

26. Are there analogies in classical physics to the quantum mechanical concept of parity?
27. Are there unbound states for a simple harmonic oscillator potential? How many bound states are there? How realistic is the potential?
28. Explain all aspects of the behavior of all the probability densities of Table 6-2; in particular explain the probability density for the barrier potential with energy above the top.
29. What are the other significant features of the systems of Table 6-2?
30. Considering separately each system treated in this chapter, state which of its properties agree, and disagree, with classical mechanics in the microscopic limit. Which agree, and disagree, with classical wave motion in that limit? Make the same classifications for the properties of the systems in the macroscopic limit.
31. The eigenvalues in Figure 6-35 are equally spaced, but the lowest eigenvalues in Figure 6-22 come in closely spaced pairs. By considering the effect of a large bump in a potential well on the eigenvalues for symmetric versus antisymmetric eigenfunctions, explain the tendency for the eigenvalues to come in pairs in Figure 6-22.

PROBLEMS

1. Show that the step potential eigenfunction, for $E < V_0$, can be converted in form from the sum of two traveling waves, as in (6-24), to a standing wave, as in (6-29).
2. Repeat the step potential calculation of Section 6-4, but with the particle initially in the region $x > 0$ where $V(x) = V_0$, and traveling in the direction of decreasing x towards the point $x = 0$ where the potential steps down to its value $V(x) = 0$ in the region $x < 0$. Show that the transmission and reflection coefficients are the same as those obtained in Section 6-4.
3. Prove (6-43) stating that the sum of the reflection and transmission coefficients equals one, for the case of a step potential with $E > V_0$.
4. Prove (6-44) which expresses the reflection and transmission coefficients in terms of the ratio E/V_0 .
5. Consider a particle tunneling through a rectangular potential barrier. Write the general solutions presented in Section 6-5, which give the form of ψ in the different regions of the potential. (a) Then find four relations between the five arbitrary constants by matching ψ and $d\psi/dx$ at the boundaries between these regions. (b) Use these relations to evaluate the transmission coefficient T , thereby verifying (6-49). (Hint: First eliminate F and G , leaving relations between A , B , and C . Then eliminate B .)
6. Show that the expression of (6-49), for the transmission coefficient in tunneling through a rectangular potential barrier, reduces to the form quoted in (6-50) if the exponents are very large.
7. Consider a particle passing over a rectangular potential barrier. Write the general solutions, presented in Section 6-5, which give the form of ψ in the different regions of the potential. (a) Then find four relations between the five arbitrary constants by matching ψ and $d\psi/dx$ at the boundaries between these regions. (b) Use these relations to evaluate the transmission coefficient T , thereby verifying (6-51). (Hint: Note that the four relations become exactly the same as those found in the first part of Problem 5, if k_{II} is replaced by ik_{III} . Make this substitution in (6-49) to obtain directly (6-51).)
8. (a) Evaluate the transmission coefficient for an electron of total energy 2 eV incident upon a rectangular potential barrier of height 4 eV and thickness 10^{-10} m, using (6-49) and then using (6-50). Repeat the evaluation for a barrier thickness of (b) 9×10^{-9} m and (c) 10^{-9} m.
9. A proton and a deuteron (a particle with the same charge as a proton, but twice the mass) attempt to penetrate a rectangular potential barrier of height 10 MeV and thickness 10^{-14} m. Both particles have total energies of 3 MeV. (a) Use qualitative arguments to predict which particle has the highest probability of succeeding. (b) Evaluate quantitatively the probability of success for both particles.

