MAPA DE MEYER

$$\begin{cases} X_1 = X_0 - P_0 \\ P_1 = P_0 - \delta + (X_0 - P_0)^2 \end{cases}$$

Obs. $\boxed{\delta \equiv -\epsilon}$

$$F \equiv \begin{pmatrix} \frac{\partial X_1}{\partial X_0} & \frac{\partial X_1}{\partial P_0} \\ \frac{\partial P_1}{\partial X_0} & \frac{\partial P_1}{\partial P_0} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2(X_0 - P_0) & 1 - 2(X_0 - P_0) \end{pmatrix}$$

$\det(F) = 1 \Rightarrow$ o mapa preserva área

Pontos Fixos

$X_1 = X_0 \Rightarrow P = 0$

$P_1 = P_0 \Rightarrow (X - P)^2 = \delta \Rightarrow X = \pm\sqrt{\delta}$

Estabilidade

$$tr(F) = 2 - 2(X - P)$$

- $x_+ = \sqrt{\delta}$, $p_+ = 0$ $\quad tr(F) = 2(1-\sqrt{\delta})$

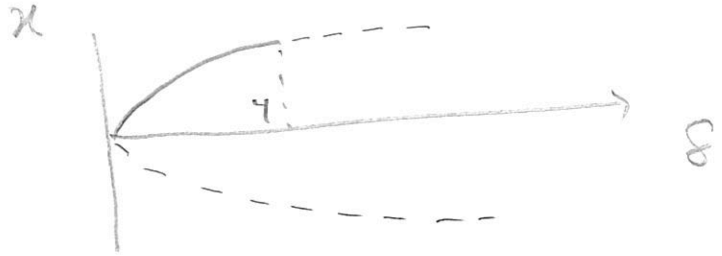
(x_+, p_+) é estável se $-2 < tr(F) < 2$

$\Rightarrow$ estável p/ $0 < \delta < 4$.

instável p/ $\delta > 4$

- $x_- = -\sqrt{\delta}$, $p_- = 0$ $\quad tr(F) = 2(1+\sqrt{\delta}) >$

$\Rightarrow$ (x_-, p_-) é instável p/ $\delta > 0$.



Pontos Fixos de Período 2

$$X_2 = X_1 - P_1 = [x_0 - p_0] - [p_0 - \delta + (x_0 - p_0)^2]$$

$$= x_0 - 2p_0 + \delta - (x_0 - p_0)^2$$

$$P_2 = P_1 - \delta + (X_1 - P_1)^2 = p_0 - \delta + (x_0 - p_0)^2 - \delta + [x_0 - p_0 - p_0 + \delta - (x_0 - p_0)^2]^2$$

$$= p_0 - 2\delta + (x_0 - p_0)^2 + [x_0 - 2p_0 + \delta - (x_0 - p_0)^2]^2$$

$$X_2 = X_0 \Rightarrow (X_0 - P_0)^2 = -2P_0 + \delta$$

$$P_2 = P_0 \Rightarrow 2\delta = (X_0-P_0)^2 + \left[(X_0-P_0) - P_0 + \delta - (X_0-P_0)^2\right]^2$$

$$x_0 - P_0 \equiv u$$

$$\begin{cases} u^2 = \delta - 2P_0 \\ 2\delta = u^2 + \left[u - P_0 + \delta - u^2\right]^2 \end{cases}$$

$$2\delta = \delta - 2P_0 + \left[u - P_0 + \delta - \delta + 2P_0\right]^2$$

$$\delta = -2P_0 + \left[u + P_0\right]^2 = -2P_0 + u^2 + 2uP_0 + P_0^2$$

$$= -2P_0 + \delta - 2P_0 + 2uP_0 + P_0^2$$

$$4P_0 = 2uP_0 + P_0^2 \Rightarrow \boxed{P_0 = 0} \quad \text{ou}$$

$4 - P_0 = 2u$ elevando ao quadrado,

$$16 - 8P_0 + P_0^2 = 4u^2 = 4\delta - 8P_0$$

$$16 + P_0^2 = 4\delta \qquad P_0^2 = 4(\delta - 4) \qquad \boxed{P_0 = \pm 2\sqrt{\delta - 4}}$$

$\Rightarrow$ pontos de período 2 só existem se $\delta > 4$.

