keywords billiards; Fermi acceleration; chaos; hamiltonian mechanics

Suppression of Fermi acceleration in composite particles

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Abstract

We study the motion of a composite particle in a one-dimensional billiard with a moving wall. The particle is modelled by two point masses coupled by a harmonic spring. We show that the energy gained by the composite particle is greatly reduced with respect to a single point particle. We show that the amount of energy transferred to the system at each collision with the walls is independent of the spring constant. However, the presence of the spring is responsible for the energy suppression because it diminishes the number of collisions by storing part of the system's energy and reducing the velocity of the particle's center of mass.

PACS numbers: 05.45,05.45Pq,05.40.Jc

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I. INTRODUCTION

Billiards with moving boundaries have been studied in the context of physics since 1949. In this year Fermi presented a model that was later modified by Ulam and that came to be known as "Fermi-Ulam model" (or FU model). The model was originally proposed as a way of explaining the high energy with which some charged particles of cosmic rays arrived in Earth. Fermi believed that these particles could accelerate as a result of successive collisions with structures in a moving magnetic field [1]. Ulam proposed a mechanical analogue of this model, the FU model, in which a particle moves between two walls of infinite mass, one fixed and one moving according to some rule. The particle gains energy in head on collisions and loses it in overtaking ones [2, 3]. According to Fermi hypothesis if the oscillations of the wall were random one would expect that on average the particle would gain more energy than it lost and, therefore, it would accelerate. This phenomenon in which a particle gains unlimited amounts of energy due to collisions with a moving wall is called Fermi Acceleration (FA) [1, 4].

Later work showed that the Fermi hypothesis was correct only when the movement of the wall was random. Lieberman e Lichtenberg [5, 6], showed that if the motion of the wall is periodic in time the FU model could be interpreted as an autonomous Hamiltonian system with two degrees of freedom. In this case, if the motion of the wall is sufficiently smooth, KAM theory can be applied and due to KAM curves the energy of the system remains bounded and Fermi acceleration is not observed. Since then more realistic models were presented to explain the behaviour of cosmic rays. But the study of billiards with moving walls continued in the context of dynamical systems and it has become a field of its own [7].

Billiards are simple systems that can have unexpectedly complex behaviour. Even when the walls are not moving there is a wide range of possible dynamics that can be found in two or higher dimensional billiards. Depending on the shape of the billiard domain the dynamics can be regular, for integrable billiards, it can have a mixed phase or it can be fully chaotic [8–10]. When the walls are put into motion there is, in addition, the possibility of adding or removing energy from the system. For billiards with two or higher dimensions the average energy gained (or lost) depends on the shape of the boundary and on the protocol used for the wall motion. Depending on the billiard it is possible to find protocols where the billiard

ball gains energy indefinitely, that is, FA is achieved. [11].

For two dimensional billiards a conjecture by Loskutov, Ryabov and Akinshin (LRA conjecture) states that if the static billiard has a chaotic phase space then FA can be achieved with moving walls [12]. This conjecture has been shown correct in the annular and stadium billiards [13, 14]. However it has also been shown integrable billiards, such as the ellipse, can lead to FA for certain protocols of wall movement [15].

An important question regarding billiards that has not received much attention is how a particle's internal degrees of freedom would affect its behavior in a billiard with moving walls. Internal degrees of freedom often couple with the center of mass during collisions and the consequences of this coupling can be observed even in double slit experiments in quantum mechanics [16]. However, to the best of our knowledge, no work has yet explored the influence of the internal degrees of freedom of a composite particle in the phenomenon of Fermi acceleration.

The purpose of this work is to evaluate how a particle with internal degrees of freedom behaves in a billiard with moving walls. For simplicity we will consider a one-dimensional billiard with a single moving wall and model the composite particle as two point masses coupled by a harmonic spring. We will show that the particle's energy gain is significantly diminished with respect to the corresponding single particle and we will describe the mechanism that leads to that. In the next section we present the model used and in section III we show and discuss the results of numerical simulations for billiards with fixed and moving walls.

II. MODEL

Our model consists of two point particles connected by an harmonic spring and moving in a one dimensional infinite well that has one moving boundary. The two particles and the spring can be seen as a single composite particle with internal degrees of freedom. The vibrational modes of the system can be excited when either particle collides with the walls.

The system is described by the Hamiltonian

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}k(x_1 - x_2)^2 + V_{ext}^1(t) + V_{ext}^2(t) \tag{1}$$

where p_i, m_i and x_i are, respectively, the momentum, mass and position of the particle

i. The two point particles interact via an harmonic potential with elastic constant k. The equilibrium distance between these particles is zero and they collide elastically at zero length.

V^i_{ext} describes the environment in which the particles are allowed to move.

$$V^i_{ext}(t) = \begin{cases} 0, & 0 \leq x_i \leq X_w(t) \\ \infty, & \text{otherwise} \end{cases} \tag{2}$$

as illustrated in Fig. 1.

We assume that the collisions between the particles and between each particle and the walls are elastic. We also assume that one particle cannot go through the other. This restricts the movement of the two particles to the interval $0 \leq x_1 \leq x_2 \leq X_w(t)$. Therefore particle 1 collides only with the fixed wall and particle 2 with the moving wall.