10. A fusion reaction important in solar energy production (see Question 16) involves capture of a proton by a carbon nucleus, which has six times the charge of a proton and a radius of $r' \simeq 2 \times 10^{-15}$ m. (a) Estimate the Coulomb potential V experienced by the proton if it is at the nuclear surface. (b) The proton is incident upon the nucleus because of its thermal motion. Its total energy cannot realistically be assumed to be much higher than $10 kT$, where k is Boltzmann's constant (see Chapter 1) and where T is the internal temperature of the sun of about 10^7 °K. Estimate this total energy, and compare it with the height of the Coulomb barrier. (c) Calculate the probability that the proton can penetrate a rectangular barrier potential of height V extending from r' to $2r'$, the point at which the Coulomb barrier potential drops to $V/2$. (d) Is the penetration through the actual Coulomb barrier potential greater or less than through the rectangular barrier potential of part (c)?
11. Verify by substitution that the standing wave general solution, (6-62), satisfies the time-independent Schrodinger equation, (6-2), for the finite square well potential in the region inside the well.
12. Verify by substitution that the exponential general solutions, (6-63) and (6-64), satisfy the time-independent Schrodinger equation (6-13) for the finite square well potential in the regions outside the well.
13. (a) From qualitative arguments, make a sketch of the form of a typical unbound *standing wave* eigenfunction for a finite square well potential. (b) Is the amplitude of the oscillation the same in all regions? (c) What does the behavior of the amplitude predict about the probabilities of finding the particle in a unit length of the x axis in various regions? (d) Does the prediction agree with what would be expected from classical mechanics?
14. Use the qualitative arguments of Problem 13 to develop a condition on the total energy of the particle, in an unbound state of a finite square well potential, which makes the probability of finding it in a unit length of the x axis the same inside the well as outside the well. (Hint: What counts is the relation between the de Broglie wavelength inside the well and the width of the well.)
15. (a) Make a quantitative calculation of the transmission coefficient for an unbound particle moving over a finite square well potential. (Hint: Use a trick similar to the one indicated in Problem 7.) (b) Find a condition on the total energy of the particle which makes the transmission coefficient equal to one. (c) Compare with the condition found in Problem 14, and explain why they are the same. (d) Give an example of an optical analogue to this system.
16. (a) Consider a one-dimensional square well potential of finite depth V_0 and width a . What combination of these parameters determines the "strength" of the well—i.e., the number of energy levels the well is capable of binding? In the limit that the strength of the well becomes small, will the number of bound levels become 1 or 0? Give convincing justification for your answers.
17. An atom of the noble gas krypton exerts an attractive potential on an unbound electron, which has a very abrupt onset. Because of this it is a reasonable approximation to describe the potential as an attractive square well, of radius equal to the 4×10^{-10} m radius of the atom. Experiments show that an electron of kinetic energy 0.7 eV, in regions outside the atom, can travel through the atom with essentially no reflection. The phenomenon is called the *Ramsauer effect*. Use this information in the conditions of Problem 14 or 15 to determine the depth of the square well potential. (Hint: One de Broglie wavelength just fits into the width of the well. Why not one-half a de Broglie wavelength?)
18. A particle of total energy $9V_0$ is incident from the $-x$ axis on a potential given by

$$\begin{array}{ll}
 8V_0 & x < 0 \\
 V = 0 & 0 < x < a \\
 5V_0 & x > a
 \end{array}$$

Find the probability that the particle will be transmitted on through to the positive side of the x axis, $x > a$.

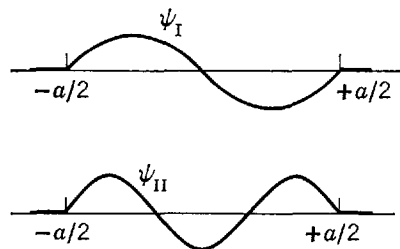


Figure 6-37 Two eigenfunctions considered in Problem 20.

19. Verify by substitution that the standing wave general solution, (6-67), satisfies the time-independent Schroedinger equation, (6-2), for the infinite square well potential in the region inside the well.
20. Two possible eigenfunctions for a particle moving freely in a region of length a , but strictly confined to that region, are shown in Figure 6-37. When the particle is in the state corresponding to the eigenfunction ψ_I , its total energy is 4 eV. (a) What is its total energy in the state corresponding to ψ_{II} ? (b) What is the lowest possible total energy for the particle in this system?
21. (a) Estimate the zero-point energy for a neutron in a nucleus, by treating it as if it were in an infinite square well of width equal to a nuclear diameter of 10^{-14} m. (b) Compare your answer with the electron zero-point energy of Example 6-6.
22. (a) Solve the classical wave equation governing the vibrations of a stretched string, for a string fixed at both its ends. Thereby show that the functions describing the possible shapes assumed by the string are essentially the same as the eigenfunctions for an infinite square well potential. (b) Also show that the possible frequencies of vibration of the string are essentially different from the frequencies of the wave functions for the potential.
23. (a) For a particle in a box, show that the fractional difference in the energy between adjacent eigenvalues is

