MFT - 2nd Q VII - 6

2 partículas diferentes

$$H = H_0 + V_{int}$$

$$H_0 = \sum_\nu \xi_\nu^a a_\nu^\dagger a_\nu + \sum_\mu \xi_\mu^b b_\mu^\dagger b_\mu$$

$$V_{int} = \sum_{\nu\nu'\mu\mu'} V_{\nu\mu,\nu'\mu'} a_\nu^\dagger b_\mu^\dagger b_{\mu'} a_{\nu'}$$

lembrando

$$V_{\nu\mu,\nu'\mu'} = \int d\vec{r}_1 d\vec{r}_2 \, \psi_\nu^\dagger(\vec{r}_1) \varphi_\mu^\dagger(\vec{r}_2) V(\vec{r}_1,\vec{r}_2) \psi_{\nu'}(\vec{r}_1) \varphi_{\mu'}(\vec{r}_2)$$

$\psi^\dagger \to a^\dagger$

$\varphi^\dagger \to b^\dagger$

$a_\nu^\dagger a_{\nu'}$ e $b_\mu^\dagger b_{\mu'}$ desviam pouco da média

$$d_{\nu\nu'} = a_\nu^\dagger a_{\nu'} - \langle a_\nu^\dagger a_{\nu'} \rangle$$

$$e_{\nu\nu'} = b_\mu^\dagger b_{\mu'} - \langle b_\mu^\dagger b_{\mu'} \rangle$$

$$\Rightarrow H = H_0 + V_{MF} + \sum_{\nu\nu'\mu\mu'} \underbrace{V_{\nu\mu,\nu'\mu'} d_{\nu\nu'} e_{\mu\mu'}}_{\text{peq-eno}}$$

$$V_{MF} = \sum_{\nu\nu'\mu\mu'} V_{\nu\mu\nu'\mu'} \left[a_\nu^\dagger a_{\nu'} \langle b_\mu^\dagger b_{\mu'} \rangle + b_\mu^\dagger b_{\mu'} \langle a_\nu^\dagger a_{\nu'} \rangle \right]$$

$$- \sum_{\nu\nu'\mu\mu'} V_{\nu\mu\nu'\mu'} \langle a_\nu^\dagger a_{\nu'} \rangle \langle b_\mu^\dagger b_{\mu'} \rangle$$

$H_{MF} = H_0 + V_{MF}$

$\downarrow$

hamilt. de 1 partícula $\rightarrow$ diagonalizável simples

— o —

Formulação MF:

$H_{AB} = AB$

$\langle H_{AB} \rangle = \langle A \rangle \langle B \rangle$

previne dupla contagem

$\downarrow$

$\Rightarrow H_{AB}^{MF} = A \langle B \rangle + \langle A \rangle B - \langle A \rangle \langle B \rangle$

como calcular

$\langle a_\nu^\dagger a_{\nu'} \rangle$?

$\langle b_\mu^\dagger b_{\mu'} \rangle$?

Método 1 auto consistente

$$\langle n^a_{\nu\nu'} \rangle = \langle a^\dagger_\nu a_{\nu'} \rangle_{MF} = \frac{1}{Z_{MF}} \, Tr\left(e^{-\beta H_{MF}} a^\dagger_\nu a_{\nu'}\right)$$

$$Z_{MF} = Tr\left(e^{-\beta H_{MF}}\right)$$

$\langle n^b_{\mu\mu'} \rangle \rightarrow$ similar

$\rightarrow$ eqs. autoconsistentes

Método 2 Minimização da energia livre F_{MF}

$$F_{MF} = -\frac{1}{\beta} \ln Z_{MF}$$

$$\frac{d}{d\langle n^a_{\nu\nu'}\rangle} F_{MF} = 0$$

$$\Rightarrow \frac{1}{Z_{MF}} Tr\left(e^{-\beta H_{MF}} \frac{d}{d\langle n^a_{\nu\nu'}\rangle} H_{MF}\right)$$

$$= \frac{1}{Z_{MF}} Tr\left[e^{-\beta H_{MF}} \left(\sum_{\mu\mu'} V_{\nu\mu\nu'\mu'} \left(b^\dagger_\mu b_{\mu'} - \langle n^b_{\mu\mu'}\rangle\right)\right)\right]$$

como deve valer p/ tal por $\nu\nu'$,

$$\langle n^b_{\mu\mu'} \rangle = \langle b^\dagger_\mu b_{\mu'} \rangle_{MF} =$$

$$= \frac{1}{Z_{MF}} \, Tr\left(e^{-\beta H_{MF}} b^\dagger_\mu b_{\mu'} \right)$$

