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Particle-Spin Coupling in a Chaotic System: Localization-Delocalization in the Husimi Distributions.

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Abstract. – The wave functions of the Dicke Hamiltonian, describing a spin coupled to a bosonic mode, are studied via Husimi distributions. A classical analogue of this system is also obtained. For several energy ranges studied, the Husimi distribution of the wave functions show the scar of simple periodic orbits when projected into the boson phase space. Surprisingly, these same distributions, when projected into the spin phase space, are spread through large regions. An explanation of this fact is given in terms of semiclassical theory and border effects associated with nonsemiclassical behaviour.

In Hamiltonian systems describing the motion of particles, it is known that in the semiclassical limit groups of wave functions tend to concentrate along the classical periodic orbits. These results were obtained analytically [1-3] and tested numerically for particle systems [4]. Phase space for particle systems are generally infinite and some very important theoretical results so far obtained strongly hinge upon this fact. Hamiltonians with compact phase spaces were recently studied [5]. However, these systems have only a finite number of eigenstates which restricts, among other things, statistical analysis. When we consider a nonintegrable Hamiltonian where a spin is coupled to a particle, we obtain a system which allows for an infinite number of states together with a phase space which is finite in the spin degree of freedom.

The Hamiltonian we are going to study in this letter is well known from quantum optics as the Dicke Hamiltonian [6], and reads

$$H = \varepsilon(a^\dagger a + J_z) + \frac{G}{\sqrt{N}}(J_+ a + J_- a^\dagger) + \frac{G'}{\sqrt{N}}(J_- a + J_+ a^\dagger), \tag{1}$$

($) Supported in part by CNPq and FINEP.
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where $a^\dagger$, a are the Bose operators of a harmonic-oscillator mode with frequency ε, J_z and $J_+(J_-)$ are the usual z-component and raising (lowering) spin operators. Throughout this paper we shall use $\varepsilon = 1$.

The fluctuation properties of the level distributions for the case $J = 9/2$ were extensively studied [7]. In particular, for the case $G' = 0$, it has been shown that the system already behaves semiclassically in the sense that it approaches the mean-field approximation (which is exact for $J \to \infty$) [8].

To this quantum Hamiltonian we have associated a classical one via coherent states:

$$H_{cl}(Q_1, P_1, Q_2, P_2) = \langle zw|H|zw\rangle, \tag{2}$$

where $|zw\rangle$ is the direct product of the oscillator and spin normalized coherent states:

$$|zw\rangle = \frac{\exp[-z\bar{z}/2]}{(1+w\bar{w})^J}\exp[z\hat{a}^\dagger + wJ_+]|0, -J\rangle, \qquad w = \frac{1}{\sqrt{2}}(P_1 + iQ_1) \tag{3}$$

and

$$z = \frac{1}{\sqrt{2}}(P_2 + iQ_2).$$

The classical Hamiltonian (eq. (2)) can be rewritten as

$$H_{cl}(q_1, p_1, q_2, p_2) =$$

$$= \frac{1}{2}(p_1^2 + q_1^2) + \frac{1}{2}(p_2^2 + q_2^2) - J + \frac{\sqrt{2J - (1/2)(p_1^2 + q_1^2)}}{\sqrt{2J}}(G_+ p_1 p_2 + G_- q_1 q_2), \tag{4}$$