The collision between particle 1 and the fixed wall only changes the direction of the velocity of this particle, $v_1 \longrightarrow -v_1$. The collision between particle 2 and the moving boundary is a perfect reflection if seen in the frame of the moving wall. Therefore,

$$v_2 \longrightarrow -v_2 + 2V_w(t). \tag{3}$$

Finally, the collision between particles 1 and 2 is elastic. Therefore, the particle velocities before (u_i) and after (v_i) the collision must satisfy the relations:

$$\begin{cases} m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \\ \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2. \end{cases} \tag{4}$$

By introducing the variables $\bar{x_i} = \sqrt{m_i}x_i$, $\bar{u_i} = \sqrt{m_i}u_i$ and $\bar{v_i} = \sqrt{m_i}v_i$ we can interpret this model in an alternative way. Instead of two particles moving in one dimension, we can think of the system as a single particle moving in a two dimensional space. In these variables the system's configuration space becomes a triangle (see Fig. 2) where the diagonal is given by $\bar{x_1}/\sqrt{m_1} = \bar{x_2}/\sqrt{m_2}$. In the new variables the reflection rules between the particles can be written as [17]:

$$\begin{cases} \sqrt{m_1}\bar{u_1} + \sqrt{m_2}\bar{u_2} = \sqrt{m_1}\bar{v_1} + \sqrt{m_2}\bar{v_2} \\ \bar{u_1}^2 + \bar{u_2}^2 = \bar{v_1}^2 + \bar{v_2}^2. \end{cases} \tag{5}$$

In this representation the single particle moves inside the triangle and undergoes elastic collisions at its boundaries. The spring between the two particles is now interpreted as an external potential for this one particle. The movement of the right wall corresponds to the movement of the top side of the triangle.

The region near the acute angles of the triangle can be a source of numerical errors when the equations of motion are solved numerically. In order to avoid this problem we use a geometrical tool known as unfolding. Since the collision between the particle and the diagonal of the triangle is elastic, and therefore the angle of incidence equals the angle of reflection, we can reflect the entire triangle instead of reflecting the particle (Fig. 2(B)). In the unfolded scheme, the movement of the wall is equivalent to the joint movement of the top and right sides of the square. The advantage here is that it is no longer necessary to consider the collisions between the particle and the diagonal (that is, the collision between the two original particles). Note that the diagonal of the original triangle is a trajectory corresponding to the particles moving together.

III. RESULTS AND DISCUSSION

A. Behavior for fixed walls

Previous studies of billiards with moving walls have shown that the existence, or non-existence, of Fermi acceleration depends strongly on the dynamical properties of the corresponding static billiard. In particular, numerical experiments show that particle acceleration is more likely to be observed in chaotic billiards [12].

When the walls are fixed at $x = 0$ and $x = 1$ the system is conservative and its dynamics can be studied with the help of Poincare maps. In this case the total energy is simply the sum of the kinetic energy of each particle with the elastic energy of the spring. Fig. 3 shows the momentum and position of particle 2 when particle 1 is colliding with the wall at $x = 0$ for several values of the total energy. For these simulations we used $k = 2$, $m_1 = m_2 = 1$ so that:

$$E = \frac{v_1^2}{2} + \frac{v_2^2}{2} + (x_1 - x_2)^2 \tag{6}$$

For small energies, such as for $E = 0.125$, the system is mostly chaotic. In this case the maximum extent to which the spring can stretch is smaller than the width of the billiard. There are only two stable fixed points and they correspond to the motion along the line where $x_1 = x_2$, that is, when the two particles move together inside the billiard. As the energy increases the Poincare map becomes more regular, as more islands and stable fixed

points are visible. For $E = 1$ the structure is particularly rich. This is the case in which the system has just enough energy to stretch the spring to the full extension of the well. At higher energies the Poincare Map exhibits invariant curves that become more and more straight and the map becomes increasingly more similar to the map for a square billiard. [17]

B. Energy gain due to the movement of the wall

Here we will show that, for a fixed protocol of wall motion, the rate of energy gain for a structured particle is smaller than for a single particle and for two non-interacting particles.

The left wall was kept fixed at $x = 0$ and we studied the behavior of the system for three different velocities of the right wall. When the right wall, originally at $x = 1$, is allowed to oscillate, the total energy is not conserved and the system passes through the different regimes shown in Fig.3.

In order to maximize the energy gain by the composite particle we chose the following protocol for the movement of the wall: it starts at the maximum amplitude $1 + \epsilon$ and moves left with constant speed $V_w = 2\epsilon/T_w$. When the wall reaches $1 - \epsilon$ it abruptly moves back right to the starting position and repeats the cycle. The position of the wall is given by

$$X_w(t) = 1 + \epsilon \left[1 - \frac{2}{T_w} mod(t, T_w) \right] \tag{7}$$

where $mod(t, T_w) = t - T_w \lfloor t/T_w \rfloor$ and $\lfloor x \rfloor$ stands for the integer part of x.

This protocol was chosen to ensure that FA would always be observed. For the FU-Model with a single particle, Fermi acceleration does not happen if the movement of the wall is smooth and periodic [5]: as the energy of the particle increases it approaches a more regular region of the phase space and the energy of the particle remains bounded. For our model (with a composite particle) FA is also not expected if the wall moves smoothly. The regular behaviour seen at high energies (see Fig.3) would hinder FA according to the LRA conjecture. With the chosen protocol all the collisions with the wall provide energy to the particle (composite or simple) and FA is, necessarily, present. For Figs. 4 to 6 below we used $\epsilon = 0.01$.