Significado físico MF:

$$\langle V_{int} \rangle \approx \sum_{\nu\nu'\mu\mu'} V_{\nu\nu'\mu\mu'} \langle a^\dagger_\nu a_{\nu'} \rangle \langle b^\dagger_\mu b_{\mu'} \rangle$$

$a^\dagger$ e $b^\dagger$ não correlacionados!

$$\langle V_{int} \rangle_{MF} = \frac{1}{Z_{MF}} \, Tr\left(e^{-\beta H_{MF}} V_{int} \right)$$

mas $H_{MF} = H^a_{MF} + H^b_{MF}$

$$\Rightarrow \langle V_{int} \rangle_{MF} = \sum_{\nu\nu'\mu\mu'} V_{\nu\mu\nu'\mu'} \langle a^\dagger_\nu a_{\nu'} \rangle_{MF} \langle b^\dagger_\mu b_{\mu'} \rangle_{MF}$$

MF $\longrightarrow$ correlações não são importantes

Vamos olhar o caso de uma única partícula

$$H = H_0 + V_{int}$$

$$H_0 = \sum_{\nu} \xi_\nu c_\nu^\dagger c_{\nu'}$$

$$V_{int} = \frac{1}{2} \sum_{\nu\nu'\mu\mu'} V_{\nu\mu\nu'\mu'} c_\nu^\dagger c_\mu^\dagger c_{\mu'} c_{\nu'}$$

$$\rho_{\mu\mu'} = c_\mu^\dagger c_{\mu'} \longrightarrow \text{grande qd. } \langle \rho_{\mu\mu'} \rangle \neq 0$$

maior parte dos casos $\langle \rho_{\mu\mu'} \rangle \to 0$

Seguindo os mesmos passos

$$c_\nu^\dagger \left(c_\mu^\dagger c_{\mu'} - \langle c_\mu^\dagger c_{\mu'} \rangle \right) c_{\nu'} + c_\nu^\dagger c_{\nu'} \langle c_\mu^\dagger c_{\mu'} \rangle$$

<u>Se</u> $\nu' \neq \mu$ podemos comutar e

$$c_\nu^\dagger c_{\nu'} \left(c_\mu^\dagger c_{\mu'} - \langle c_\mu^\dagger c_{\mu'} \rangle \right) + c_\nu^\dagger c_{\nu'} \langle c_\mu^\dagger c_{\mu'} \rangle$$

$\downarrow$

mesma substituição

$$\left(c_\nu^\dagger c_{\nu'} - \langle c_\nu^\dagger c_{\nu'} \rangle \right) \left(c_\mu^\dagger c_{\mu'} - \langle c_\mu^\dagger c_{\mu'} \rangle \right)$$

$$+ c_\nu^\dagger c_{\nu'} \langle c_\mu^\dagger c_{\mu'} \rangle + c_\mu^\dagger c_{\mu'} \langle c_\nu^\dagger c_{\nu'} \rangle$$

$$- \langle c_\mu^\dagger c_{\mu'} \rangle \langle c_\nu^\dagger c_{\nu'} \rangle$$

desprezando δ^2 (~~...~~ 1º termo)

$$\Rightarrow V_{int}^{Hartree} = \frac{1}{2}\sum V_{\nu\mu\nu'\mu'} \langle n_{\mu\mu'} \rangle c_\nu^+ c_{\nu'} +$$

$$+\frac{1}{2}\sum V_{\nu\mu\nu'\mu'} \langle n_{\nu\nu'} \rangle c_\mu^+ c_{\mu'}$$

$$-\frac{1}{2}\sum V_{\nu\mu\nu'\mu'} \langle n_{\mu\mu'} \rangle \langle n_{\nu\nu'} \rangle$$

O que falta? as partículas são <u>indistinguíveis</u> e o resultado obtido trata $\mu\mu'$ e $\nu\nu'$ como partículas distintas.

o que acontece qd. $\langle c_\nu^\dagger c_{\mu'} \rangle$ é finito?