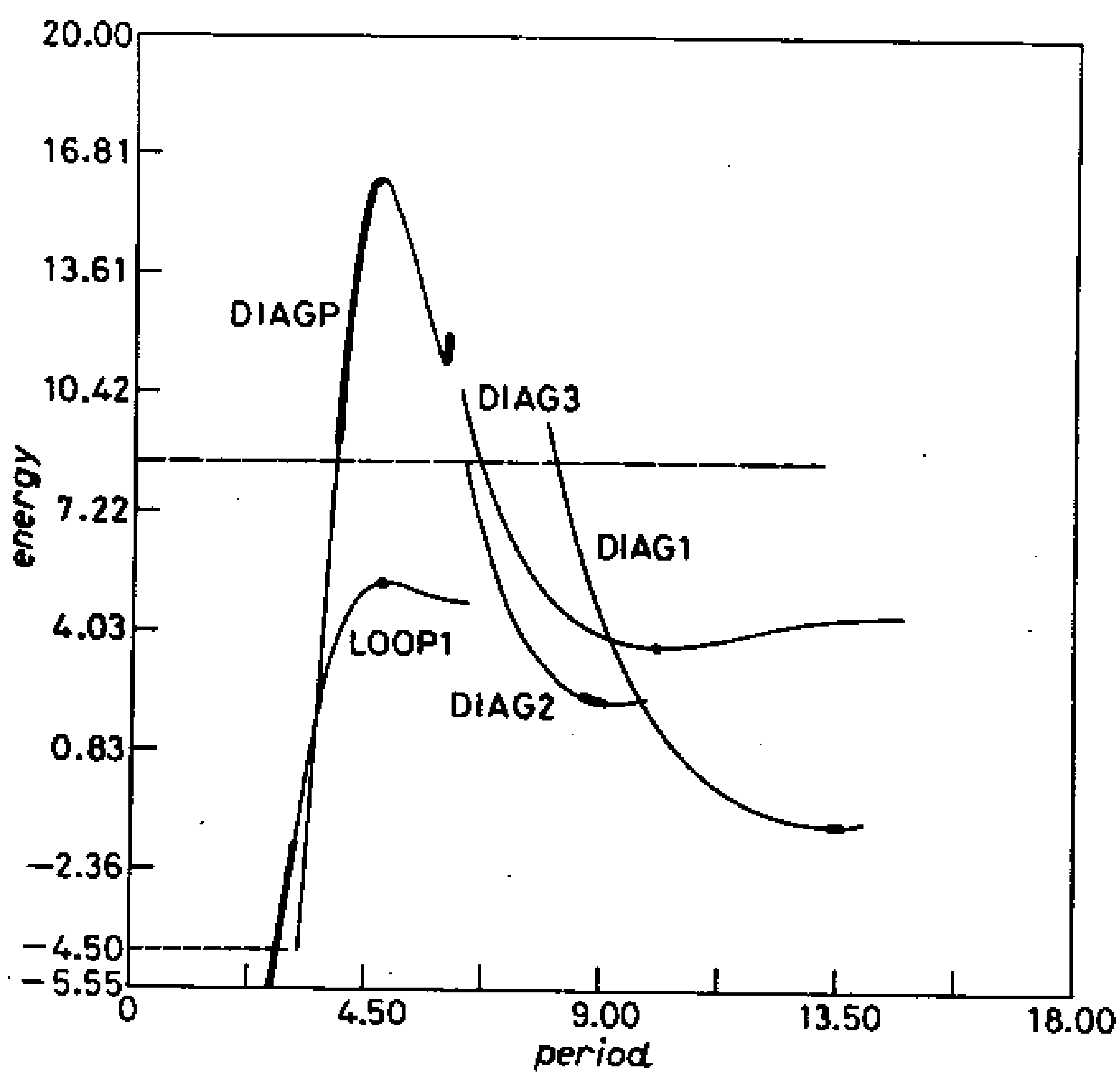

Fig. 1. – Energy *vs.* period for the three simplest families of periodic orbits. Thick (thin) lines correspond to stable (unstable) orbits.

where

$$G_\pm = G \pm G', \quad p_1 = P_1\sqrt{\frac{4J}{2+P_1^2+Q_1^2}}, \quad q_1 = Q_1\sqrt{\frac{4J}{2+P_1^2+Q_1^2}}, \quad p_2 = P_2, \quad q_2 = Q_2.$$

The phase space of H_{cl} is limited by a circle of radius $\sqrt{4J}$ in the (p_1, q_1) variables (which we shall call «the border»), plus a regular infinite space in (p_2, q_2).

