 GeV^2

Belle S. Uehara et. al., Phys. Rev. D86 092007 (2012). 4.0 – 40.0 GeV^2

Introduction

The $\boxed{\gamma^*\gamma \to \pi^0}$ transition form factor:





Lowest order in perturbation theory and the leading twist asymptotic QCD calculation:

G.P. Lepage, and S.J. Brodsky, Phys. Rev. D22, 2157 (1980).

Introduction

H. L. L. Roberts, et al.,
Phys. Rev. C 82 (2010) 065202.

S. J. Brodsky, F.-G. Cao, G. F. de Teramond,
Phys. Rev. D 84 (2011) 033001.

A. P. Bakulev, S. V. Mikhailov, A. V. Pimikov, N. G. Stefanis,
Phys. Rev. D 84 (2011) 034014.

S. J. Brodsky, F.-G. Cao, G. F. de Teramond,
Phys. Rev. D 84 (2011) 075012.

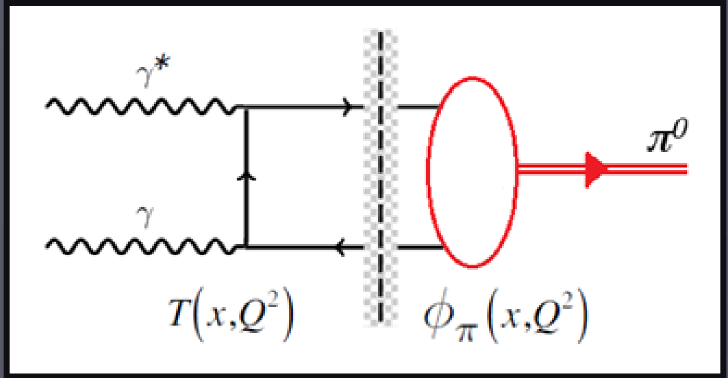
A. Bakulev, S. Mikhailov, A. Pimikov, N. Stefanis,
Phys. Rev. D 86 (2012) 031501.

B. El-Bennich, J. P. B. C. de Melo, T. Frederico,
Few-Body Systems 54 (2013) 1851–1863.

Introduction

Transition form factor is the correlator of two currents:

Collinear factorization:



$$\int d^4z\, \mathrm{e}^{-iq\cdot z} \;< \pi^0(P)|\mathrm{T}\{j_\mu(z)j_\nu(0)\}|0> \;=\; i\epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta G^{\gamma^*\gamma^*\pi}(Q^2,q^2)$$

$$G^{\gamma^*\gamma^*\pi}(Q^2,q^2) = N\int_0^1 dx\, T(Q^2,q^2,\mu^2,x)\, \phi_\pi(x,\mu^2) + \mathcal{O}(1/q^4)$$

T: hard scattering amplitude with quark gluon sub-processes.

$\phi_\pi(x,\mu^2)$ is the pion distribution amplitude:

Introduction

$$G^{\gamma^*\gamma^*\pi}(Q^2, q^2) = N \int_0^1 dx\, T(Q^2, q^2, \mu^2, x)\, \phi_\pi(x, \mu^2) + \mathcal{O}(1/q^4)$$

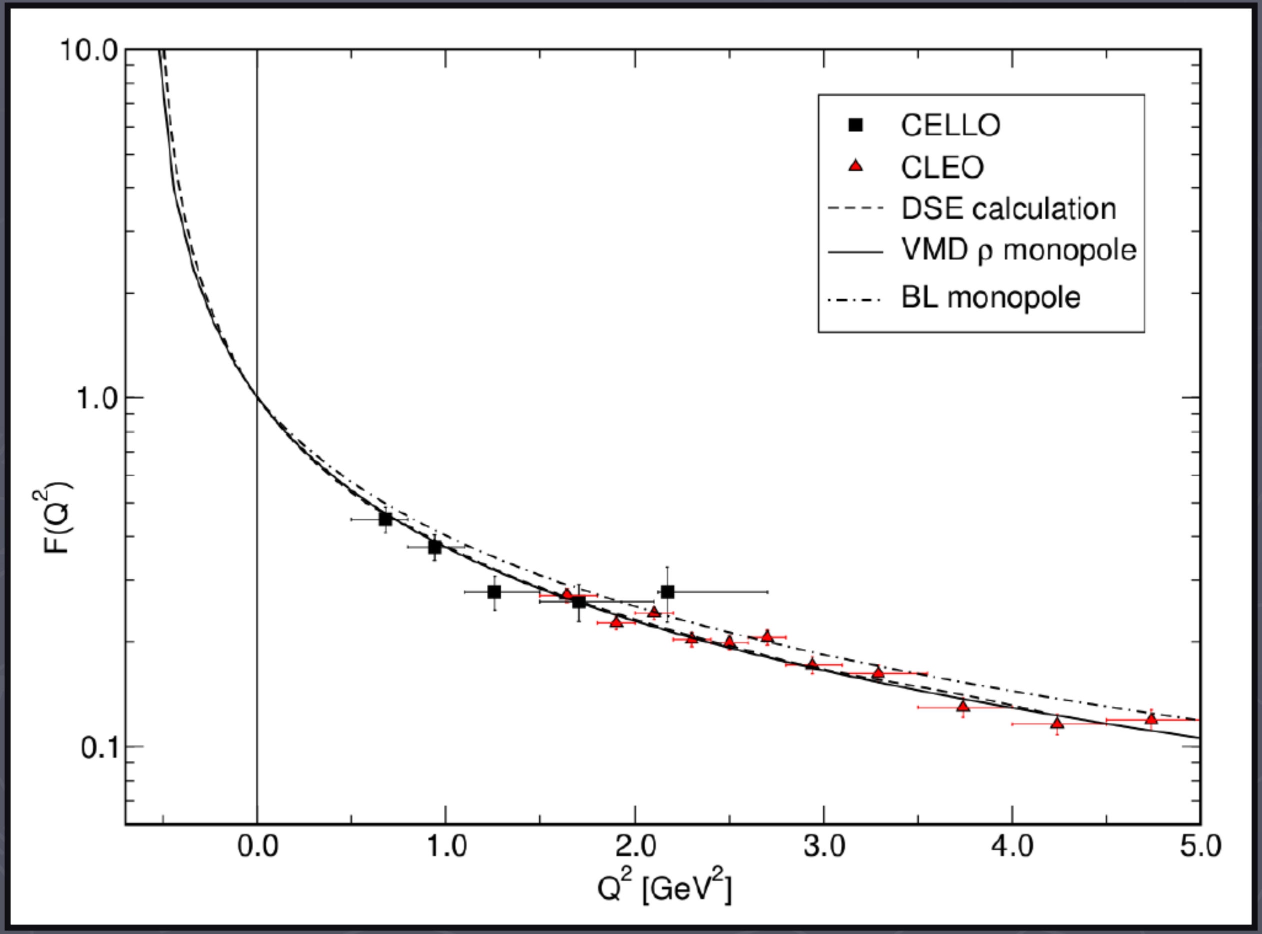
$\phi_\pi(x, \mu^2)$ is the pion distribution amplitude:

$$\phi_\pi(x, \mu^2) = \psi_0(x) + \Sigma_{n=2,4,\cdots}\, a_n(\mu^2)\, \underbrace{\psi_n(x)}_{\text{Gegenbauer harmonics}}$$

In asymptotic QCD: $\phi_\pi^{asym}(x, \mu^2) = \psi_0(x) = 6x(1-x)$

Introduction

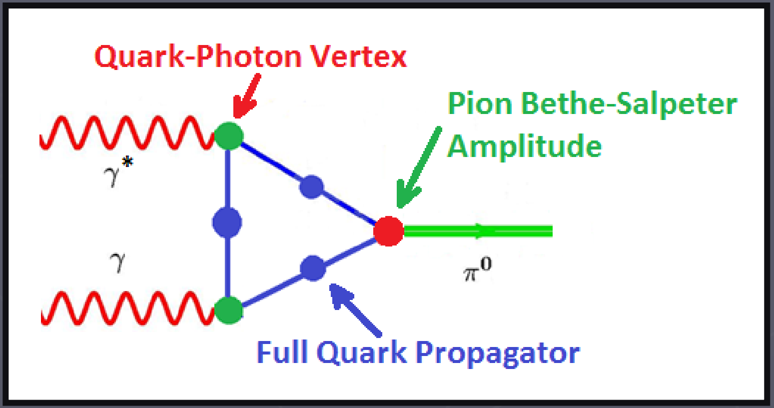
BSE: Integration domain hits singularities at large Q^2.



