Problems

1. Obtain the "scattering states" (energy eigenstates with $E \geq 0$) for a one-dimensional delta-function potential, $g\delta(x)$. Calculate the matrix elements $\langle k'|S|k\rangle$ and verify the unitarity of the S matrix. Obtain the transmission coefficient, and compare with Eq. (6.19) and Exercise 6.13. Perform the calculations in both the coordinate and momentum representations.

2. Use the Born approximation to calculate the differential and total cross sections for the elastic scattering of electrons by a hydrogen atom that is in its ground state. Approximate the interaction between the continuum electron and the atom by the static field of the atom and neglect exchange phenomena.

3. The cross section for two-quantum annihilation of positrons of velocity v with an electron at rest has, for $v \ll c$, the "classical" value

$$\sigma = \pi\left(\frac{e^2}{m_e c^2}\right)\frac{c}{v}$$

Use this information to derive the annihilation probability per unit time from a plane wave state and (assuming that annihilation occurs only if the two particles are at the same place) estimate the decay probability in the singlet ground state of positronium.

4. Using the Born approximation, and neglecting relativistic effects, express the differential cross section for scattering of an electron from a spherically symmetric charge distribution $\rho(r)$ as the product of the Rutherford scattering cross section for a point charge and the square of a *form factor* F. Obtain an expression for the form factor and evaluate it as a function of the momentum transfer for (a) a uniform charge distribution of radius R, and (b) a Gaussian charge distribution with the same root-mean-square radius.

5. If the nonlocal *separable* scattering potential

$$\langle \mathbf{r}'|V|\mathbf{r}''\rangle = \lambda u(r')u(r'')$$

is given, work out explicitly and solve the integral equation for $\Psi^{(+)}$. Obtain the scattering amplitude, and discuss the Born series for this exponential.