The basic ingredient for studying the semiclassical limit is the classical periodic orbits. We have calculated the simplest periodic orbits of eq. (4) using an extension of the method developed by Baranger *et al.* [9, 10] and we found three main families as shown in fig. 1 in an energy x period plot. We have also calculated Poincaré sections, at several energies and found that a considerable amount of chaos is present [10]. Notice that as the border is approached the motion becomes chaotic. Also, the equations of motion become divergent in this limit:

$$\left\{\begin{aligned}
\dot{q}_1 &= -p_1 - \frac{G_+ p_2\sqrt{2J-H_1}}{\sqrt{2J}} + \frac{P_1}{2\sqrt{2J}\sqrt{2J-H_1}}(G_+p_1p_2 + G_-q_1q_2),\\
\dot{p}_1 &= q_1 + \frac{G_- q_2\sqrt{2J-H_1}}{\sqrt{2J}} - \frac{q_1}{2\sqrt{2J}\sqrt{2J-H_1}}(G_+p_1p_2 + G_-q_1q_2),\\
\dot{q}_2 &= -p_2 - \frac{G_+ p_1\sqrt{2J-H_1}}{\sqrt{2J}},\\
\dot{p}_2 &= +q_2 + \frac{G_- \overset{q_1}{\cancel{p_1}}\sqrt{2J-H_1}}{\sqrt{2J}},
\end{aligned}\right. \tag{5}$$

where $H_1 = (p_1^2 + q_1^2)/2$.

In order to compare classical *vs.* quantum features, we have calculated the Husimi distribution for several eigenfunctions:

$$h(z, w) = |\langle zw|\psi_l\rangle|^2, \tag{6}$$

where $|\psi_l\rangle$ is an exact eigenfunction of Hamiltonian (1). The Hamiltonian (1) has been diagonalized in the basis $|nm\rangle = |n\rangle \otimes |m\rangle$, where $|n\rangle$ are the eigenstates of $a^\dagger a$ and $|m\rangle$ the eigenfunctions of J^2 and J_z:

$$|\psi_l\rangle = \sum_{n=0}^{\infty}\sum_{m=-J}^{J} C_{nm}^l |nm\rangle. \tag{7}$$

The behaviour of the particle and spin degrees of freedom are best visualized by the following projections:

$$h(z) = \int h(z, w)\, d[\mathrm{Re}(w)]\, d[\mathrm{Im}(w)]\, \frac{(2J+1)}{\pi(1+w\bar{w})^2} = \sum_{nn'm} C_{n'm}^{l*} C_{nm}^l \langle z|n\rangle\langle n'|z\rangle \tag{8}$$

and

$$h(w) = \int h(z, w)\frac{d[\mathrm{Re}(z)]\, d[\mathrm{Im}(z)]}{\pi} = \sum_{nmm'} C_{nm'}^{l*} C_{nm}^l \langle w|m\rangle\langle m'|w\rangle. \tag{9}$$