P. Maris and P.C. Tandy., Phys. Rev. C65 045211 (2002).

Introduction

- The triangle diagram



- Schwinger-Dyson Equations

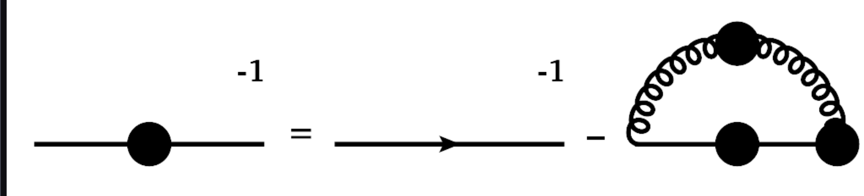
  Quark Propagator: Quark Mass Function

  Pion Bethe-Salpeter Amplitude

  Quark-Photon Vertex

The Quark Propagator

Quark propagator:



$$S_B^{-1}(p,\Lambda) = S_0^{-1}(p) + \int d^4q\; g_B^2(\Lambda) D^B_{\mu\nu}(p-q,\Lambda)\frac{\lambda^a}{2}\gamma_\mu S_B(q;\Lambda)\Gamma^a_{B\nu}(q,p;\Lambda)$$

$$\begin{aligned} g_B(\Lambda) &= \mathcal{Z}_g g(p-q,\mu) \\ D^B_{\mu\nu}(p-q,\Lambda) &= \mathcal{Z}_3 D_{\mu\nu}(p-q,\mu) \\ S_B(q;\Lambda) &= \mathcal{Z}_{2F} S(p,\mu) \\ \Gamma^a_{B\nu}(q,p;\Lambda) &= \mathcal{Z}_{1F}^{-1}\Gamma(p,q,\mu) \end{aligned}$$

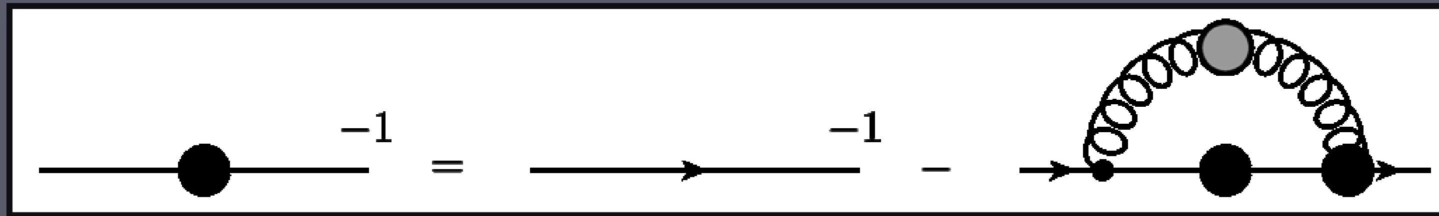
$$\frac{\mathcal{Z}_1}{\mathcal{Z}_3} = \frac{\tilde{\mathcal{Z}}_1}{\tilde{\mathcal{Z}}_3} = \frac{\mathcal{Z}_5}{\mathcal{Z}_1} = \frac{\mathcal{Z}_{1Fj}}{\mathcal{Z}_{2Fj}}$$

$$S^{-1}(p,\mu) = \mathcal{Z}_{2F} i\gamma\cdot p + \mathcal{Z}_4 m(\mu) + \mathcal{Z}_{1F}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(p,\mu)\Gamma(p,q,\mu)$$

$$S^{-1}(p,\mu) = \mathcal{Z}_{2F} S_0^{-1}(p) + \frac{\tilde{\mathcal{Z}}_1 \mathcal{Z}_{2F}}{\tilde{\mathcal{Z}}_3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(p,\mu)\Gamma(p,q,\mu)$$

The Quark Propagator

Quark propagator



$$S^{-1}(p;\mu) = Z_{2F}(\mu,\Lambda) i\gamma\cdot p + Z_4(\mu,\Lambda) m(\mu)$$
$$+ Z_{1F}(\mu,\Lambda) \int d^4q\, g^2 D_{\mu\nu}(p-q;\mu)\, \frac{\lambda^a}{2}\gamma_\mu\, S(q;\mu)\, \Gamma^a_\nu(q,p;\mu)$$

Quark propagator:

$$S^{-1}(p,\mu) = i\,\gamma\cdot p\; A(p^2,\mu^2) + B(p^2,\mu^2) = \frac{i\,\gamma\cdot p + M(p^2)}{Z(p^2,\mu^2)}$$

$$S(p,\mu) = -i\,\gamma\cdot p\;\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)$$

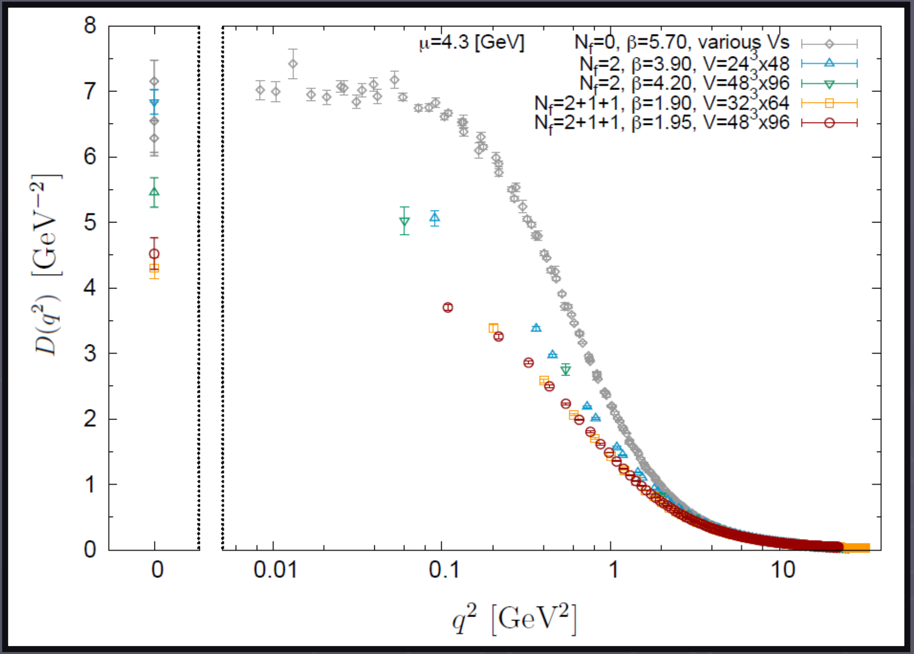
- The gluon propagator and the quark-gluon vertex are directly responsible for the quark gap equation to have infrared enhanced kernel.