In Fig. 4 we studied three different situations: a single particle (with mass $M = 2$), two 'free particles' that only interact via direct collisions, and two particles interacting via an

harmonic potential (with masses $m_1 = m_2 = 1$). These values were chosen to evaluate if the system with two particles with masses m_1 and m_2 could behave as a single particle with mass $M = m_1 + m_2$. For the third case we used three different values of spring constant, k (2, 20 and 200). We also used three different periods, T, for the wall motion (0.1, 1 and 10) and the same initial energy for all situations ($E = 2$). For the single particle we used 18 different initial positions for the particle and the starting velocity was $\sqrt{2}$. For the cases where two particles were involved we chose 5 initial conditions in which both particles are at the same position with velocities (v_{10} and v_{20}) given by:

$$\begin{cases} v_{10} = v_0 \cos[\pi(j-3)/10] \\ v_{20} = v_0 \sin[\pi(j-3)/10] \end{cases} \tag{8}$$

where $v_0 = \sqrt{2}$ and $j = 1, ..., 5$. Note that for $j = 3$ only particle 1 is moving initially.

The equations of motion were integrated numerically using the function $NDSolve$ from the software *Mathematica*® (version 9.0) [18]. Maximum numbers of steps was set to 400000 and the default setting was used for the remaining parameters.

Fig. 4 shows the total energy of the system (kinetic plus potential) as a function of the time. For $T = 0.1$ and $T = 1$ we collected data during 290 cycles of wall movement and for $T = 10$ we used 100 cycles. In all cases there is a net energy absorption from the moving wall. However, for a given wall period, the rate of energy gain for the system is not the same. The energy gained by the two non-connected particles ($k = 0$) is the largest and has about the same value as the energy gained by the single particle with mass equal to the sum of the masses of the two particles. As k increases less energy is absorbed per unit time by the system. Fermi acceleration still occurs but at a smaller rate. We emphasize that this suppression is not merely due to the energy transferred to the spring, since we are looking at the total energy of the composite particle.

For the special cases of a single particle and of two non-interacting particles, the corresponding static billiards are integrable [17]. In these cases KAM theory and LRA conjecture, respectively, would predict no FA if the wall motion was smooth in time. Since the movement of the walls is periodic but not smooth, we observe large gains of energy in both situations [5]. In our model the collisions of the particles with the walls are always head on and, therefore, energy is always provided to the system. To our knowledge the square

billiard, itself, has not yet been studied in the context of FA. However, this is a separable billiard that can be decomposed into two 1D billiards and, hence, the qualitative properties of the FU-model and square billiard are essentially the same.

We also note that having two particles with $k = 0$ is not the same as having a single particle in the billiard: collisions between the two particles effectively decrease the space available for each particle, increasing the number of collisions with the moving wall (see Fig. 5(A)).

C. Why is there a reduction of the rate of energy gain due to the spring?

Fig. 4 shows that the presence of the spring clearly affects the energy gain of the system. In this subsection we will show that a decreased rate of collisions with the moving wall is the main mechanism by which the inter-particle potential affects the rate of energy gain.

The amount of energy transferred at each collision depends solely on the velocity of the wall and on the velocity of the particle colliding with it. For a single particle with mass M the energy, E_{1p}, accumulated after N collisions with the wall is:

$$E_{1p} = \frac{1}{2} M \left(v_0 + 2N \frac{2\epsilon}{T_W} \right)^2 \tag{9}$$

where v_0 is the initial speed of the particle [19]. For large N the behaviour of this function is such that:

$$\log E_{1p} \to \log(N^2) + 2 \log \left(\frac{4\epsilon}{T_w} \right) \tag{10}$$

This equation corresponds to the solid line shown on the first column of Fig. 5 and, as expected, it reproduces well the behaviour of the energy for a single particle as N gets larger.

When two particles are involved the situation is a bit more tricky even when there is no interaction between the particles. When the particles collide they exchange velocities (if $m_1 = m_2$) and, therefore, the velocity particle 2 has when it collides with the moving wall can be different from the velocity it acquired on the previous collision with the wall. Using the unfolded domain of figure 2 the moving wall would correspond to the right and upper walls of the domain. Collisions with each of these walls will result in a different energy gain.

If we have n collisions with the right wall and m collisions with the upper wall the energy, E_{2p}, gained will be:

$$E_{2p} = \frac{1}{2}\left(v_{10} + 2n\frac{2\epsilon}{T_W}\right)^2 + \frac{1}{2}\left(v_{20} + 2m\frac{2\epsilon}{T_W}\right)^2 \tag{11}$$

where v_{j0} is the initial velocity of particle j and we already used $m_1 = m_2 = 1$. For large n or m the behaviour of this function is such that:

$$\log E_{2p} \to \log(n^2 + m^2) + 2\log\left(\frac{4\epsilon}{T_w}\right) - \log(2) \leq \log(N^2) + 2\log\left(\frac{4\epsilon}{T_w}\right) - \log(2) \equiv \log E_{2p}^{max} \tag{12}$$

where $N = n + m$ is the total number of collisions with the wall. The equality holds when only one of the particles are moving at each time. This situation corresponds to the initial conditions depicted in green in Figs. 4 and 5. In the column A of Fig. 5 the dashed line corresponds to $\log E_{2p}^{max}$. Here we can see that, as expected, when there is no interaction between the particles, $\log E_{2p}^{max}$ captures the behaviour of the green line, (0), for large values of N.

The behaviour of E as a function of N^2 is essentially the same for two non-interacting particles and for $k = 2$ and $k = 20$ (see Fig 5-Column A). At large N there is an upper bound for the energy given by $\log E_{2p}^{max}$. For $k = 200$ the comparison can not be properly made because for the number of cycles considered this function was still in its initial transient state. These results show that the amount of energy gained by the system at each collision does not depend on the interaction between the two particles. Therefore the altered rate of energy gain cannot be attributed to a smaller energy transfer at each collision.