$$V_{int} = \frac{1}{2} \sum V_{\nu\mu\nu'\mu'} \left[-c_\nu^\dagger c_{\mu'} c_\mu^\dagger c_{\nu'} + c_\nu^\dagger c_{\nu'} \right]$$

$$-c_\nu^\dagger c_{\mu'} c_\mu^\dagger c_{\nu'} = -\left(c_\nu^\dagger c_{\mu'} - \langle c_\nu^\dagger c_{\mu'} \rangle\right) c_\mu^\dagger c_{\nu'}$$

$$- \langle c_\nu^\dagger c_{\mu'} \rangle c_\mu^\dagger c_{\nu'}$$

$$= \underbrace{-\left(c_\nu^\dagger c_{\mu'} - \langle c_\nu^\dagger c_{\mu'} \rangle\right)\left(c_\mu^\dagger c_{\nu'} - \langle c_\mu^\dagger c_{\nu'} \rangle\right)}_{\sim 0}$$

$$- \langle c_\nu^\dagger c_{\mu'} \rangle c_\mu^\dagger c_{\nu'} - c_\nu^\dagger c_{\mu'} \langle c_\mu^\dagger c_{\nu'} \rangle$$

$$+ \langle c_\nu^\dagger c_{\mu'} \rangle \langle c_\mu^\dagger c_{\nu'} \rangle$$

desprezando $\mu = \mu'$ (i.e., $c_\nu^\dagger c_{\nu'}$)

temos

$$V_{int}^{Fock} = -\frac{1}{2} \sum V_{\nu\mu\nu\mu'} \langle n_{\nu\mu'} \rangle c_\mu^\dagger c_{\nu'}$$

$$- \frac{1}{2} \sum V_{\nu\mu\nu\mu'} \langle n_{\nu'\mu} \rangle c_{\mu'}^\dagger c_\mu$$

$$+ \frac{1}{2} \sum V_{\nu\mu\nu\mu'} \langle n_{\nu\mu'} \rangle \langle n_{\nu'\mu} \rangle$$

Ex: gas de e^- homogénea

$$H^{HF} = \sum_{k\sigma} \xi_k^{HF} c_{k\sigma}^{\dagger} c_{k\sigma}$$

$$\xi_k^{HF} = \xi_{k^{-1}} + \sum_{k'\sigma'} \left[V(0) - \delta_{\sigma\sigma'} V(k^{-1} - k'^{-1}) \right] n_{k'^{-1}\sigma'}$$

$$= \xi_{k^{-1}} + V(0) N - \sum_{k'\sigma'} V(k^{-1} - k'^{-1}) n_{k'^{-1}\sigma'}$$

cancela el int. c/ fondo positivo

fermo de trasc

Quebra de simetria

Ex: cristal

Operador translacional: $T(\vec{R})$

$$[H, T(\vec{R})] = 0$$

$$T = e^{i\vec{P}\cdot\vec{R}}$$

$\vec{P}$ = total momento $= \hbar \sum_{k\sigma} \vec{k}\, c^{\dagger}_{k\sigma} c_{k\sigma}$

soma de estados c/ $\vec{P}$ definido

cristal parâmetro de ordem onda de densidade:

$$\rho(\vec{Q}) = \sum_{k\sigma} c^{\dagger}_{k\sigma} c_{k+\vec{Q}}$$

$$\langle c^{\dagger}_{k\sigma} c_{k+\vec{Q}} \rangle = \frac{1}{Z_{MF}} \sum_{\vec{P}} e^{-\beta E_P} \langle \vec{P} | c^{\dagger}_{k\sigma} c_{k+\vec{Q}} | P \rangle \neq 0$$

$$c^{\dagger}_{k} c_{k+\vec{Q}} |P\rangle \rightarrow |P+\vec{Q}\rangle \Rightarrow \text{ortogonal} \neq |\vec{P}\rangle !$$

Ex:

$$\langle c_{\vec{k}}^{\dagger} c_{\vec{k}'} \rangle = \int d\vec{r} \int d\vec{r}' \, e^{-i\vec{k}\cdot\vec{r}'} e^{i\vec{k}\cdot\vec{r}} \langle \psi^{\dagger}(\vec{r}) \psi(\vec{r}') \rangle$$

homogeneas

$$\langle \psi(\vec{r}) \psi^{\dagger}(\vec{r}') \rangle = f(\vec{r}-\vec{r}') \Rightarrow \langle c_{\vec{k}}^{\dagger} c_{\vec{k}'} \rangle = \langle n_{\vec{k}} \rangle \delta_{\vec{k}\vec{k}'}$$

mas se há quebra espontânea de simetria

$$\langle \psi(\vec{r}) \psi(\vec{r}') \rangle \not\rightarrow f(\vec{r}-\vec{r}')$$

$$\Downarrow$$

$$h(\vec{r}, \vec{r}')$$

Ex: cristal $h(\vec{r}, \vec{r}') = h(\vec{r}+\vec{R}, \vec{r}'+\vec{R})$

temos que observar que a simetria é quebrada!