The Quark Propagator

The Gluon Propagator:

$$\Delta^{ab}_{\mu\nu}(q) \;=\; \delta^{ab}\, D(q^2)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$

Several SDE and lattice results support decoupling solution for the gluon propagator.

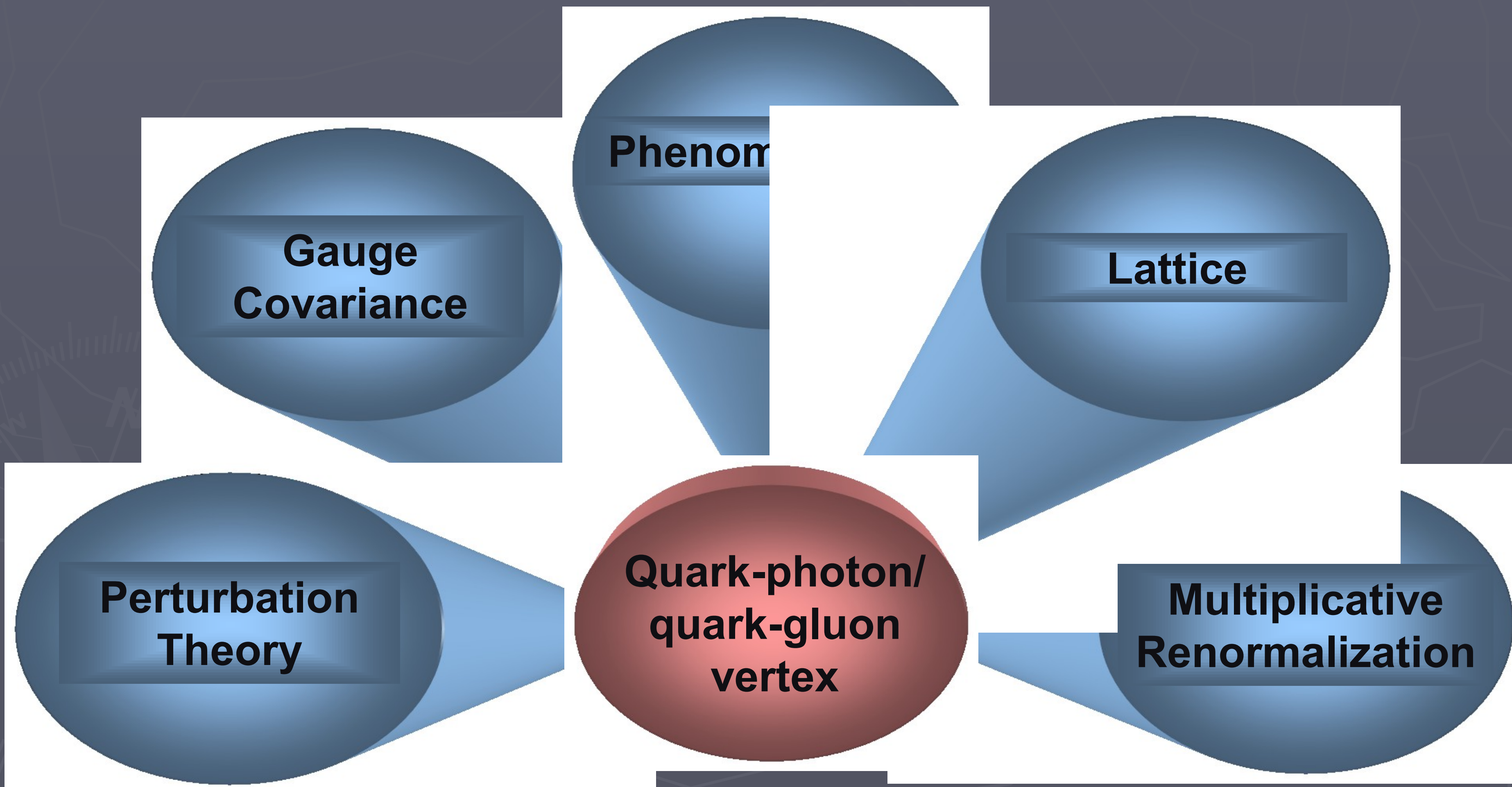


Momentum dependent gluon mass is reminiscent of the momentum dependent quark mass function.
It is also in accord with the revised GZ-picture.

$$D^{\mathrm{RGZ}}(q^2) \;=\; \frac{q^2 + M^2}{q^4 + q^2(m^2 + M^2) + 2g^2 N_c \gamma^2 + M^2 m^2}$$

The Quark Propagator

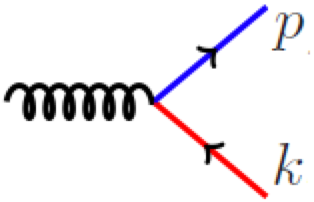
The Quark-Gluon Vertex:



The Quark Propagator

The Quark-Gluon Vertex:

- The quark-gluon vertex can be expanded in terms of the the following quantities.



$$\gamma_\mu \qquad k_\mu \qquad p_\mu$$

$$1 \qquad \gamma\cdot k \qquad \gamma\cdot p \qquad \gamma\cdot k\,\gamma\cdot p$$

- Quark gluon vertex consists of 12 linearly independent Dirac structures.
- 5 of these 12 structures are generated dynamically in the chiral limit.
- Thus DCSB manifests itself not only in the quark propagator but also the quark-gluon vertex.

The Quark-Gluon Vertex

The Quark-Gluon Vertex:

- The rainbow ladder truncation and the Ball-Chiu vertices describe π and the ρ mesons well but fail to do so for their parity partners σ and a_1.