The presence of the spring diminishes the number of collisions between the particles and the moving wall because part of the energy gets stored in the spring, decreasing the velocity of the center of mass. Column B shows that, except for the single particle case, the energy gain is proportional to the square of the number of collisions. Hence this smaller rate of collisions with the moving wall seems to be the main mechanism responsible for a smaller rate of energy gain.

We note that the value of k is important only at low energies, since the elastic energy can not be larger than $k/2$. However, at a given time, the amount of energy acquired by the system depends on the value of k even for large energies. This shows that the transient

phase, in which the system is at low energies, can impact the amount of energy absorbed by the system.

Initial conditions also play an important role. For the cases (0), (2) and (20) the largest growth corresponds to the initial condition displayed in the curve drawn in magenta ($j = 5$), but for the case (200) it is the yellow curve ($j = 2$) that grows faster. The total energy is initially the same for all the curves but the energy of the center of mass is not. The initial condition with $j = 5$ is associated with the largest initial velocity for the center of mass. Since more collisions are expected for larger velocities of the center of mass it is natural to expect the curve in magenta to grow faster than the others. It is not clear why this doesn't happen for the case (200). More detailed numerical investigations would be necessary to understand how the evolution of individual initial conditions is affected by the value of the spring constant.

Finally we note that that the number of collisions for two non-interacting particles, (0), is larger than for a single particle, (1p), even though the energy gained is the same. For (0) the initial energy is divided between the two particles but, as mentioned before, each one has an effectively smaller distance to travel before hitting the wall or the other particle, which results in a larger number of collisions. In spite of that, the energy gained at a given time is the same because this energy depends on the mass of the particle colliding with the wall, which is $m_2 = 1$ for (0) and $M = 2$ for (1p).

D. Energy gain for the center of mass

In the previous section we discussed how the total energy of the system changes as a function of the number of collisions. Now we will address the energy growth for the center of mass of the system and show that it behaves in a slightly different way.

Fig 5, column (C), shows the energy of the center of mass as a function of the square of the number of collisions, N^2. In this figure the initial energy for the center of mass is equal for equal colors. We can see that for some initial conditions the energy of the center of mass does not strictly grow with time. In (0), for example, the initial condition shown in magenta alternates between two increasing curves. This can be understood with the help of the unfolded domain in Fig. 2. Consider that initially the particles have velocities v_{10} and v_{20}. Collisions with the right (upper) wall change only v_1 (v_2). After n (m) collisions with

the right (upper) wall these velocities will be:

$$\begin{cases} v_1 = v_{10} + 2n\frac{2\epsilon}{T_W} \\ v_2 = v_{20} + 2m\frac{2\epsilon}{T_W}. \end{cases} \tag{13}$$

Suppose that before the collisions with the upper wall v_1 and v_2 have the same sign and that before the collisions with the right wall v_1 and v_2 have opposite signs (for non-interacting particles this will always be the case if one of these conditions is satisfied for the first collision). In this situation if the last collision happened with the right wall the energy of the center of mass would be:

$$E_{cmR} = \frac{1}{4}\left(v_1 + v_2\right)^2 = \frac{1}{4}\left(v_{10} + v_{20} + N\frac{4\epsilon}{T_W}\right)^2 \tag{14}$$

where $N = n + m$ for $m_1 = m_2 = 1$. For large N this equation becomes:

$$\log E_{cmR} \to \log(N^2) + 2\log\left(\frac{4\epsilon}{T_w}\right) - \log(4) \tag{15}$$

This equation corresponds to the line in Fig. 5, column B. We can see that for all initial conditions one of the curves for E_{cm} agrees with this function for large N.

If, on the other hand, the last collision happened with the upper wall the energy of the center of mass would be:

$$E_{cmU} = \frac{1}{4}\left(v_1 - v_2\right)^2 = \frac{1}{4}\left(v_{10} - v_{20} + (n-m)\frac{4\epsilon}{T_W}\right)^2 \tag{16}$$

If for every collision with the upper wall there are α collisions with the right wall, then $(n - m) = N(\alpha - 1)/(\alpha + 1)$. As N becomes larger the equation becomes:

$$\log E_{cmU} \to \log(N^2) + 2\log\left(\frac{4\epsilon}{T_w}\right) + 3\log\left(\frac{\alpha - 1}{\alpha + 1}\right) - \log(4) \leq \log E_{cmR} \tag{17}$$

This explains the second growth curve present in Fig. 5(B).

In none of the cases depicted in Fig 5 the center of mass behaved as a single particle. The green initial condition is the only one where a single curve is present. But this case already corresponds to having only one of the particles moving at each time and only colliding with either the upper or right.

E. Can a structured particle behave as a point particle?

We have seen that for large values of total energy the two interacting particles behave similarly to non-interacting particles. Is there a situation in which the two particles behave as a single particle? For large values of the spring constant, for example, the composite particle becomes increasingly more rigid and, therefore, it might behave more like a single particle. Does this happen? Furthermore, does the center or mass of the two particles behave as a single particle for some k?

In order to gain some insight as to how the value of k affects the energy gain we collected the value of the total energy, E_{tot}, energy for the center of mass, E_{cm}, and number of collisions, N, for k ranging from 2 to 200. The data was collected after one, ten or 100 cycles of wall movement. We used the same initial energy for the center of mass ($E_{cm} = 4$) and initially the two particles were moving with the same velocity ($v_1 = v_2 = v_{cm}$). For each k the initial distance between the particles was chosen so that the potential energy was equal to 0.5. For each value of k we used up to 16 different initial positions for the particles. The results obtained are depicted in Fig. 6.