$$\frac{\Delta E_n}{E_n} = \frac{2n+1}{n^2}$$

- (b) Use this formula to discuss the classical limit of the system.
24. Apply the normalization condition to show that the value of the multiplicative constant for the $n = 3$ eigenfunction of the infinite square well potential, (6-79), is $B_3 = \sqrt{2/a}$.
25. Use the eigenfunction of Problem 24 to calculate the following expectation values, and comment on each result: (a) \bar{x} , (b) \bar{p} , (c) $\bar{x^2}$, (d) $\bar{p^2}$.
26. (a) Use the results of Problem 25 to evaluate the product of the uncertainty in position times the uncertainty in momentum, for a particle in the $n = 3$ state of an infinite square well potential. (b) Compare with the results of Example 5-10 and Problem 13 of Chapter 5, and comment on the relative size of the uncertainty products for the $n = 1$, $n = 2$, and $n = 3$ states. (c) Find the limits of Δx and Δp as n approaches infinity.
27. Form the product of the eigenfunction for the $n = 1$ state of an infinite square well potential times the eigenfunction for the $n = 3$ state of that potential. Then integrate it over all x , and show that the result is equal to zero. In other words, prove that

$$\int_{-\infty}^{\infty} \psi_1(x) \psi_3(x) dx = 0$$

(Hint: Use the relation: $\cos u \cos v = [\cos(u+v) + \cos(u-v)]/2$.) Students who have worked Problem 36 of Chapter 5 have already proved that the integral over all x of the $n = 1$ eigenfunction times the $n = 2$ eigenfunction also equals zero. It can be proved that the integral over all x of any two different eigenfunctions of the potential equals zero. Furthermore, this is true for any two different eigenfunctions of any other potential. (If the eigenfunctions are complex, the complex conjugate of one is taken in the integrand.) This property is called *orthogonality*.

28. Apply the results of Problem 20 of Chapter 5 to the case of a particle in a three-dimensional box. That is, solve the time-independent Schroedinger equation for a particle

- moving in a three-dimensional potential that is zero inside a cubical region of edge length a , and becomes infinitely large outside that region. Determine the eigenvalues and eigenfunctions for the system.
29. Airline passengers frequently observe the wingtips of their planes oscillating up and down with periods of the order of 1 sec and amplitudes of about 0.1 m. (a) Prove that this is definitely not due to the zero-point motion of the wings by comparing the zero-point energy with the energy obtained from the quoted values plus an estimated mass for the wings. (b) Calculate the order of magnitude of the quantum number n of the observed oscillation.
 30. The restoring force constant C for the vibrations of the interatomic spacing of a typical diatomic molecule is about 10^3 joules/m². Use this value to estimate the zero-point energy of the molecular vibrations. The mass of the molecule is 4.1×10^{-26} kg.
 31. (a) Estimate the difference in energy between the ground state and first excited state of the vibrating molecule considered in Problem 30. (b) From this estimate determine the energy of the photon emitted by the vibrations in the charge distribution when the system makes a transition between the first excited state and the ground state. (c) Determine also the frequency of the photon, and compare it with the classical oscillation frequency of the system. (d) In what range of the electromagnetic spectrum is it?
 32. A pendulum, consisting of a weight of 1 kg at the end of a light 1 m rod, is oscillating with an amplitude of 0.1 m. Evaluate the following quantities: (a) frequency of oscillation, (b) energy of oscillation, (c) approximate value of quantum number for oscillation, (d) separation in energy between adjacent allowed energies, (e) separation in distance between adjacent bumps in the probability density function near the equilibrium point.
 33. Devise a simple argument verifying that the exponent in the decreasing exponential, which governs the behavior of simple harmonic oscillator eigenfunctions in the classically excluded region, is proportional to x^2 . (Hint: Take the finite square well eigenfunctions of (6-63) and (6-64), and treat the quantity $(V_0 - E)$ as if it increased with increasing x in proportion to x^2 .)
 34. Verify the eigenfunction and eigenvalue for the $n = 2$ state of a simple harmonic oscillator by direct substitution into the time-independent Schroedinger equation, as in Example 6-7.