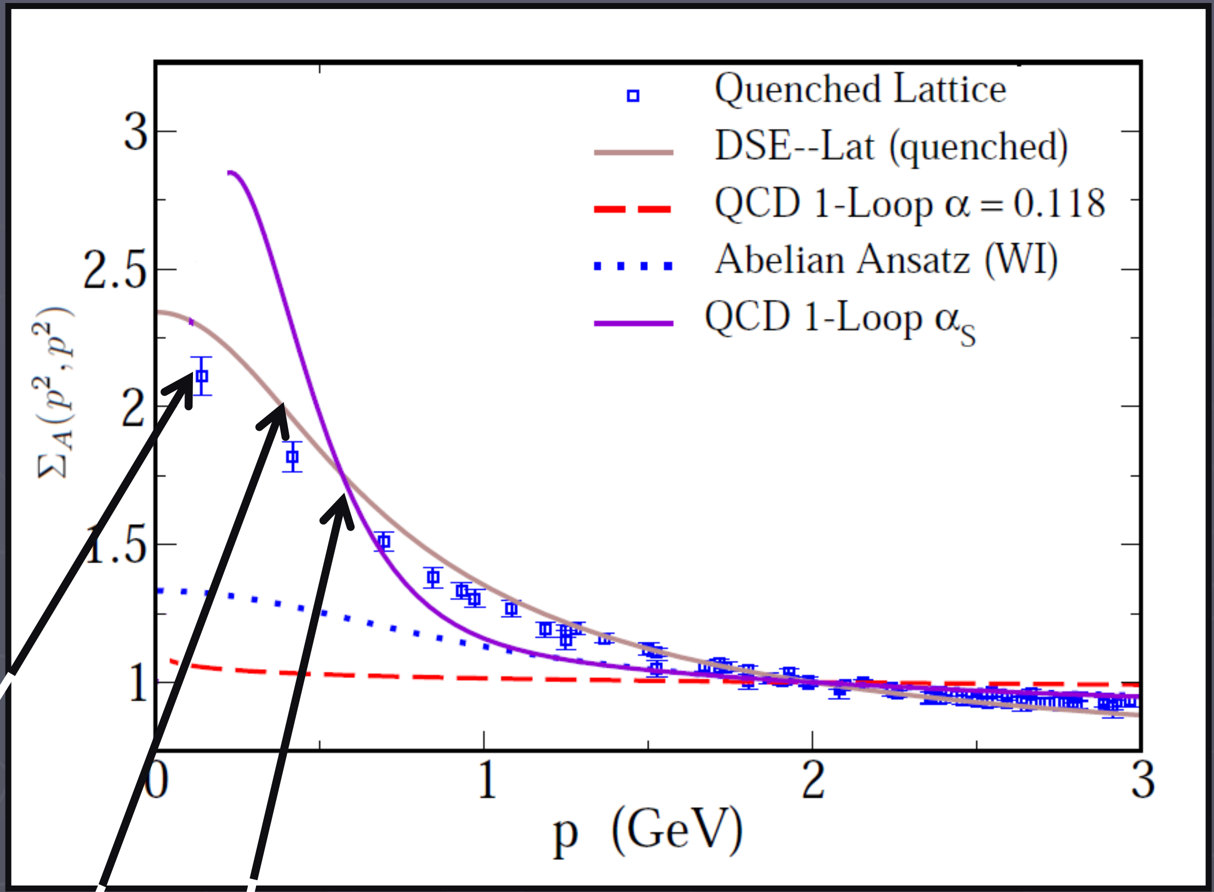
MeV	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1020	1280
ρ	770	644	764	800	840
Mass splitting	455	115	121	220	440

- The non perturbative details of the vertex explain the mass splitting of the parity partner mesons through the generation of quark anomalous chromo-magnetic moment.

L. Chang, C.D. Roberts, Phys. Rev. Lett. 103 081601 (2009)

- The corrections cancel in the pseudoscalar and vector channels but add in the scalar and axial vector channels.

The Quark Propagator



The Quark-Gluon Vertex:

One of the 12 form factors

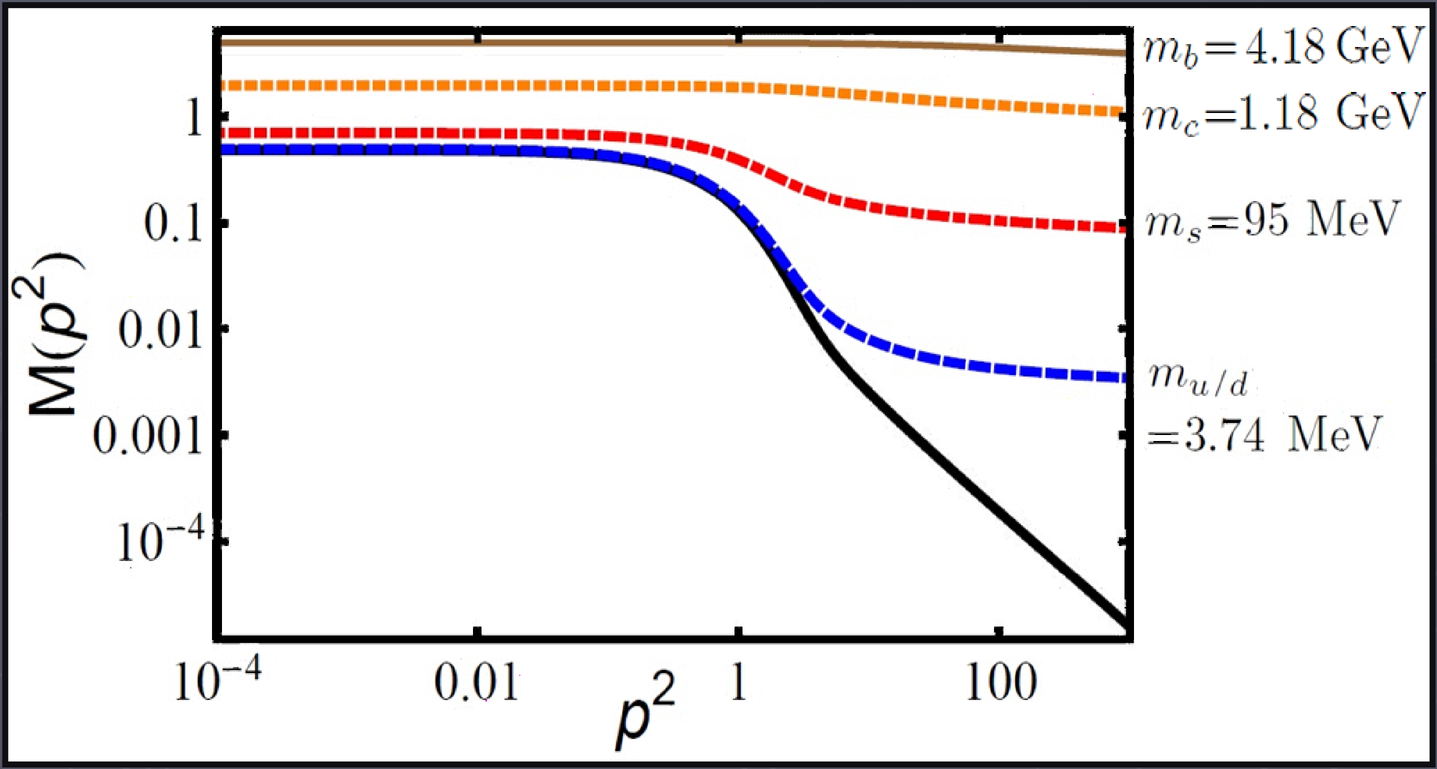
J. Skullerud, P. Bowman, A. Kizilersu, D. Leinweber, A. Williams, J. High Energy Phys. 04 047 (2003)

M. Bhagwat, M. Pichowsky, C. Roberts, P. Tandy, Phys. Rev. C68 015203 (2003).

AB, L. Gutiérrez, M. Tejeda, AIP Conf. Proc. 1026 262 (2008).

The Quark Propagator

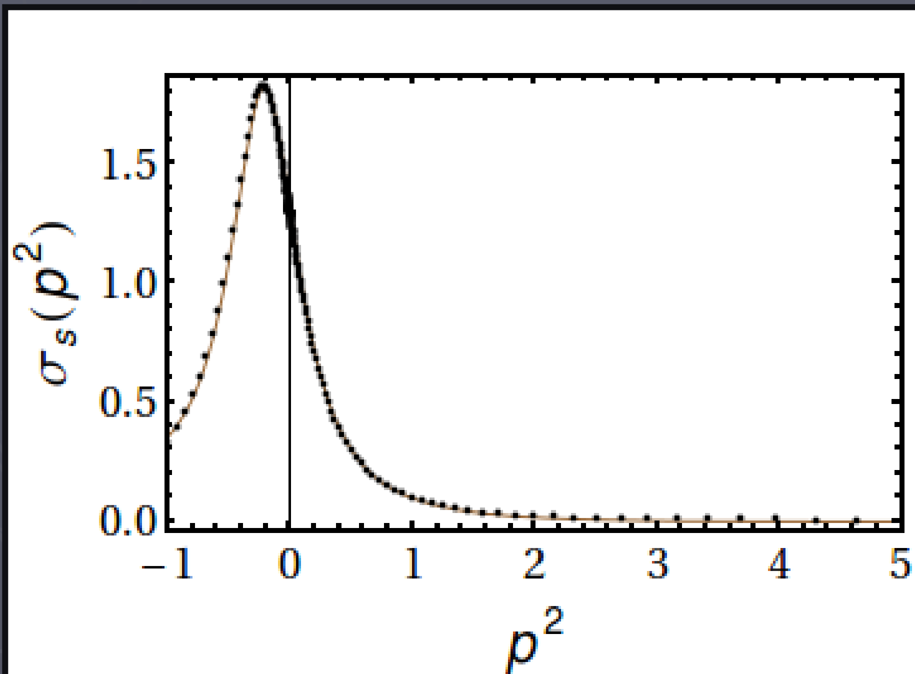
$$S(p,\mu) = -i\,\gamma\cdot p\,\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)$$