The red lines in Fig. 6 correspond to the results obtained for a single particle. The initial conditions were such that the particle's initial energy was equal to 4 (the initial value of E_{cm} in the simulations involving two particles). We used 9 different starting positions for the single particle. Fig. 6 shows that, at a fixed time, different initial conditions will have very similar values for the energy and number of collisions. In this case, of course $E_{cm}/E_{tot} = 1$. Is there a k that has any of these properties?

After one period the total energy of the two interacting particles does not change much from its initial value (4.5) since the particles have collided with the moving wall only two to three times.

The center of mass energy after one period, on the other hand, can be either slightly larger or quite smaller than its initial value. As discussed before the energy of the center of mass can follow two different growth curves (see Fig. 5). The initial conditions we used correspond to starting at different points within the triangle from Fig. 2(A) and moving towards the upper wall with a trajectory that is, initially, parallel to the diagonal of the square. For non-interacting particles this would lead to a situation where the system alternates between collisions with the upper and right wall of Fig. 2(B). So, after an even

number of collisions the center of mass energy would be given by E_{cmR} (see equation 15) and should be larger than its initial value. This does not happen for interacting particles. For some initial conditions the particle will collide with the right wall before colliding with the upper wall and the center of mass energy will be given by E_{cmU} after two collisions and, therefore, be smaller than its initial value.

After 10 periods the number of collisions is different for different k's and for different initial conditions. As a consequence there is a broader range of values for E_{tot}. However, at this early time there is no well defined relation between E_{tot} and k. After 100 periods, this situation changes and for most initial conditions N and E_{tot} are smaller for large k. However there is still a broad range of values that these variables can assume. Also, after 10 and 100 periods the energy of the center of mass can be any fraction of the total energy of the system. Therefore, within the assumptions of our model, the limit of large k cannot reproduce the behaviour of a single particle.

To summarize, we have shown that a system composed of two point particles connected by a spring moving on a square well with an oscillating wall will absorb energy at a lower rate than what would be observed for a single unstructured particle. The reason for the smaller gain is the reduction in the number of collisions with the moving wall, which in turn is due to the slow down of the center of mass of the system. For a fixed integration time, the larger the k, the lower the amount of energy absorbed. However, for fixed k, as the energy of the system becomes larger than $k/2$ (the maximum elastic energy than can be stored by the spring) the behaviour of the point masses approaches that of two non-interacting particle, although this regime might only be reached after very long integration times.

Acknowledgements

M.A.M.A. acknowledges financial support from CNPq and FAPESP. K.M.S. was supported by FAPESP.

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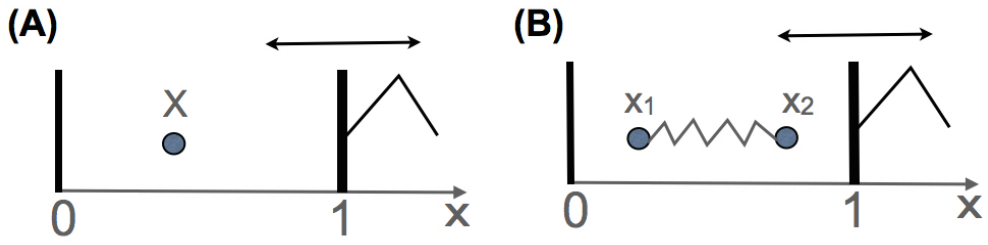

FIG. 1. Schematic illustration of the (A) FU model; (B) our model

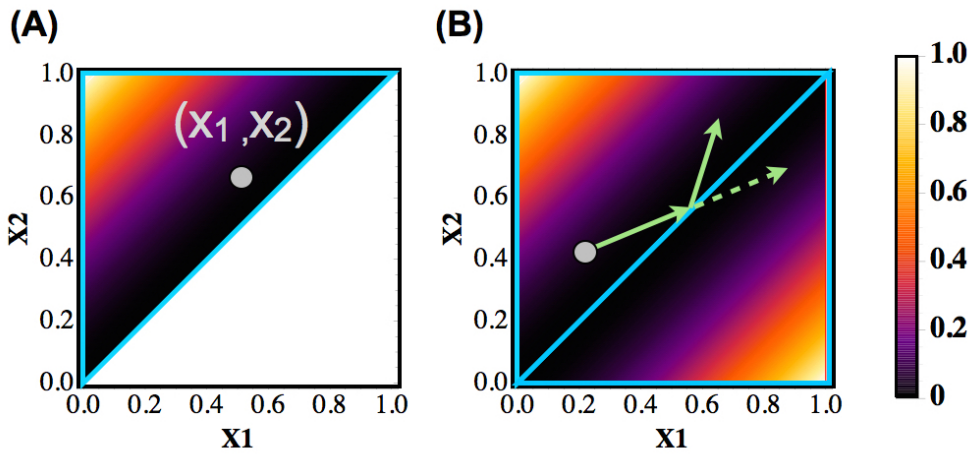

FIG. 2. (A) - Reinterpretation of the system as a single particle moving in a triangular domain. The harmonic spring is seen as an external potential. Parameters used: $k = 2$, $m_1 = m_2 = 1$. (B)-Unfolding of the domain.