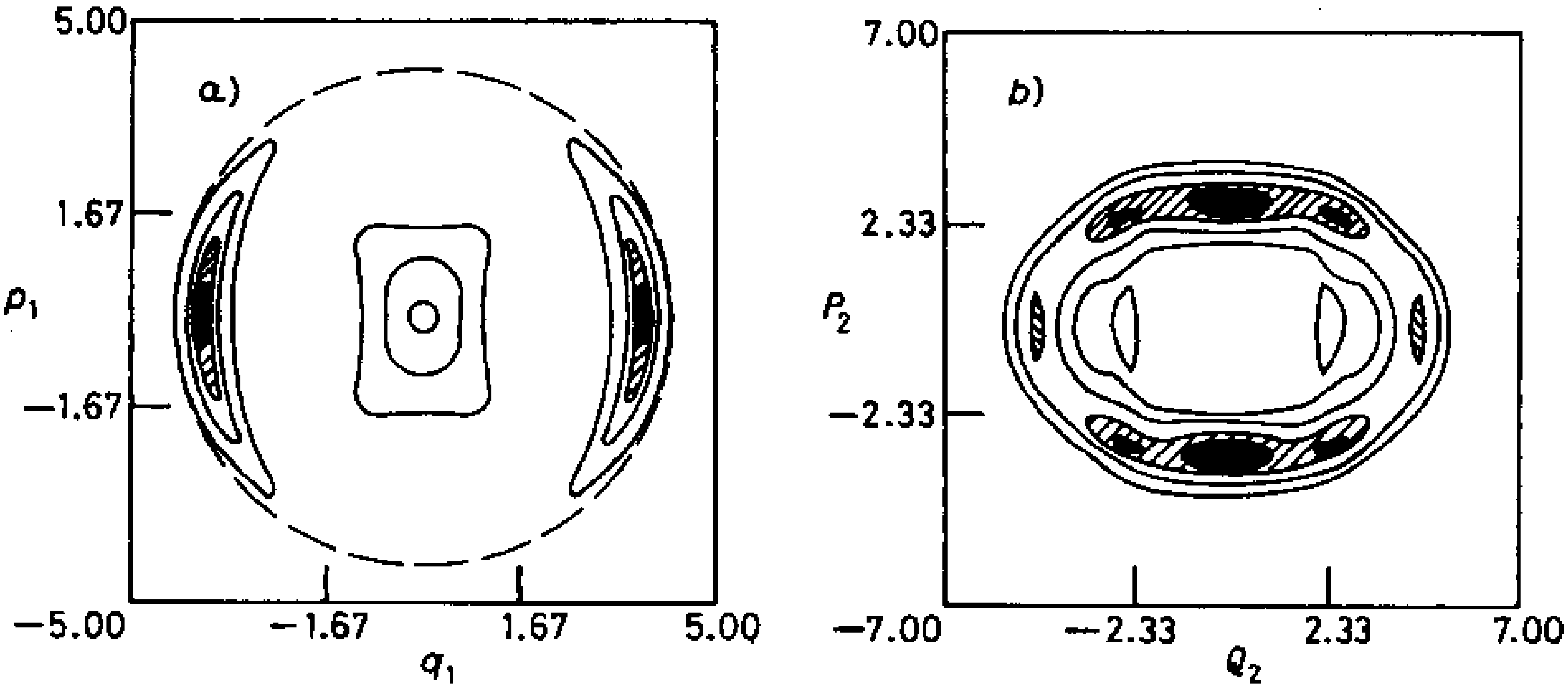

Fig. 2. – *a*) Contour plots of $h(w)$ for the eigenstate no. 55, energy $E = 8.19$. The blackened area represents the distribution at 95% of its maximum; the shaded area from 95% to 90%. Contours at 80% and 70% have been drawn. *b*) The same for $h(z)$.

We have selected three most representative wave functions. Further details of both classical and quantum calculations shown here will appear elsewhere [10].

Let us analyse what happens to the particle degree of freedom: from fig. 2, 3 and 4 (part *b*)) we notice that the particle's Husimi distribution concentrates along the classical orbits of smaller periods, DIAGP, DIAG2 and DIAG3 (see fig. 1 and 5). The behaviour of the oscillator's Husimi distribution is not new, it has been observed for several other particle systems and can be understood in the context of presently available semiclassical theories [3]. Surprisingly the same Husimi distribution when projected into the spin phase space are spread through large regions. The presence of scars here is by far not as conspicuous (see part *a*) of the same figures).

An explanation of this fact can be given in terms of semiclassical theory: consider the Poincaré section of a coherent-state wave packet as in eq. (3) centred on a periodic orbit. Then propagate this packet using classical dynamics and study its subsequent overlaps with initial packet. According to Heller's semiclassical arguments [2], the scar of this orbit will be found in the eigenstates if these overlaps are large for several iterations of the map. In fig. 6 we show the evolution of such a packet centred on the orbit DIAGP at $E = 8.6$ for both spin and oscillator degrees of freedom (actually only the 80% and 90% contour levels are shown—see captions). Notice from fig. 1 that at this energy, all calculated periodic orbits