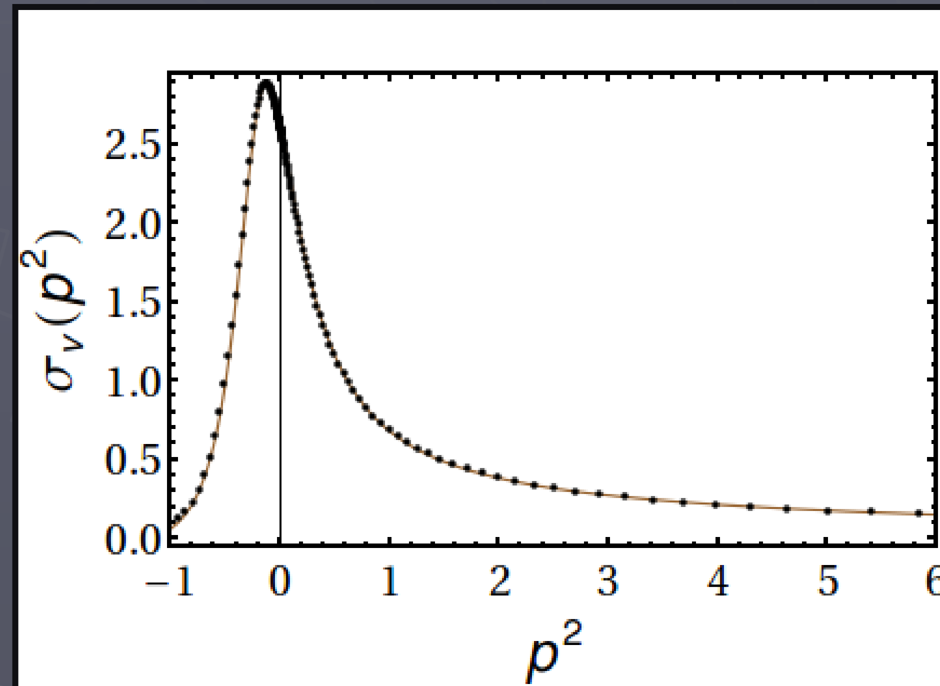


$$S^{-1}(p;\mu) = Z^{-1}(p^2;\mu^2)(i\gamma\cdot p + M(p^2))$$

The Quark Propagator

$$\boxed{S(p,\mu) = -i\,\gamma\cdot p\,\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)}$$





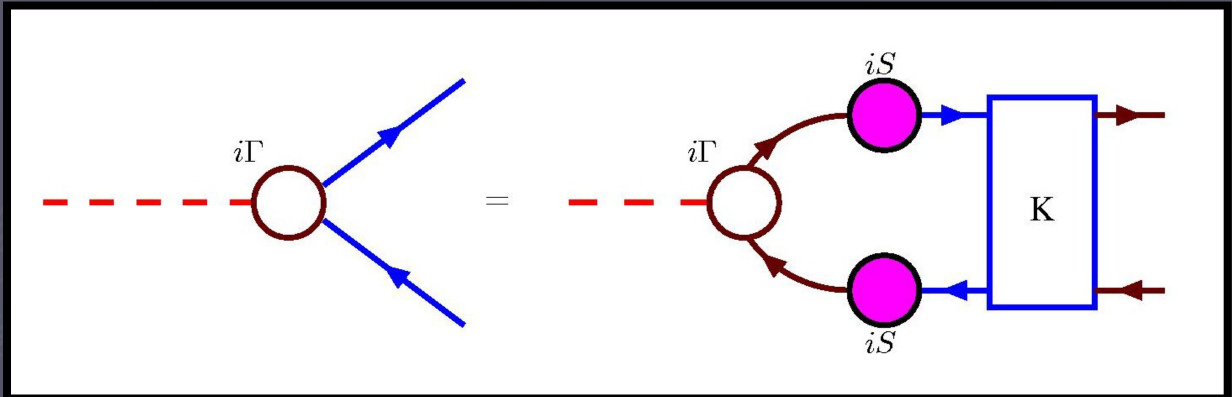
Complex conjugate pole representation.

All quantities are in MeV			
m=4.8	E amplitude	E,F,G amplitudes	Experiment
m_π	133	138	138.5
f_π	88	92	92

The Bethe-Salpeter Amplitudes

Bethe-Salpeter amplitude for the pion:

$$\Gamma_\pi(k,P) = \gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P G_\pi(k;P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k;P) \right]$$

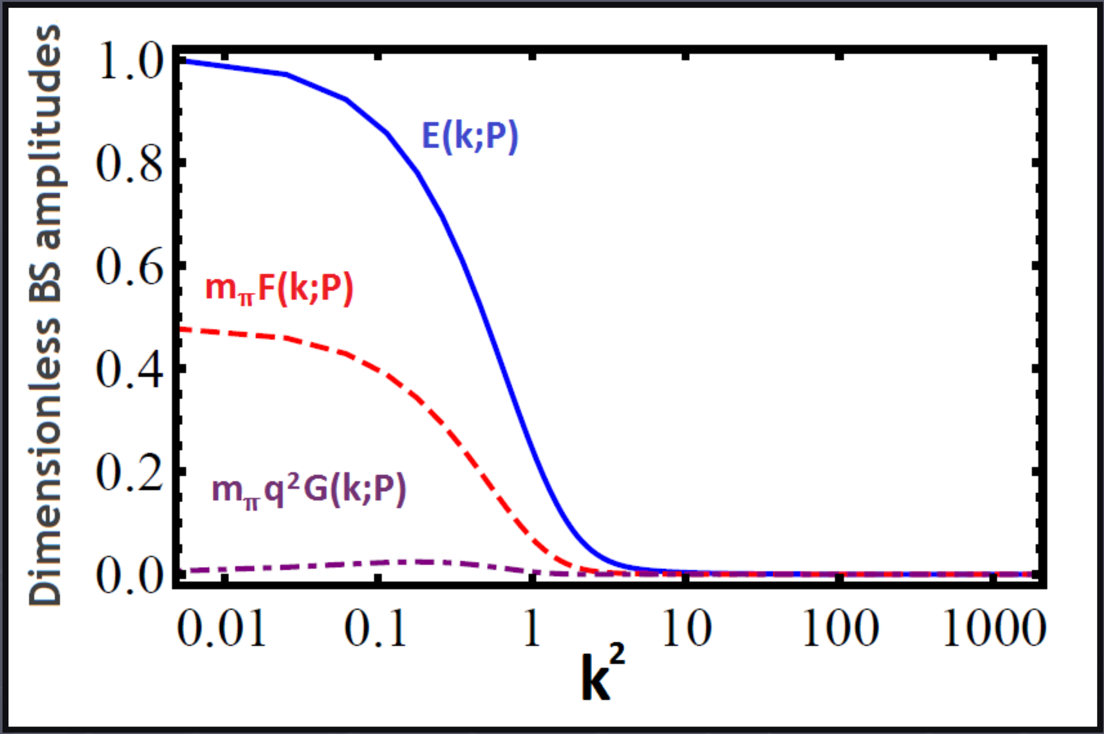


$$S(p) = \frac{1}{i\gamma\cdot p A(p^2) + B(p^2)}$$

Goldberger-Triemann relations:

$$\begin{aligned}
f_\pi\ E_\pi(k;P=0) &= B(k^2) \\
F_R(k;0) + 2f_\pi\ F_\pi(k;0) &= A(k^2) \\
G_R(k;0) + 2f_\pi\ G_\pi(k;0) &= 2A'(k^2) \\
H_R(k;0) + 2f_\pi\ H_\pi(k;0) &= 0
\end{aligned}$$