Ex: modelo de Ising

$$H = -2J \sum_{\langle ij \rangle} \hat{\sigma}_i \hat{\sigma}_j$$

$$H \sim H_{MF} = -2J \sum_{\langle ij \rangle} \langle \sigma_i \rangle \sigma_j - 2J \sum_{\langle ij \rangle} \sigma_i \langle \sigma_j \rangle$$

$$+ 2J \sum_{\langle ij \rangle} \langle \sigma_i \rangle \langle \sigma_j \rangle$$

$\langle \sigma_i \rangle \neq 0$!

$$m = 2J \sum_j \langle \sigma_j \rangle = 2 z J \langle \sigma \rangle$$

$$\Rightarrow H_{MF} = -2 \sum_i m \sigma_i + m N \langle \sigma \rangle$$

$$Z_{MF} = \left(e^{2\beta m} + e^{-2\beta m}\right)^N e^{\beta N m \langle \sigma \rangle}$$

$$= \left[\left(e^{2\beta m} + e^{-2\beta m}\right) e^{\beta m^2 / 2zJ}\right]^N$$

$$\frac{\partial F_{MF}}{\partial m} = -\frac{1}{\beta} \frac{\partial}{\partial m} \ln Z_{MF} = -N \left(\frac{e^{2\beta m} + e^{-2\beta m}}{e^{2\beta m} + e^{-2\beta m}} \right) +$$

$$+ N \frac{m}{z} = 0$$

eq. transc.

$$\alpha = \operatorname{tgh} b\alpha$$

$$\alpha = \frac{m}{zJ}$$

$$b = zJ\beta$$

ρ / α pegen

$$\alpha \approx b\alpha - \frac{1}{3}(b\alpha)^3$$

$$\Rightarrow k_B T_c = zJ$$

$$\alpha \approx \frac{1}{b}\sqrt{\frac{3(b-1)}{b}}$$

$$\Rightarrow m \approx zJ\sqrt{\frac{3(T_c - T)}{T_c}}$$

Modelo de Stoner p/ magnetismo

(1)

Magnetismo: camada d

s-p deslocalizados → não geram interação suficiente

Gás de elétrons pode desempenhar um papel

→ magnetismo localizado x deslocalizado.

Modelo: Hubbard

interação $U\delta(\vec{r}) \longrightarrow U \equiv$ cte (no espaço $\vec{k}$)

$$H_H = \sum_{\vec{k}\sigma} \xi_{\vec{k}} c^{\dagger}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \frac{U}{2V} \sum_{\vec{k}\vec{k}'\sigma\sigma'} c^{\dagger}_{\vec{k}+\vec{q}\sigma} c^{\dagger}_{\vec{k}'-\vec{q}\sigma'} c_{\vec{k}'\sigma'} c_{\vec{k}\sigma}$$

HF mas com valores médios dependentes do spin:

$$\langle c^{\dagger}_{\vec{k}\uparrow} c_{\vec{k}'\uparrow} \rangle = \delta_{\vec{k}\vec{k}'} \bar{n}_{\vec{k}\uparrow}$$

$$\langle c^{\dagger}_{\vec{k}\downarrow} c_{\vec{k}'\downarrow} \rangle = \delta_{\vec{k}\vec{k}'} \bar{n}_{\vec{k}\downarrow}$$

$$\Rightarrow \text{MFT: } V^{MF}_{int} = \frac{U}{V} \sum_{\vec{k}\vec{k}'\vec{q}\sigma\sigma'} c^{\dagger}_{\vec{k}+\vec{q}\sigma} \langle c^{\dagger}_{\vec{k}'-\vec{q}\sigma'} c_{\vec{k}'\sigma'} \rangle c_{\vec{k}\sigma}$$

