$H(t_n)a_n$; então o pulso completo será: $\sum H(t_n)a_n$.

Cada degrau tem uma solução do tipo

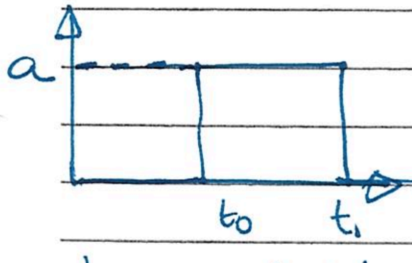
$$x_n(t) = \frac{a_n}{\omega_0^2}\left\{1 - e^{-\gamma(t-t_n)}\left[\cos\left(\omega_1(t-t_n)\right) + \frac{\gamma}{\omega_1}\operatorname{sen}\left(\omega_1(t-t_n)\right)\right]\right\}$$

p/ $t_n < t < t_{n+1}$

Sendo esta a solução em cada intervalo. A solução geral de cada contribuição será

$$x(t) = \sum_{n=0}^{n} x_n(t)$$; pois a equação é linear

<u>Exemplo Importante</u>: Um pulso quadrado



O pulso será dado então por
$H(t_0)a + H(t_1)(-a)$

$$H(t_0)a = \begin{cases} 0, & t < t_0 \\ a; & t \geq t_0 \end{cases}$$

$$H(t_1)(-a) = \begin{cases} 0, & t < t_1 \\ -a, & t \geq t_1 \end{cases}$$

$$H(t_0)a + H(t_1)(-a) = \begin{cases} 0, & t < t_0 \\ a, & t_0 \leq t \leq t_1 \\ 0, & t > t_1 \end{cases}$$

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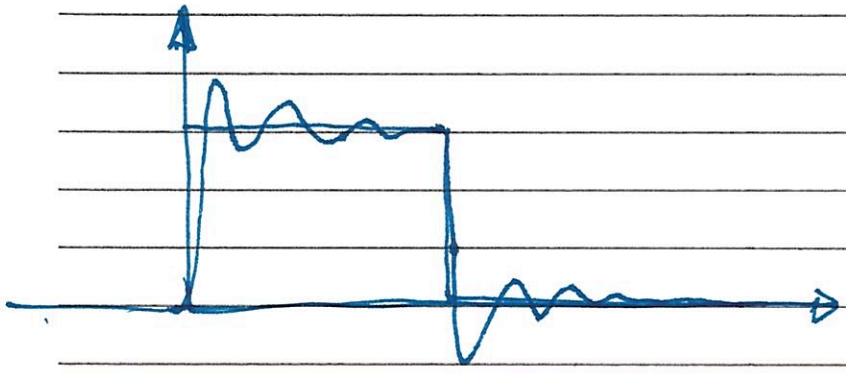
p/ $t > t_1$, a solução será:

$$x(t) = \sum_{m=0}^{1} x_m(t) = \frac{a}{\omega_0^2}\left\{ e^{-\gamma(t-t_1)}\left[\cos(\omega_1(t-t_1)) + \frac{\gamma}{\omega_1}\operatorname{sen}(\omega_1(t-t_1))\right] - e^{-\gamma(t-t_0)}\left[\cos(\omega_1(t-t_0)) + \frac{\gamma}{\omega_1}\operatorname{sen}\omega_1(t-t_0)\right]\right\}$$

Se $t_1 - t_0 = \tau$

$$x(t) = \frac{a}{\omega_0^2}\left\{ e^{-\gamma(t-t_0-\tau)}\left[\cos(\omega_1(t-t_0-\tau)) + \frac{\gamma}{\omega_1}\operatorname{sen}(\,)\right] - e^{-\gamma(t-t_0)}\left[\cos(\omega_1(t-t_0)) + \frac{\gamma}{\omega_1}\operatorname{sen}(\omega_1(t-t_0))\right]\right\}$$

$$\therefore x(t) = \frac{a e^{-\gamma(t-t_0)}}{\omega_0^2}\left[e^{\gamma\tau}\cos(\omega_1(t-t_0-\tau)) - \cos(\omega_1(t-t_0)) + \frac{\gamma e^{\gamma\tau}}{\omega_1}\sin(\omega_1(t-t_0-\tau)) - \frac{\gamma}{\omega_1}\sin(\omega_1(t-t_0))\right], \quad t > t_1$$



Um caso interessante será p/ a função pulso quando

$t_1 - t_0 \equiv \tau \to 0 \quad \therefore$

a Solução anterior nos fica fazendo $t - t_0 \equiv t'$ e

lembrando que $\cos\omega_1(t'-\tau) = \cos\omega_1 t' \cos\tau\omega_1 + \operatorname{sen}\omega_1 t' \operatorname{sen}\tau\omega_1$