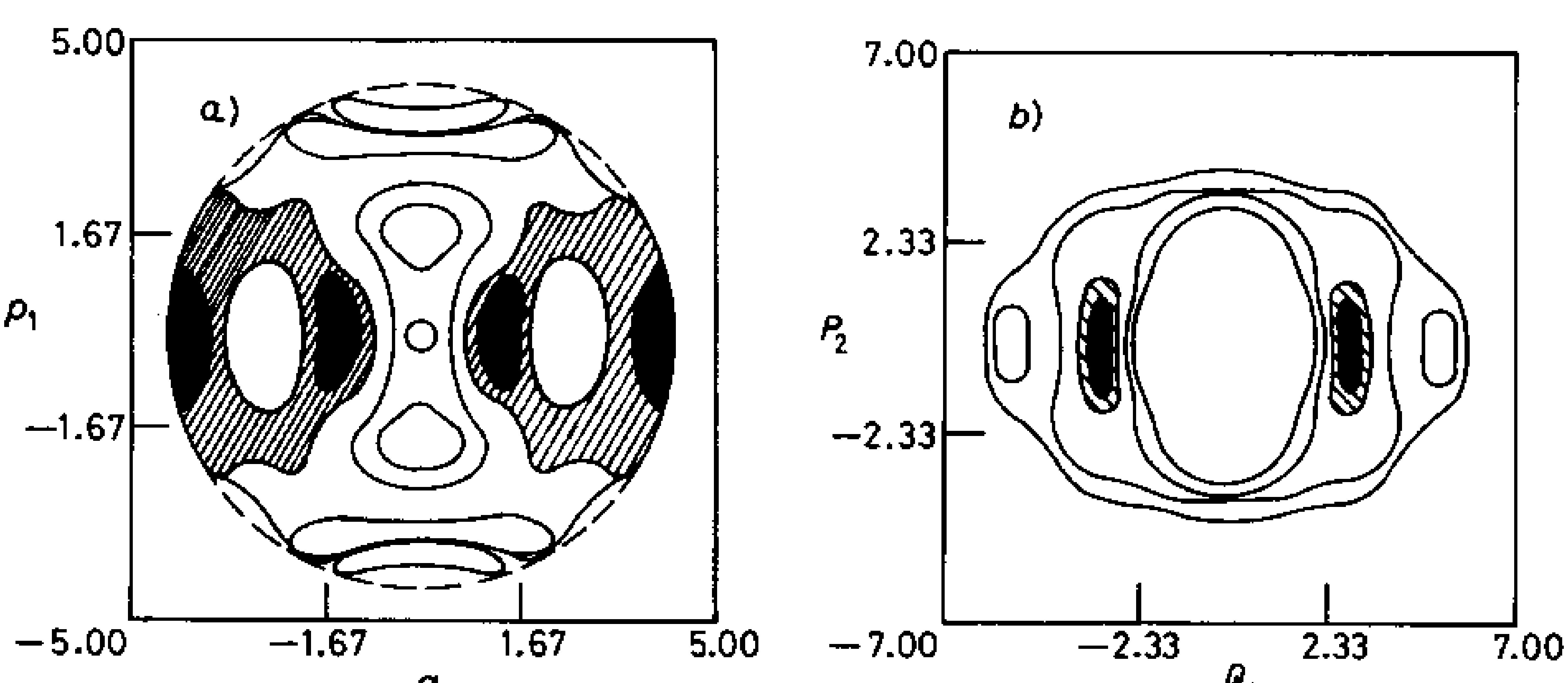

Fig. 3. – Idem for eigenstate no. 58, $E = 8.93$.