$$- \frac{U}{V} \sum_{\dots} \langle c^{\dagger}_{\vec{k}+\vec{q}\sigma} c_{\vec{k}'\sigma'} \rangle c^{\dagger}_{\vec{k}'-\vec{q}\sigma'} c_{\vec{k}\sigma}$$

$$- \frac{U}{2V} \sum_{\dots} \Big[\langle c^{\dagger}_{\vec{k}+\vec{q}\sigma} c_{\vec{k}\sigma} \rangle \langle c^{\dagger}_{\vec{k}'-\vec{q}\sigma} c_{\vec{k}'\sigma'} \rangle -$$

$$- \langle c^{\dagger}_{\vec{k}+\vec{q}\sigma} c_{\vec{k}'\sigma'} \rangle \langle c^{\dagger}_{\vec{k}'-\vec{q}\sigma'} c_{\vec{k}\sigma} \rangle \Big]$$

Com as approx. MFT,

$$V_{int}^{MF} = U \sum_{k\sigma\sigma'} c_{k\sigma}^{+} \sigma_{k\sigma} \left[\bar{n}_{\sigma'} - \bar{n}_{\sigma} \delta_{\sigma\sigma'} \right] - UV \sum_{\sigma\sigma'} \bar{n}_{\sigma} \bar{n}_{\sigma'}$$
$$+ UV \sum_{\sigma} \bar{n}_{\sigma}^{2}$$

onde

$$\bar{n}_{\sigma} = \frac{1}{V} \sum_{\vec{k}} \langle c_{\vec{k}\sigma}^{+} c_{\vec{k}\sigma} \rangle$$

$$\Rightarrow H^{MF} = \sum_{\vec{k}\sigma} \xi_{\vec{k}\sigma}^{MF} c_{k\sigma}^{+} c_{k\sigma} - \frac{UV}{2} \sum_{\sigma\sigma'} \bar{n}_{\sigma} \bar{n}_{\sigma'} + \frac{UV}{2} \sum_{\sigma} \bar{n}_{\sigma}^{2}$$

onde $\xi_{k\sigma}^{MF} = \xi_{\vec{k}} + U(\bar{n}_{\uparrow} + \bar{n}_{\downarrow} - \bar{n}_{\sigma}) = \xi_{\vec{k}} + U\bar{n}_{\bar{\sigma}}$

$\cancel{\frac{\partial H^{MF}}{\partial \bar{n}_{\sigma}}}$ $\cancel{\bar{F}} =$

$$F_{MF} = -\frac{1}{B} \ln Z_{MF} = -\frac{1}{B} \ln \left(\mathrm{Tr}\, e^{-BH_{MF}} \right)$$

$$\bar{n}_{\sigma} = \frac{1}{V} \sum_{\vec{k}} \langle c_{\vec{k}\sigma}^{+} c_{\vec{k}\sigma} \rangle_{MF} = \frac{1}{Z_{MF}} \frac{1}{V} \sum_{\vec{k}} \left(\mathrm{Tr}\, e^{-BH_{MF}} c_{k\sigma}^{+} c_{\vec{k}\sigma} \right)$$

$$\frac{\partial F_{MF}}{\partial \bar{n}_{\sigma}} = \frac{d}{d\bar{n}_{\sigma}} -\frac{1}{B} \ln \left(\mathrm{Tr}\, e^{BH_{MF}} \right) = \frac{1}{Z_{MF}} \mathrm{Tr} \left(e^{-BH_{MF}} \frac{d}{d\bar{n}_{\sigma}} H_{MF} \right)$$

$$\cancel{\frac{d H_{MF}}{d\bar{n}_{\sigma}} = -\frac{UV}{2} \Sigma}$$

$$H_{MF} = \sum_{\vec{k}} \left(\xi^{MF}_{k\uparrow} c^{+}_{k\uparrow} c_{k\uparrow} \cancel{- \frac{UV}{2}} + \xi^{MF}_{k\downarrow} c^{+}_{k\downarrow} c_{k\downarrow} \right)$$

$$- \frac{UV}{2} \left(\cancel{\bar{n}_\uparrow \bar{n}_\uparrow} + \bar{n}_\uparrow \bar{n}_\downarrow + \bar{n}_\downarrow \bar{n}_\uparrow + \cancel{\bar{n}_\downarrow \bar{n}_\downarrow} \right) + \frac{UV}{2} \left(\cancel{\bar{n}_\uparrow^2} + \cancel{\bar{n}_\downarrow^2} \right)$$