$\operatorname{sen}\omega_1(t'-\tau) = \operatorname{sen}\omega_1 t' \cos\tau\omega_1 - \operatorname{sen}\omega_1\tau \cos\omega_1 t'$

$$\boxed{x(t) = \frac{a}{\omega_0^2} e^{-\gamma t'} \Big\{ e^{\gamma\tau}\left[\cos\omega_1 t' \cos\omega_1\tau + \operatorname{sen}\omega_1 t' \operatorname{sen}\tau\omega_1\right] + \frac{\gamma}{\omega_1} e^{\gamma\tau}\left[\operatorname{sen}\omega_1 t' \cos\omega_1\tau - \operatorname{sen}\omega_1\tau \cos\omega_1 t'\right] - \cos\omega_1 t' - \frac{\gamma}{\omega_1}\operatorname{sen}\omega_1 t' \Big\}}$$

Fazendo

$$\boxed{\begin{array}{ll} e^{\gamma\tau} \sim 1 + \gamma\tau & \\ \cos\omega_1\tau \sim 1 & \omega_1\tau \ll 2\pi \\ \operatorname{sen}\omega_1\tau \sim \omega_1\tau & \end{array}}$$

Temos:

$$x(t) = \frac{a}{\omega_0^2} e^{-\gamma t'} \Big\{ (1+\gamma\tau)\left[\cos\omega_1 t' + \omega_1\tau \operatorname{sin}\omega_1 t'\right] + \frac{\gamma}{\omega_1}(1+\gamma\tau)\left[\operatorname{sen}\omega_1 t' - \omega_1\tau\cos\omega_1 t'\right] - \cos\omega_1 t' - \frac{\gamma}{\omega_1}\operatorname{sen}\omega_1 t' \Big\}$$

$$x(t) = \frac{a}{\omega_0^2} e^{-\gamma t'} \Big\{ \cancel{\cos \omega_1 t'} + \omega_1 \tau \sin \omega_1 t' + \cancel{\gamma \tau \cos \omega_1 t'} + \underbrace{\cancel{\gamma \tau^2 \omega_1 \sin \omega_1 t'}}_{0} + \cancel{\frac{\gamma}{\omega_1} \sin \omega_1 t'} +$$

$$- \cancel{\gamma \tau \cos \omega_1 t'} + \frac{\gamma^2 \tau}{\omega_1} \operatorname{sen} \omega_1 t' - \underbrace{\cancel{\gamma^2 \tau^2 \cos \omega_1 t'}}_{0} \cancel{- \cos \omega_1 t'}$$

$$- \cancel{\frac{\gamma}{\omega_1} \operatorname{sen} \omega_1 t'} \Big\}$$

Desprezando termos em segundo ordem em τ^2 ou superiores pois se $\tau \to 0$; $\tau^2 \to 0$ mais rapidamente...

$$x(t) = \frac{a\, e^{-\gamma t'}}{\omega_0^2} \Big\{ \frac{\gamma^2 \tau}{\omega_1} \sin \omega_1 t' + \omega_1 \tau \sin \omega_1 t' \Big\}$$

$$x(t) = \frac{a \tau e^{-\gamma t'}}{\cancel{\omega_0^2}} \frac{\operatorname{sen} \omega_1 t'}{\omega_1} \Big[\underbrace{\cancel{\gamma^2 + \omega_1^2}}_{\omega_0^2} \Big]$$

$$\therefore \boxed{x(t) = \frac{a \tau}{\omega_1} e^{-\gamma t'} \operatorname{sen} \omega_1 t'}$$

veja página 82 das notas de aula!

~~Esta~~ A solução de um pulso de Largura τ com $\tau \to 0$ e $\omega_1 \tau \ll 2\pi$; ocorrendo em

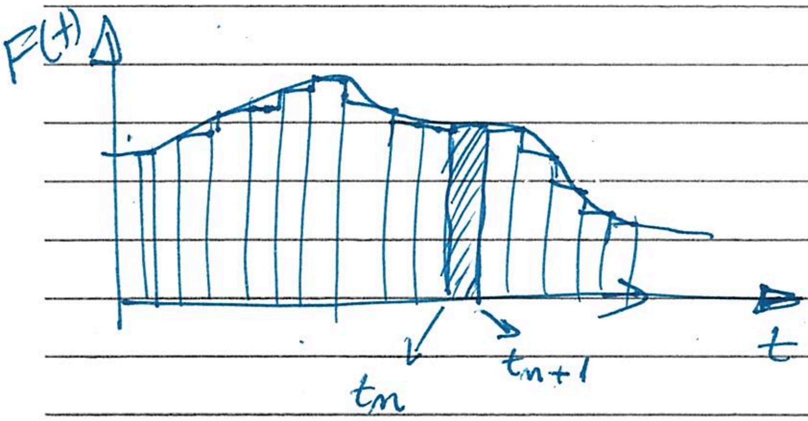
$t = t_0$ e terminando em $t \geq t_0 + \tau$ será:

$$\boxed{x(t) = \frac{a\tau}{\omega_1} e^{-(t-t_0)} \sin \omega_1 (t - t_0)}$$

$t > t_0 + \tau$

$\tau \to 0$

Podemos usar a função impulso como uma forma de resolver forças generalizadas $\frac{F(t)}{m}$ como uma soma de Impulsos



$$\ddot{x} + 2\sigma \dot{x} + \omega_0^2 x = \sum_{n=-\infty}^{\infty} F_n(t) = \sum_{n=-\infty}^{\infty} I_m(t) = F(t)$$

$$I_n(t) = I(t_n, t_{n+1}) = \begin{cases} a_n(t_n) \,; & t_n < t < t_{n+1} \\ 0 & \text{fora do intervalo.} \end{cases}$$

Se a solução de um impulso $t_n - t_{n+1} = \tau$, $\tau \ll \frac{2\pi}{\omega_1}$

$$x_n(t) = \frac{a_n(t_n)}{\omega_1} \tau e^{-\sigma(t-t_n)} \sin \omega_1 (t - t_n) \quad t \geq t_n + \tau$$

A solução p/ todos os impulsos até N será a soma:

$$\boxed{x(t) = \sum_{n=-\infty}^{N} \frac{a_n(t_n)\tau}{\omega_1} e^{-\gamma(t-t_n)} \sin(\omega_1 (t-t_n))}$$

$$, \quad t_n < t < t_{n+1}$$

Fazendo $\tau \to 0$ $\sum \to \int$

$$\boxed{x(t) = \int_{-\infty}^{t} \frac{a(t')}{\omega_1} e^{-\gamma(t-t')} \sin\omega_1(t-t')\, dt'}$$

Se definirmos

$$G(t,t') \equiv \begin{cases} \frac{1}{m\omega_1} e^{-\gamma(t-t')} \sin\omega_1(t-t') ; & t \geq t' \\ 0 & t < t' \end{cases}$$

$\Rightarrow$ função de Green

então:

$$\boxed{x(t) = \int_{-\infty}^{t} F(t') G(t,t')\, dt'}$$

$\Downarrow$

Será a solução de um oscilador

$$\boxed{m\ddot{x} + b\dot{x} + kx = F(t)}$$

Sendo esta válida p/ o caso de
$x(0) = x_0$ (equilíbrio) e $v(0) = 0$

Note que a solução de Green como foi derivada, já contém as condições iniciais.

Exemplo// Encontrar $x(t)$ p/ uma força do tipo

$F(t) = F_0 e^{-\alpha t}$ p/ $t > 0$ e começando em $t=0$

Usando o método de Green.

$$x(t) = \frac{F_0}{m\omega_1}\int_0^t e^{-\alpha t'} e^{-\gamma(t-t')} \operatorname{sen}[\omega_1(t-t')]\,dt'$$

$\therefore z = \omega_1(t-t') \Rightarrow dz = -\omega_1 dt'$

$$x(t) = \frac{-F_0}{m\omega_1^2}\int_{\omega_1 t}^{0} e^{-\alpha t} e^{\left[\frac{(\alpha-\gamma)}{\omega_1}\right]z} \operatorname{sen} z\,dz$$

Fazendo $k = \frac{\alpha-\gamma}{\omega_1}$

$$x(t) = -\frac{F_0}{m\omega_1^2} e^{-\alpha t}\int_{\omega_1 t}^{0} e^{kz} \sin z\,dz$$

$\int u\,dv = uv - \int v\,du$ $\qquad u = e^{kz}$; $dv = \operatorname{sen} z\,dz$

$$\int e^{kz} \operatorname{sen} z\, dz = -e^{kz}\cos z + k\int \cos z\, e^{kz}\, dz$$

$$'' = -e^{kz}\cos z + k e^{kz}\sin z - k^2 \int e^{kz}\sin z\, dz$$

$$\cancel{(1+k^2)} \int_{\omega_1 t}^{0} e^{kz} \operatorname{sen} z\, dz = \frac{e^{kz}}{(k^2+1)}\left[k\sin z - \cos z\right]\Big|_{\omega_1 t}^{0}$$

$$\int_{\omega_1 t}^{0} e^{kz} \operatorname{sen} z\, dz = \frac{1}{k^2+1}\left[-1 - e^{k\omega_1 t}\left(k\sin\omega_1 t - \cos\omega_1 t\right)\right]$$

$$\boxed{x(t) = \frac{F_0/m}{(\alpha-\gamma)^2+\omega_1^2}\left[e^{-\alpha t} - e^{-\gamma t}\left(\cos\omega_1 t - \left(\frac{\alpha-\gamma}{\omega_1}\right)\sin\omega_1 t\right)\right]}$$