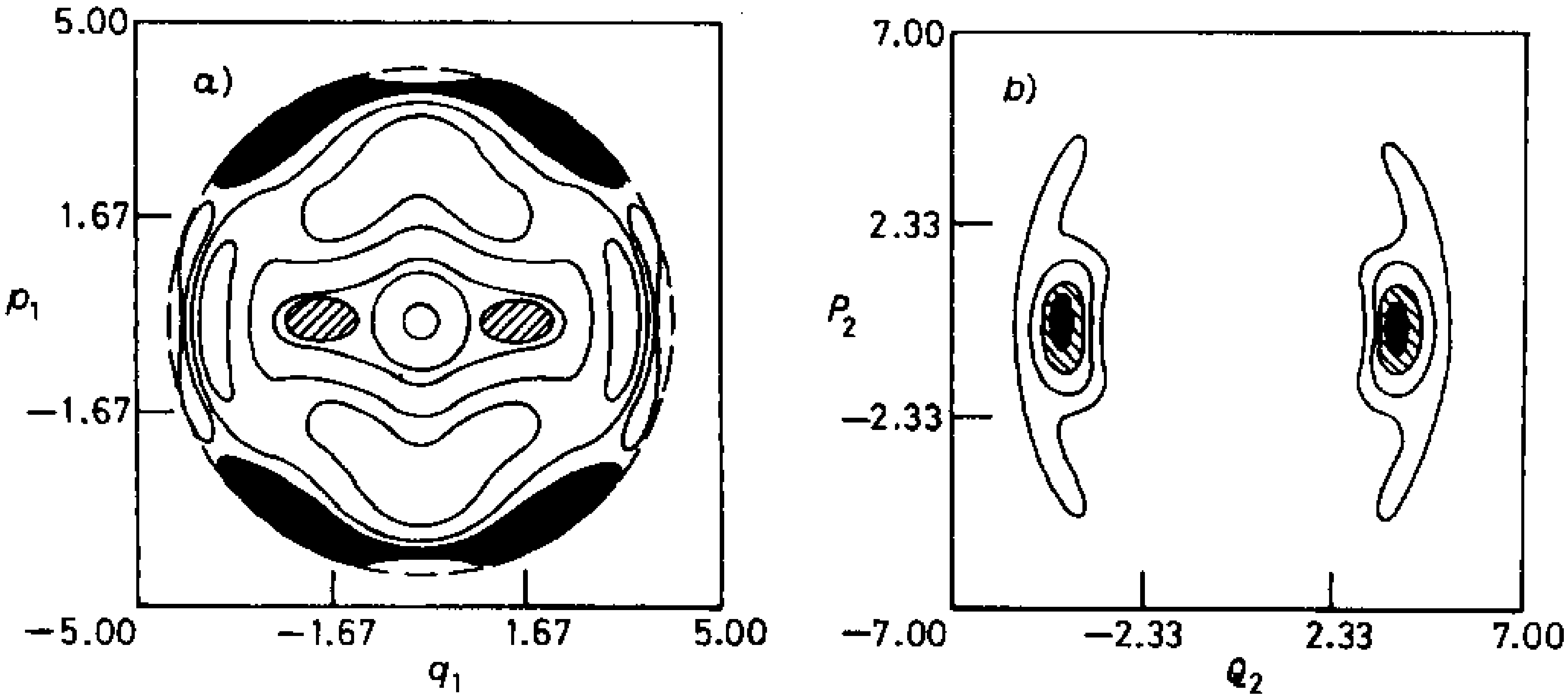

Fig. 4. – Idem for eigenstate no. 60, $E = 9.32$.

are unstable, DIAGP being the least unstable of them. From this figure it is readly seen that the wave packet stretches along the unstable manifolds after a single return, but still preserves its Gaussian shape. For other orbits, which have much larger Liapunov exponents, the wave packets get completely distorted after a single period and no considerable overlap with the initial packet results. According to this reasoning, the orbit DIAGP is the best candidate for scarring the eigenstates, and indeed, this is what happens