The Bethe-Salpeter Amplitudes



L. Chang et. al., Phys. Rev. Lett. 110 132001 (2013).

$$a(\Lambda, z) = \rho(z)\delta(\Lambda)$$

$$\rho(z) = (1 - z^2)^\nu$$

Nakanishi, Phys. Rev. 130 1230 (1963).

Parametric form of the BSAs.

$$A(k;P) = \int_{-1}^{+1} dz \int_0^\infty d\Lambda \frac{a(\Lambda, z)}{(k^2 + zk\cdot P + P^2 + \Lambda^2)^n}$$

$$A(k;P) = \int_{-1}^{+1} dz \left[\frac{\rho^{\mathrm{IR}}(z)}{(k^2 + zk\cdot P + P^2 + \Lambda_{\mathrm{IR}}^2)^{m+n}} + \frac{\rho^{\mathrm{UV}}(z)}{(k^2 + zk\cdot P + P^2 + \Lambda_{\mathrm{UV}}^2)^n}\right]$$

The Quark-Photon Vertex

To conserve current conservation and Ward identities, a proper quark-photon vertex is essential.

D.C. Curtis and M.R. Pennington Phys. Rev. D42 4165 (1990)

AB, M.R. Pennington Phys. Rev. D50 7679 (1994)

A. Kizilersu and M.R. Pennington Phys. Rev. D79 125020 (2009)

L. Chang, C.D. Roberts, Phys. Rev. Lett. 103 081601 (2009)

AB, R. Bermudez, L. Chang, C.D. Roberts, Phys. Rev. C85 045205 (2012).

It must ensure chiral anomaly at zero momentum transfer.

The quark-photon vertex must also take into account the separation of scales involved. (Gauge Technique).

R. Delbourgo, A. Salam, Phys. Rev. 135 1398 (1964).

The Quark-Photon Vertex

The vertex used satisfies the longitudinal Ward-Green-Takahashi (WGT) identity.

It is free of kinematic singularities.

It reduces to the bare vertex in the free-field limit.

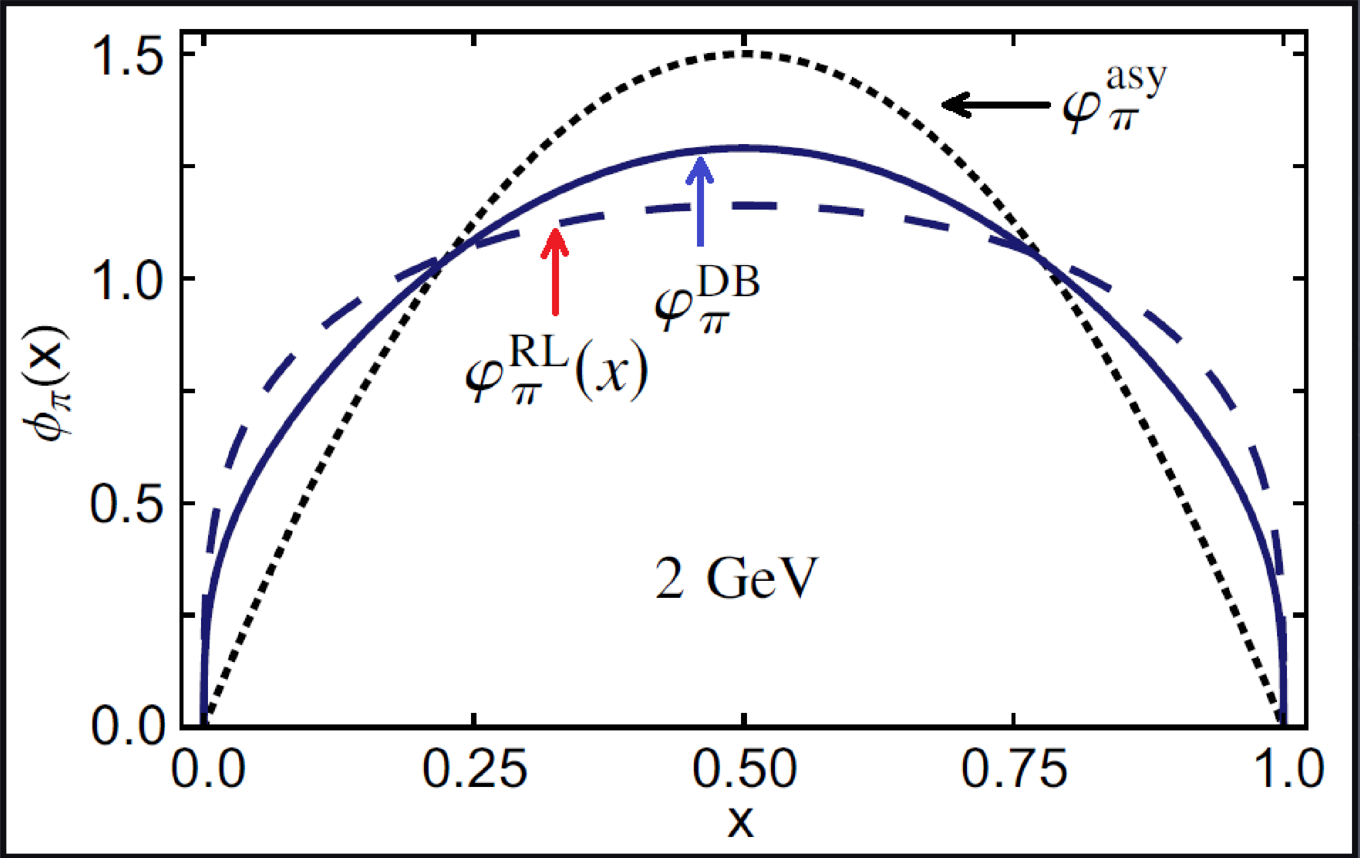
It has the same Poincare transformation properties as the bare vertex.

This *Ansatz* has the additional merit that it reproduces the correct ultraviolet limit of the photon-quark interaction when one or both photons is hard.

Pion Distribution Amplitude

$$\varphi_\pi^{\rm DB}(x) = 1.81[x(1-x)]^{\alpha_-^{\rm DB}}[1 + a_2^{\rm DB} C_2^{\alpha_{\rm DB}}(2x-1)]$$

$$\alpha_{\rm DB} = 0.81,\ a_2^{\rm DB} = -0.12$$



Pion Distribution Amplitude

This can be projected onto the Gegenbauer 3/2 basis.