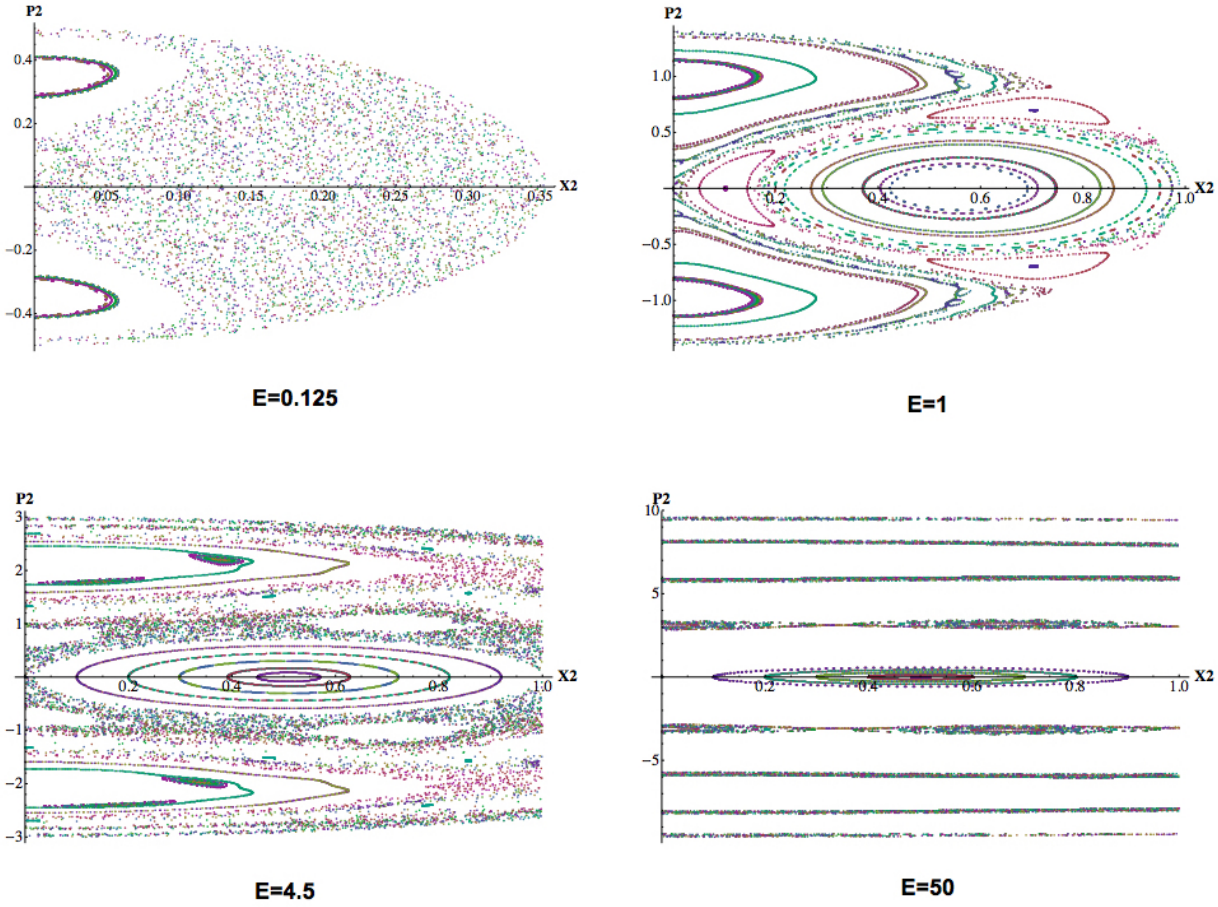

FIG. 3. Poincare maps for the system with $m_1 = m_2 = 1$ and different values of the total energy when the wall is not moving. The axes correspond to the momentum and position of particle 2 when particle 1 hits the wall.

