SOLUÇÃO DO PROBLEMA 1c

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} - q\mathcal{E}x = \frac{p^2}{2m} + \frac{m\omega^2}{2}\left(x - \frac{q\mathcal{E}}{m\omega^2}\right)^2 - \frac{q^2\mathcal{E}^2}{2m\omega^2}$$

$$\lambda \equiv \frac{q\mathcal{E}}{m\omega^2} \qquad \Delta E \equiv \frac{q^2\mathcal{E}^2}{2m\omega^2}$$

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \rightarrow H_0 |\varphi_n\rangle = E_n^0 |\varphi_n\rangle$$

$$E_n^0 = \hbar\omega(n + 1/2)$$

$$H|\xi_n\rangle = E_n|\xi_n\rangle \qquad E_n = E_n^0 - \Delta E$$

$$\xi_n(x) = \varphi_n(x-\lambda)$$

$$\boxed{|\xi_n\rangle = e^{-iP\lambda/\hbar}|\varphi_n\rangle}$$

$\boxed{\text{Para } 0 < t < \tau}$

$$|\psi(t)\rangle = \sum_n C_n e^{-iE_n t/\hbar}|\xi_n\rangle$$

$$|\psi(0)\rangle = \sum_n C_n|\xi_n\rangle = |\varphi_0\rangle = e^{i\lambda P/\hbar}|\xi_0\rangle$$

$$\Rightarrow \boxed{C_n = \langle\xi_n|e^{i\lambda P/\hbar}|\xi_0\rangle}$$

$$\langle\varphi_1|\psi(t)\rangle = \sum_n C_n e^{-iE_n t/\hbar}\langle\varphi_1|\xi_n\rangle$$

$$= \sum_n C_n e^{-iE_n t/\hbar}\langle\varphi_1|e^{-iP\lambda/\hbar}|\varphi_n\rangle$$

$$P = i\sqrt{\frac{m\hbar w}{2}}\,(a^{+} - a)$$

$$e^{-\frac{i\lambda P}{\hbar}} = e^{\frac{1}{\hbar}\frac{qE}{mw^2}\sqrt{\frac{mw\hbar}{2}}\,(a^{+}-a)} \equiv e^{\alpha(a^{+}-a)}$$

$$\boxed{\alpha = \frac{qE/w}{\sqrt{2mw\hbar}}}$$

$$e^{\alpha a^{+} - \alpha a} = e^{-\alpha^2/2}\, e^{\alpha a^{+}}\, e^{-\alpha a}$$

$$C_n = \langle E_n | e^{i\lambda P/\hbar} | \xi_0 \rangle = \langle \varphi_n | e^{i\lambda P/\hbar} | \varphi_0 \rangle$$

$$= e^{-\alpha^2/2} \langle \varphi_n | e^{\alpha a^{+}} | \varphi_0 \rangle \quad ; \quad \text{pois } e^{-\alpha a} | \varphi_0 \rangle = | \varphi_0 \rangle$$

$$= e^{-\alpha^2/2} \langle \varphi_n | \sum_{k=0}^{\infty} \frac{\alpha^k a^{+k}}{k!} | \varphi_0 \rangle$$

$$= e^{-\alpha^2/2} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} \sqrt{k!}\, \langle \varphi_n | \varphi_k \rangle = \frac{e^{-\alpha^2/2}\, \alpha^n}{\sqrt{n!}}$$

$$\boxed{C_n = \frac{e^{-\alpha^2/2}\, \alpha^n}{\sqrt{n!}}}$$

$$\gamma_n \equiv \langle \varphi_n | e^{i\lambda p/\hbar} | \varphi_1 \rangle = e^{-\alpha^2/2} \langle \varphi_n | e^{\alpha a^+} e^{-\alpha a} | \varphi_1 \rangle$$

$$e^{-\alpha a} | \varphi_1 \rangle = \left(1 - \alpha a + \frac{\alpha^2 a^2}{2!} \cdots \right) | \varphi_1 \rangle$$

$$= | \varphi_1 \rangle - \alpha | \varphi_0 \rangle$$

$$\gamma_n = e^{-\alpha^2/2} \left[\langle \varphi_n | e^{\alpha a^+} | \varphi_1 \rangle - \alpha \langle \varphi_n | e^{\alpha a^+} | \varphi_0 \rangle \right]$$