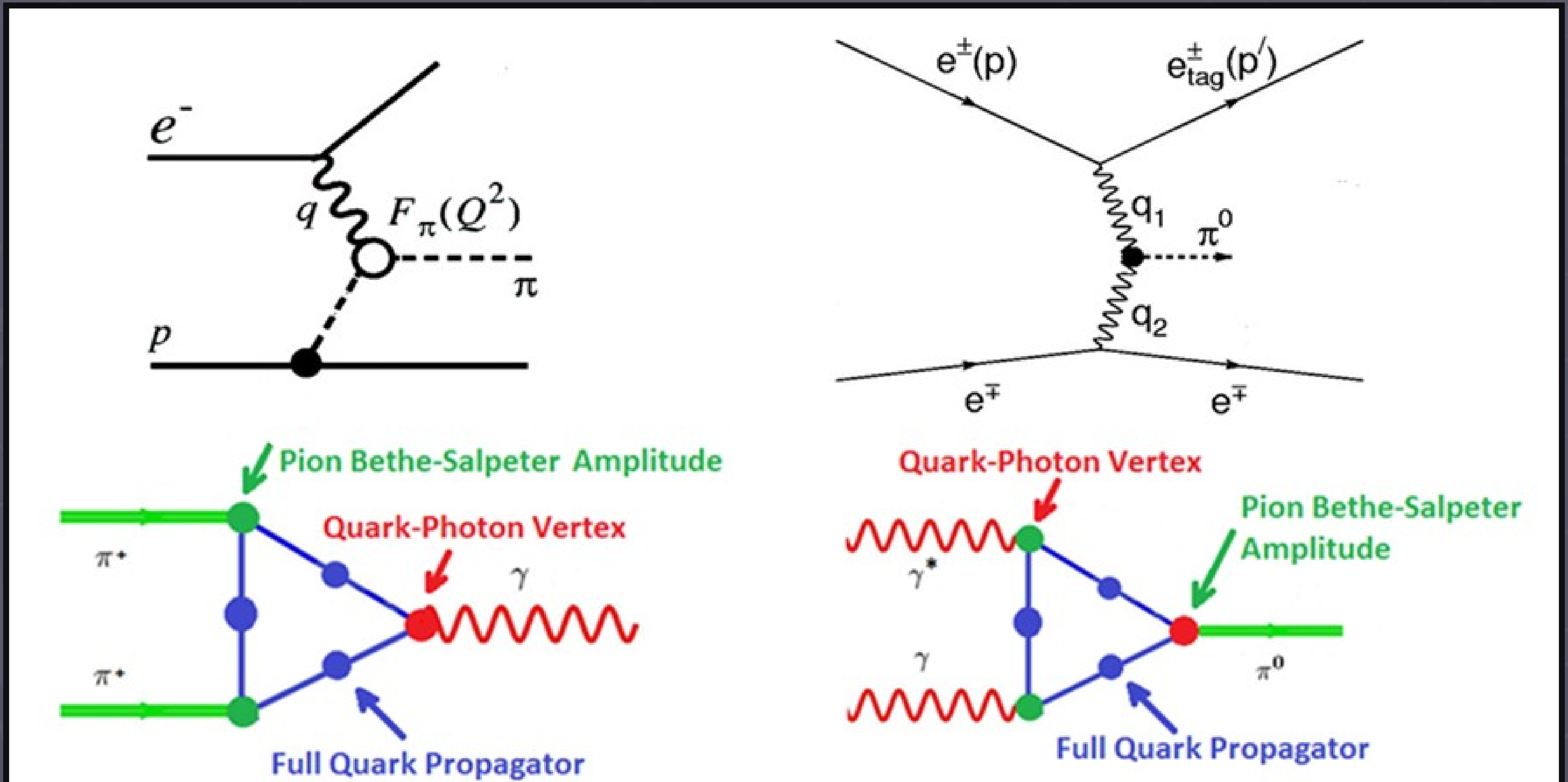
$$\varphi_\pi(x,\mu) = x(1-x)\left[1 + \sum_{j=2,4,\cdots} a_j^{3/2}(\mu) C_j^{3/2}(2x-1)\right]$$

One now evolves it to different scales and builds this evolution into the BSA amplitudes.

$$f_\pi \varphi_\pi(x) = \mathrm{tr}_{\mathrm{CD}} Z_2 \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x n\cdot P)\gamma_5 \gamma\cdot n \chi_\pi(q;P)$$

Pion Form Factors

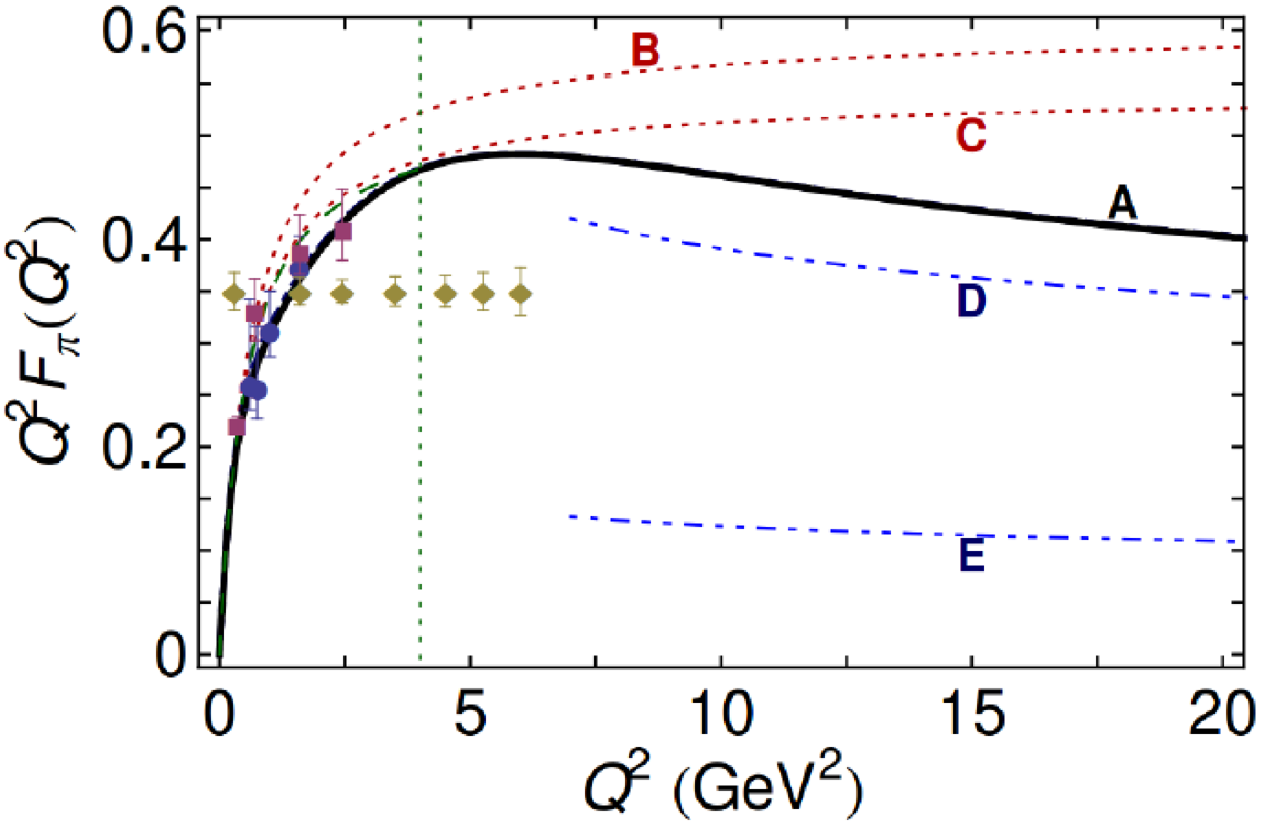
The quark propagator (NCCP), electron-photon vertex (WTI-PT), the Bethe Salpeter Amplitude (Nakanishi) and PDA evolution provide the ingredients for the pion form factor calculations.