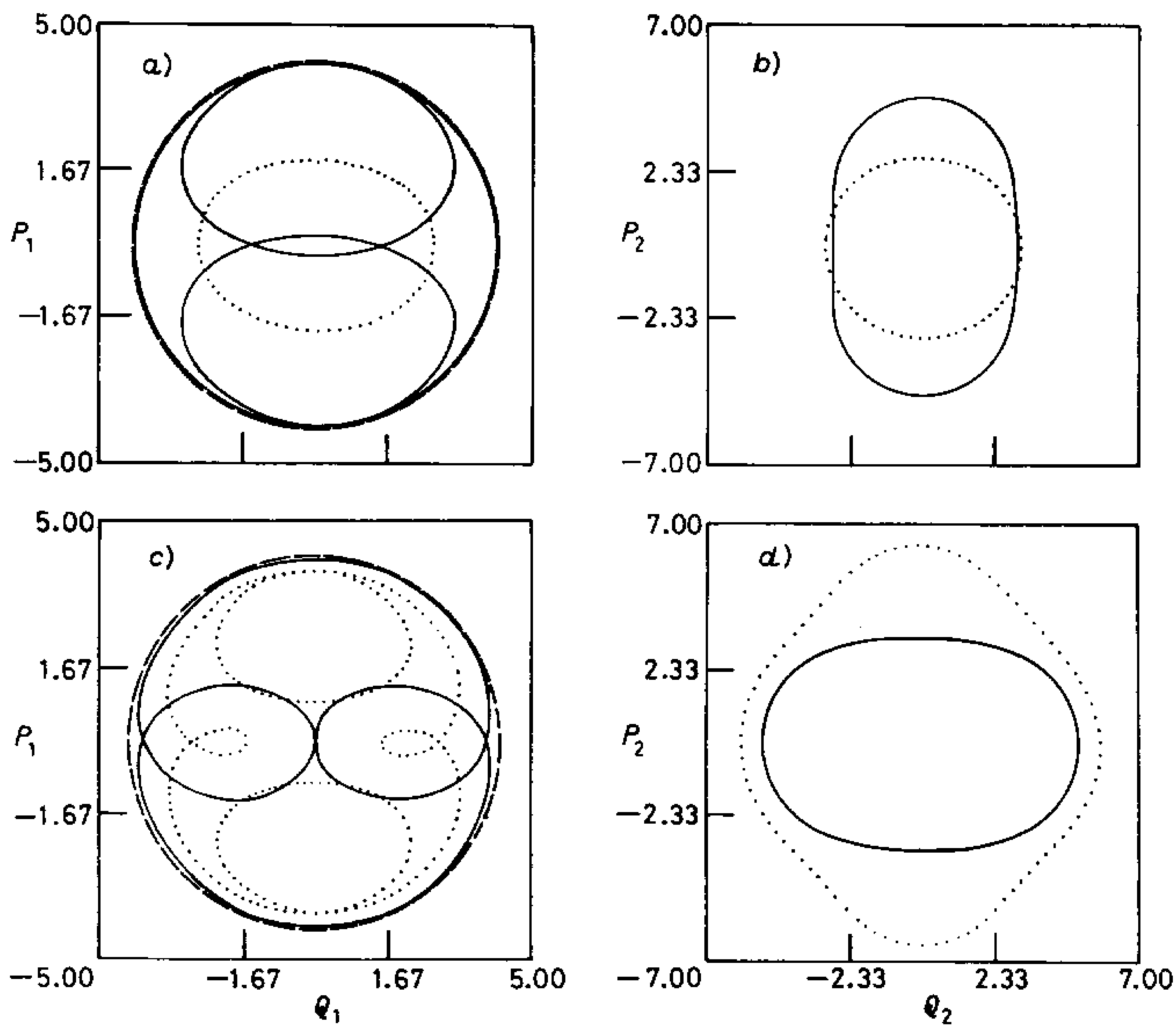

Fig. 5. – *a*) Projection of the periodic orbits DIAGP (dotted) and DIAG3 (continuous) onto the $q_1 \times p_1$ plane. The dashed circle represents the border. *b*) The same orbits projected onto the $q_2 \times p_2$ plane. *c*) Projection of DIAG1 (dotted) and DIAG2 (continuous) onto the $q_1 \times p_1$ plane. *d*) The same orbits of *c*) projected onto the $q_2 \times p_2$ plane. All orbits have energy about 8.5.