$$= e^{-\alpha^2/2} \langle \varphi_n | e^{\alpha a^+} | \varphi_1 \rangle - \alpha C_n$$

$$\gamma_n + \alpha C_n = e^{-\alpha^2/2} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} \langle \varphi_n | a^{+k} | \varphi_1 \rangle$$

$$= e^{-\alpha^2/2} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} \langle \varphi_n | \varphi_{k+1} \rangle \sqrt{(k+1)!}$$

$$= e^{-\alpha^2/2} \frac{\alpha^{n-1} \sqrt{n}}{\sqrt{(n-1)!}}$$

$$\boxed{\gamma_n = -\alpha C_n + \frac{e^{-\alpha^2/2} \alpha^{n-1} \sqrt{n}}{\sqrt{(n-1)!}}}$$

$$\langle \varphi_1 | \Psi(t) \rangle = \sum_n c_n \gamma_n e^{-iE_n t/\hbar}$$

$$= e^{-\frac{i\Delta E t}{\hbar} - \frac{i\omega t}{2}} \sum_n \left(\frac{e^{-\alpha^2/2} \alpha^n}{\sqrt{n!}} \right) \left(-\alpha \frac{e^{-\alpha^2/2} \alpha^n}{\sqrt{n!}} + \frac{e^{-\frac{\alpha^2}{2}} \alpha^{n-1} \sqrt{n}}{\sqrt{(n-1)!}} \right) \times e^{-in\omega t}$$

$$= e^{-i\beta t} e^{-\alpha^2} \sum_{n=0}^{\infty} \left[-\frac{\alpha^{2n+1}}{n!} + \frac{\alpha^{2n-1} n}{n!} \right] e^{-in\omega t} \qquad \boxed{\beta = \frac{\Delta E}{\hbar} + \frac{\omega}{2}}$$

- $$\sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{n!} e^{-i\omega t n} = \alpha \sum_{n=0}^{\infty} \frac{\left(\alpha^2 e^{-i\omega t}\right)^n}{n!} = \alpha e^{\alpha^2 e^{-i\omega t}}$$

- $$\sum_{n=1}^{\infty} \frac{\alpha^{2n-1}}{n!} n e^{-in\omega t} = \alpha \sum_{n=1}^{\infty} \frac{\alpha^{2(n-1)}}{(n-1)!} e^{-i(n-1)\omega t} e^{-i\omega t}$$

$$= \alpha e^{-i\omega t} \sum_{n=0}^{\infty} \frac{\left(\alpha^2 e^{-i\omega t}\right)^n}{n!} = \alpha e^{-i\omega t} e^{+\alpha^2 e^{-i\omega t}}$$

$$\langle \varphi_1 | \Psi(t) \rangle = e^{-i\beta t} e^{-\alpha^2} \left[-\alpha + \alpha e^{-i\omega t} \right] e^{\alpha^2 e^{-i\omega t}}$$

$$P_{0\to 1} = e^{-2\alpha^2} \alpha^2 \left| 1 - e^{-i\omega t} \right|^2 e^{2\alpha^2 \cos\omega t}$$

$$\boxed{P_{0\to 1} = 4\alpha^2 e^{-2\alpha^2} \sin^2\frac{\omega t}{2} \, e^{2\alpha^2 \cos\omega t}}$$

$$e^{2\alpha^2\cos\omega t - 2\alpha^2} = e^{-2\alpha^2(1-\cos\omega t)} = e^{-4\alpha^2\sin^2\frac{\omega t}{2}}$$

$$\boxed{P_{0\to 1} = 4\alpha^2 \sin^2\frac{\omega t}{2}\, e^{-4\alpha^2\sin^2\omega t/2}}$$

$$\alpha^2 = \frac{q^2\mathcal{E}^2}{2m\omega^3\hbar}$$

PARA $t > \tau$

$$|\psi(t)\rangle = \sum_n d_n\, e^{-iE_n^0 t/\hbar}\,|\varphi_n\rangle$$

e $\tilde{P}_{0\to 1} = |\langle\varphi_1|\psi(t)\rangle|^2 = |d_1|^2 =$ constante.

$$= P_{0\to 1}(\tau) = 4\alpha^2\sin^2\frac{\omega\tau}{2}\, e^{-4\alpha^2\sin^2\omega\tau/2}.$$