Pion Electromagnetic Form Factor

Pion to 2 photons Transition Form Factor

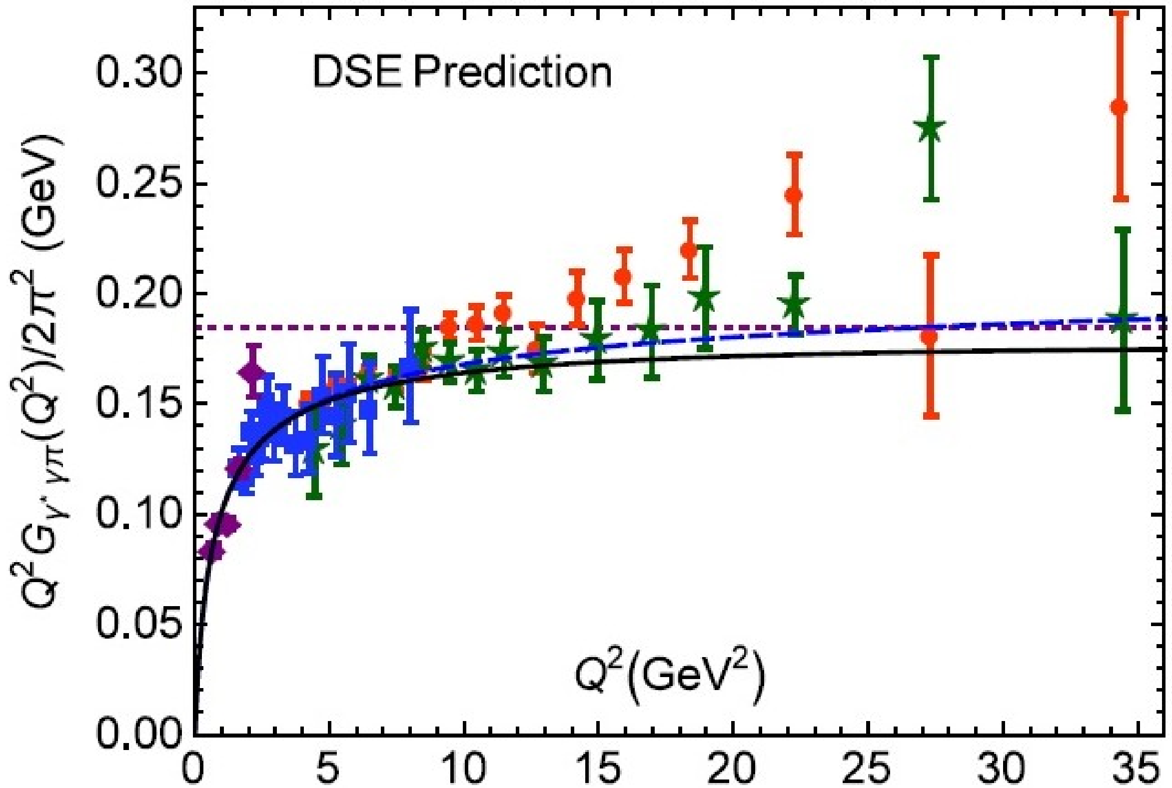
Pion Electromagnetic Form Factor



[A] SDE calculation. [B-C] Monopole fits.

[D] Finite μ limit. [E] Asymptotic limit.

$\gamma^*\pi^0\gamma$ Transition Form Factor

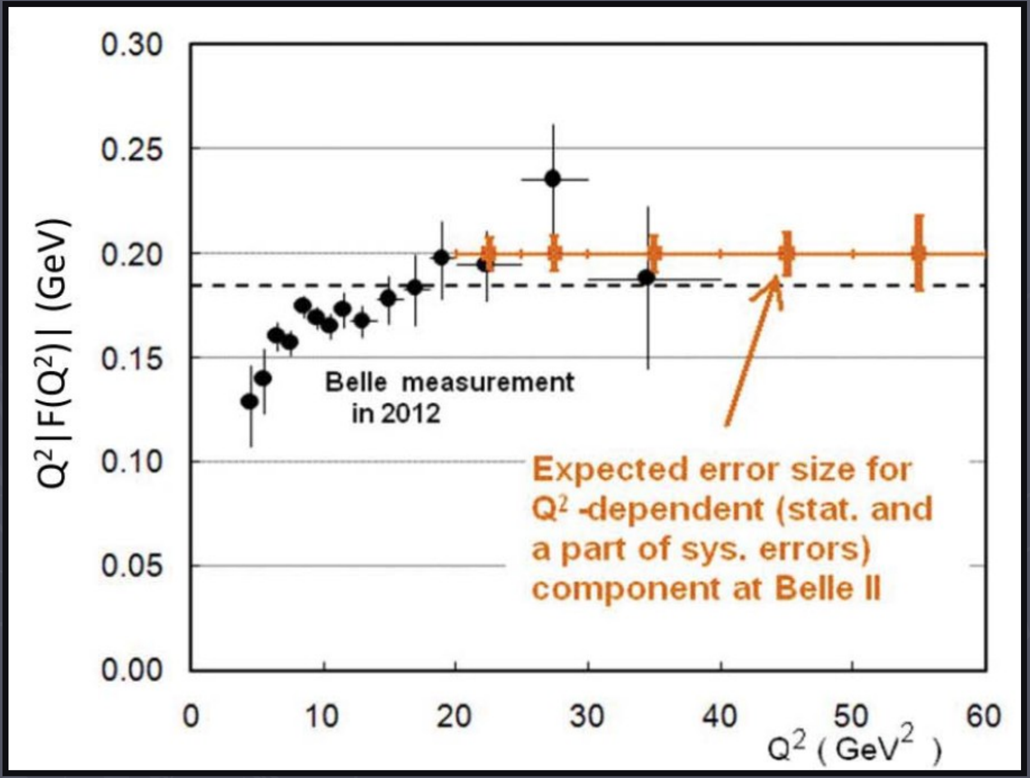


solid (black) – propagators, amplitudes and vertices, and ERBL evolution of the pion Bethe-Salpeter amplitude. long-dashed (blue) – no evolution. dotted (purple) – asymptotic limit. Data: BaBar – circles (red); Belle – stars (green)

$\gamma^*\pi^0\gamma$ Transition Form Factor

The $\boxed{\gamma^*\gamma \to \pi^0}$ transition form factor:

- Belle II will have 40 times more luminosity.



Vladimir Savinov:
5th Workshop of the APS Topical Group on Hadronic Physics, 2013.

Precise measurements at large Q^2 will provide a stringent constraint on the pattern of chiral symmetry breaking.

Conclusions

Dynamical chiral symmetry breaking and the momentum dependence of the quark mass function in QCD have experimental signals which enable us to differentiate SDE predictions from others.

In a fully consistent treatment of pion in SDE QCD, the asymptotic QCD limit of the product $Q^2G(Q^2)$ for $\gamma^*\pi^0\gamma$, seems to approach the asymptotic QCD results from below

The large Q^2 evolution of the form factors through SDEs provides us with a unified description of the physics from purely non perturbative to asymptotic region and should be systematically extended to all other form factors of interest such as N-> N*(1535).