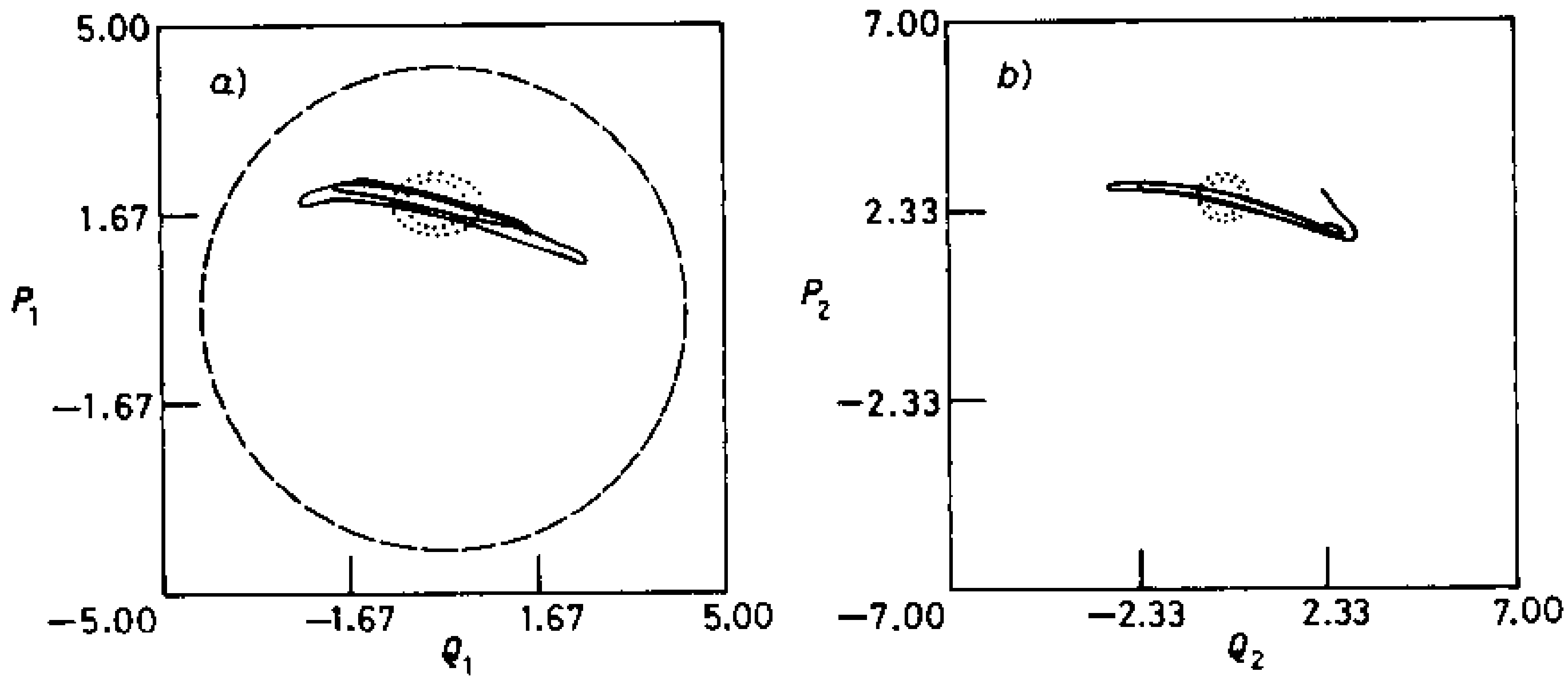

Fig. 6. – Poincaré sections of a classically evolved coherent-state wave packet centred on orbit DIAGP at $\tau = 4.34$ and $E = 8.6$. Only the 80% and 90% contours are shown. In part *a*), the section is at $q_2 = 0$, and in part *b*) at $q_1 = 0$. The dotted curves represent the initial packet, while the continuous curves show the evolved packet.

for the oscillator degree of freedom. However, no scars can be identified in the Husimi distributions for the spin degree of freedom. Therefore, we conclude that the Dicke model at $J = 9/2$ has the interesting property of exhibiting semiclassical behaviour in only one of its degrees of freedom (the oscillator). To the authors' knowledge, this peculiar behaviour was never observed before. The deviations from the classical behaviour can be attributed to the proximity of the border. Larger values of J would put the border further away and a semiclassical regime would eventually appear. In fact, a simple estimate shows that as J increases, the percentage of phase space area affected by the border decreases as J^{-1} and, therefore, the corresponding number of quantum states also diminishes: let us define the phase space area affected by the border considering that in the equations of motion the effect will be due to the divergent factor

$$f(r) = \frac{1}{\sqrt{4J - r^2}},$$

where $r^2 = p_1^2 + q_1^2$ (appearing in eq. (5)). We then choose an arbitrary radius r_0 such that $f(r) > \alpha$ ($\alpha > 0$) for $r > r_0$ and define an area A_0 between the border and the circle of radius r_0 in the plane q_1, p_1. If we compare A_0 with the corresponding total area A and take the limit $J \to \infty$ we get

$$\lim_{J\to\infty} \frac{A_0}{A} = \lim_{J\to\infty} \frac{\pi/\alpha^2}{4\pi J} = \lim_{J\to\infty} \frac{1}{\alpha^2 J} = 0.$$

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