Quantum measurement and control in optomechanical systems
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Exercises of the third lecture

1. **Markov master equation.**

   Show that the rate of change of the average photon number in the cavity is given by

$$\frac{d\langle \hat{a}^\dagger \hat{a}\rangle}{dt} = -\gamma\langle \hat{a}^\dagger \hat{a}\rangle + \gamma\bar{n}.$$

2. **Phonon number measurements.**

   For the moments of the *unconditional state*, show that

$$\begin{aligned}\frac{d\langle a\rangle}{dt} &= -i\chi\bar{n}_b - \frac{\kappa}{2}\langle a\rangle \\ \frac{d\bar{n}_b}{dt} &= -\gamma(\bar{n}_b - \bar{N})\end{aligned}$$

   where $\bar{n}_b = \langle b^\dagger b\rangle$.

3. **Phonon number measurements.**

   For the moments of the *unconditional state*, show that the solutions are

$$\begin{aligned}\bar{n}_b(t) &= \bar{n}_b(0)e^{-\gamma t} + \bar{N}(1-e^{-\gamma t}) \\ \langle a\rangle &= \langle a\rangle(0)e^{-\kappa t/2} - i\chi\left[(\bar{n}_b(0)-\bar{N})\frac{(e^{-\gamma t}-e^{-\kappa t/2})}{\kappa/2-\gamma} + \bar{N}\frac{(1-e^{-\kappa t/2})}{\kappa}\right]\end{aligned}$$

   Show that the steady state field is $\langle a\rangle_{ss} = -2i\chi\bar{N}/\kappa$.

4. **Example of conditional measurement**

   Write

$$\rho(t) = \frac{1}{2}(1 + x(t)\sigma_x + y(t)\sigma_y + z(t)\sigma_z)$$

   Show that the conditional master equation is equivalent to

$$dz(t) = 2\sqrt{\Gamma}\left(1 - z^2(t)\right)dW(t)$$

   which has two fixed points at $z = \pm 1$, the eigenstates of σ_z.