Da equação $(X_0 - P_0)^2 = -2P_0 + \delta$ temos:

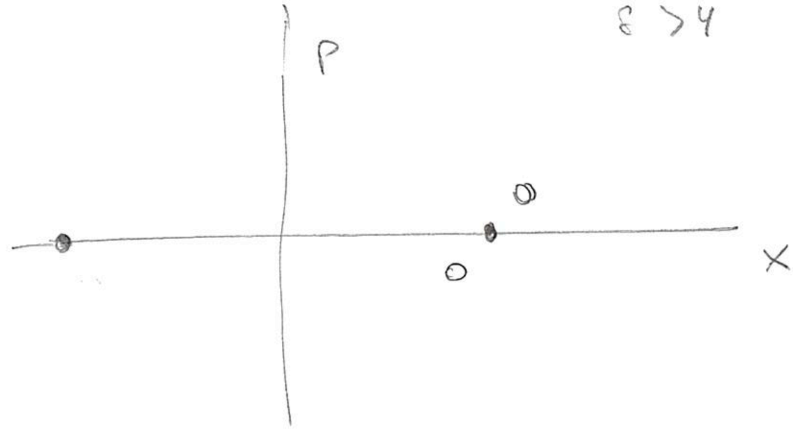
- $P_0 = 0 \Rightarrow X_0^2 = \delta \Rightarrow X_0 = \pm\sqrt{\delta}$

- $P_0 \neq 0$ podemos usar $4 - P_0 = 2u = 2x_0 - 2P_0$ ou

$$x_0 = 2 + \frac{P_0}{2}$$

$$x_0 = 2 \pm \sqrt{\varepsilon - 4}$$

Os ptos de período 2 são:



A) $(\sqrt{\varepsilon}, 0)$

B) $(-\sqrt{\varepsilon}, 0)$

C) $(2+\sqrt{\varepsilon-4}, 2\sqrt{\varepsilon-4})$

D) $(2-\sqrt{\varepsilon-4}, -2\sqrt{\varepsilon-4})$

$A \to A \to A$
$B \to B \to B$
$C \to D \to C$
$D \to C \to D$

ESTABILIDADE

$$z = \begin{pmatrix} q \\ p \end{pmatrix} \qquad z_1 = P(z_0)$$

$$z_2 = P(P(z_0)) = P(z_1)$$

$$\frac{\partial z_2}{\partial z_0} = \frac{\partial P}{\partial z_1}\frac{\partial z_1}{\partial z_0} = \frac{\partial P}{\partial z_1}\frac{\partial P}{\partial z_0}$$

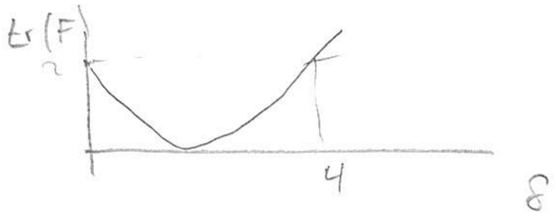
$$\Rightarrow \tilde{P} = \begin{pmatrix} 1 & -1 \\ 2(x_1-p_1) & 1-2(x_1-p_1) \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2(x_0-p_0) & 1-2(x_0-p_0) \end{pmatrix}$$

(5)

PONTO A

$$\tilde{F} = \begin{pmatrix} 1 & -1 \\ 2\sqrt{\delta} & 1-2\sqrt{\delta} \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2\sqrt{\delta} & 1-2\sqrt{\delta} \end{pmatrix} = \begin{pmatrix} 1-2\sqrt{\delta} & -2+2\sqrt{\delta} \\ 4\sqrt{\delta}-4\delta & 1+4\delta-6\sqrt{\delta} \end{pmatrix}$$

$$tr(F) = 2 - 8\sqrt{\delta} + 4\delta$$



estável se $0 < \delta < 4$

PONTO B

$$F = \begin{pmatrix} 1 & -1 \\ -2\sqrt{\delta} & 1+2\sqrt{\delta} \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -2\sqrt{\delta} & 1+2\sqrt{\delta} \end{pmatrix} = \begin{pmatrix} 1+2\sqrt{\delta} & -2-2\sqrt{\delta} \\ -4\sqrt{\delta}-4\delta & 1+4\delta+6 \end{pmatrix}$$

$$tr(\tilde{F}) = 2 + 8\sqrt{\delta} + 4\delta > 2$$

sempre instável

PONTOS C e D

$$F = \begin{pmatrix} 1 & -1 \\ 4-\rho_0^+ & 1-(4-\rho_0^+) \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 4-\rho_0^- & 1-(4-\rho_0^-) \end{pmatrix} \quad ; \quad \rho_0^{\pm} = \pm 2\sqrt{\delta-4}$$

$$tr(F) = 1-(4-\rho_0^-) - (4-\rho_0^+) + \left[1-(4-\rho_0^+)\right]\left[1-(4-\rho_0^-)\right]$$

$$= 1 - 2(4-\rho_0^-) - 2(4-\rho_0^+) + 1 + (4-\rho_0^+)(4-\rho_0^-)$$

$$= 2 + 4(P_U^- + P_U^+) - 4(P_U^- + P_U^+) + P_U^- P_U^+$$

$$= 2 + P_U^- P_U^+ = 2 - 4(\delta - 4) = 18 - 4\delta$$

p/ $\delta = 4$ $\quad tr(\tilde{F}) = 2$

p/ $\delta = 5$ $\quad tr(F) = -2$

$\Rightarrow$ C, D é estável p/ $4 < \delta < 5$