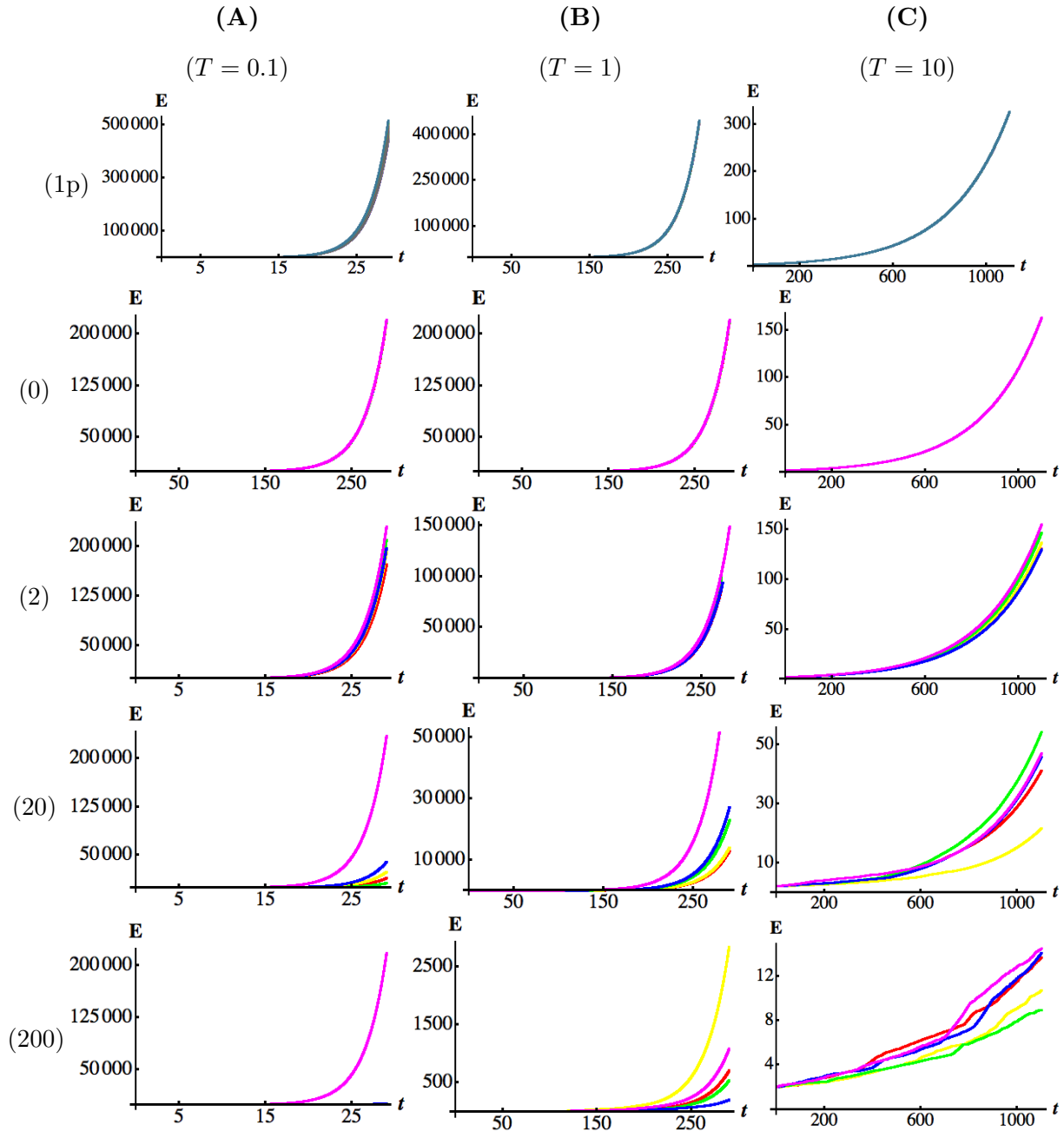

FIG. 4. Energy of the system *versus* time for a wall motion given by 7 with $\epsilon = 0.01$. Column A - $T_W = 0.1$, Column B - $T_W = 1$, Column C - $T_W = 10$; Line (1p) = a single particle (FU model), Line (0) = two non-interacting particles, Lines (2), (20) and (200) = two particles interacting via an harmonic spring with spring constant (k) equals to 2, 20 and 200, respectively. The curves correspond to 18 different initial conditions for (1p) and 5 initial conditions for (0), (2), (20) and (200). The five initial conditions are given by 8: red, yellow, green, blue, magenta correspond to $j = 1, 2, 3, 4$ and 5, respectively.

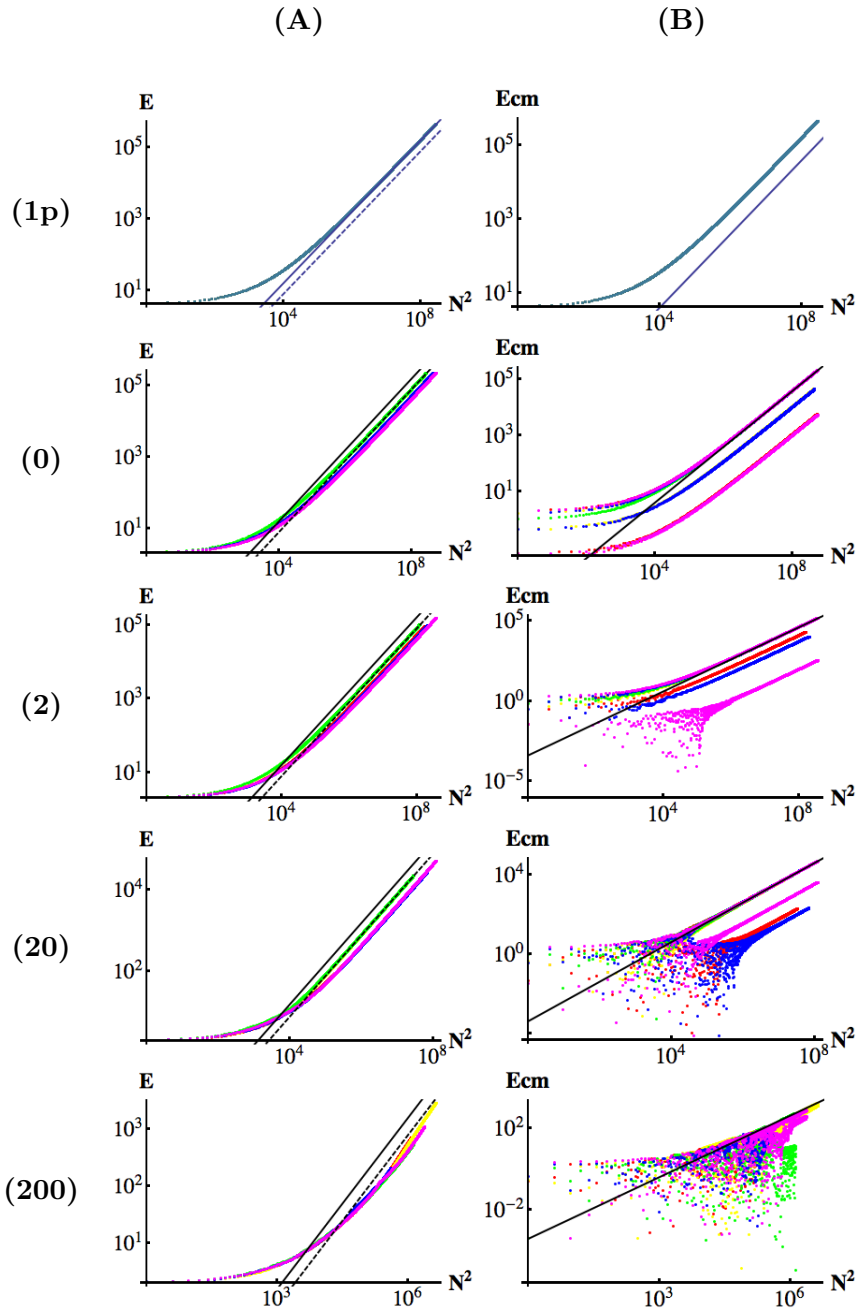

FIG. 5. Column A - Energy (E) of the system as a function of the square of the number of collisions (N^2), Column B - Energy of the system's center of mass (E_{cm}); The solid and dashed lines in column A corresponds to equation 10 and $\log E_{2p}^{max}$ in 12, respectively; The solid line in column B corresponds to equation 15. Line (1p) = one point particle, Line (0) = two non-interacting particles, Lines (2), (20) and (200) = two particles interacting via an harmonic spring with spring constant (k) equals to 2, 20 and 200, respectively. The initial conditions are the same as in Fig. 4; $\epsilon = 0.01$ and $T_W = 1$

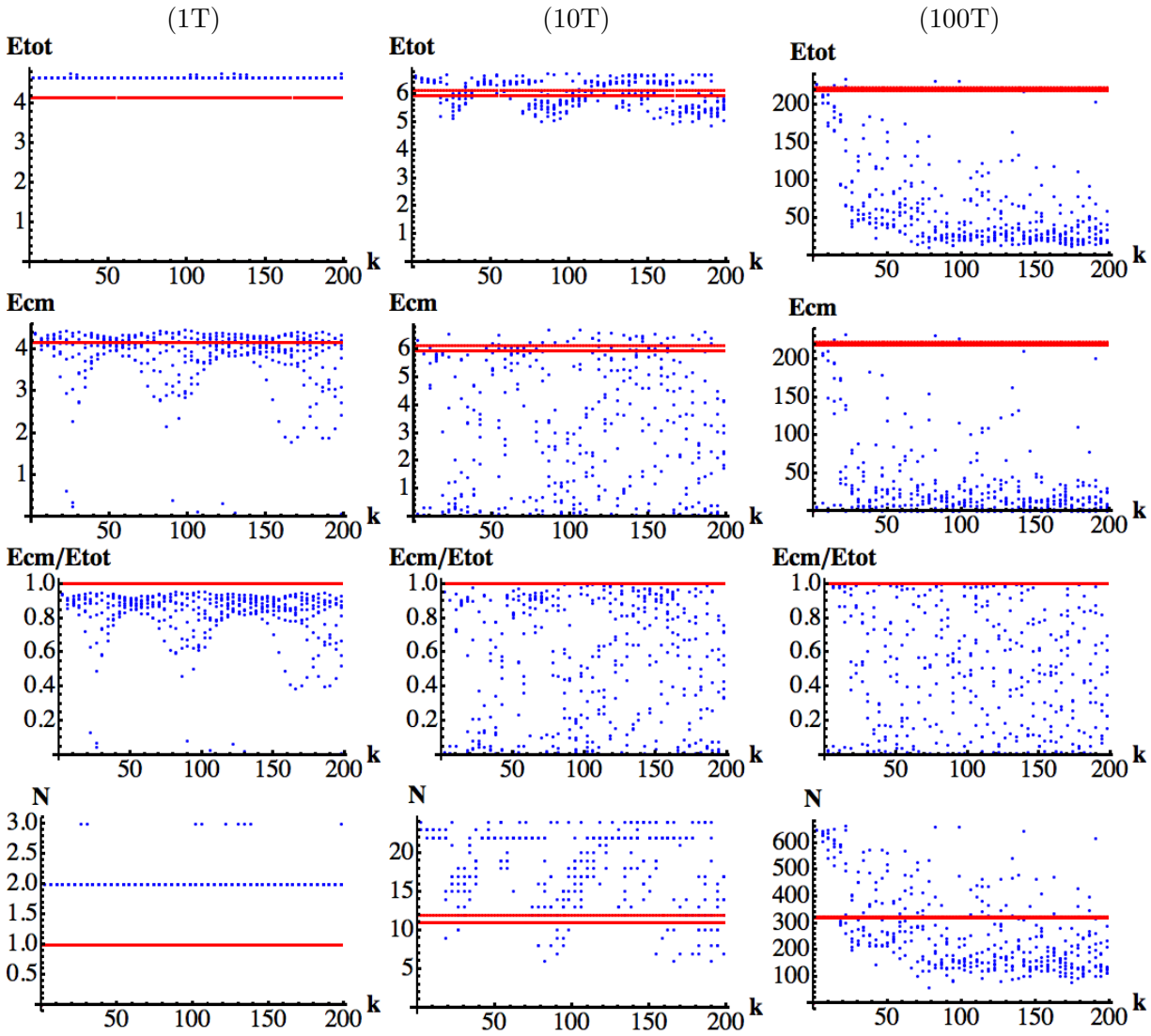

FIG. 6. Each line shows the value of the total energy of the system (E_{tot}), the energy of the center of mass (E_{cm}), the ratio E_{cm}/E_{tot} and the number of collisions N at the end of 1 (first column), 10 (second column) and 100 (third column) cycles of wall movement for a range of values of k (spring constant). The red lines are the values obtained for several initial conditions with a single particle with mass $M = 2$. In all the cases the wall is moving with amplitude $\epsilon = 0.01$ and period $T_W = 1$, and the initial energy of the center of mass of system is the same and equal to 4. When two particles were involved they initially had the same velocity ($v_1 = v_2 = v_{cm}$) and the distance between them was chosen so that the potential energy was equal to 0.5. For each value of k we used up to 16 different initial positions for the particles.