$$\xi^{MF}_{k\uparrow} = \xi_k + U \bar{n}_\downarrow$$

$$\cancel{\frac{\partial H_{MF}}{\partial \bar{n}_q} = \sum_k \xi_k U n_{k\downarrow} n}$$

$$\frac{\partial H_{MF}}{\partial \bar{n}_\uparrow} = \sum_{\vec{k}} U c^{+}_{k\downarrow} c_{k\downarrow} - UV \bar{n}_\downarrow = 0$$

$$\Rightarrow \quad n_\downarrow = \frac{1}{V} \sum_{\vec{k}} \langle c^{+}_{k\downarrow} c_{k\downarrow} \rangle_{MF}$$

A $T=0$

$$\bar{n}_\uparrow = \frac{1}{6\pi^2} k_{F\uparrow}^3 = \int \frac{d\vec{k}}{(2\pi)^3} \gamma \Big(\underbrace{\frac{\hbar^2 k_F^2}{2m} + U\bar{n}_\downarrow}_{\mu} - \frac{\hbar^2 k^2}{2m} - U\bar{n}_\downarrow \Big)$$

$$= \frac{4\pi}{(2\pi)^3} \int k^2 dk \, \gamma \left(\mu - \frac{\hbar^2 k^2}{2m} - U\bar{n}_\downarrow \right)$$

$$= \frac{1}{2\pi^2} \cdot \frac{k^3}{3} \Bigg|_0^{\frac{2m}{\hbar^2}(\mu - U\bar{n}_\downarrow)^{1/2}} = \frac{1}{6\pi^2} \left[\frac{2m}{\hbar^2} \left(\mu - U\bar{n}_\downarrow \right) \right]^{3/2} = \bar{n}_\uparrow$$

$$(6\pi^2)^{2/3} \bar{n}_\uparrow^{2/3} + U\bar{n}_\downarrow = \mu$$

$$\approx (6\pi^2)^{2/3} \bar{n}^{2/3} \ldots$$

$$\rho = \frac{n_\uparrow - n_\downarrow}{n}$$

$$\gamma = \frac{2m\, U n^{1/3}}{(3\pi^2)^{2/3}\, \hbar^2}$$

$$n = n_\uparrow + n_\downarrow$$

$$n_\uparrow^{2/3} - n_\downarrow^{2/3} = \frac{2m}{\hbar^2} \frac{1}{(6\pi^2)^{2/3}}\, U(n_\downarrow - n_\uparrow)$$

$$(1+\rho)^{2/3} - (1-\rho)^{2/3} = \frac{2m}{\hbar^2} \frac{U}{(3\pi^2)^{2/3}} \frac{(n_\downarrow - n_\uparrow)}{n^{2/3}}$$

$$= \frac{2m}{\hbar^2} \frac{U n^{1/3}}{(3\pi^2)^{2/3}}\, \rho = \gamma \rho$$

$$\Rightarrow \boxed{(1+\rho)^{2/3} - (1-\rho)^{2/3} = \gamma \rho}$$

3 tipos de solução:

$\gamma < \frac{4}{3}$: isotrópico : $\rho = 0$ → est. normal

$\frac{4}{3} < \gamma < 2^{2/3}$: polarização parcial : $0 < \rho < 1$ ferromagnético fracos

$\gamma > 2^{2/3}$: polarização completa : $\rho = 1$ ferromagn. forte

resultados: interação de curto alcance e de troca

e elétrons não podem ocupar o mesmo estado!

Como $U(\vec{r}) \to U\rho(\vec{r})$ → eles não interagem

γ depende do material.

~~$\gamma = 1$~~

(δ max) $\delta = 1 \Rightarrow 2^{2/3} = \gamma$

δ min: $\delta = 0 \Rightarrow$?

δ muito pequeno:

$$(1+\delta)^{2/3} \cong 1 + \frac{2}{3}\delta$$

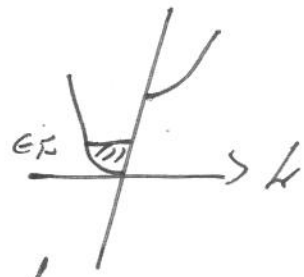
$$1 + \frac{2}{3}\delta - \left(1 - \frac{2}{3}\delta\right) = \delta\gamma$$

$$\Rightarrow \frac{4}{3}\delta = \delta\gamma \Rightarrow \gamma = \frac{4}{3}$$

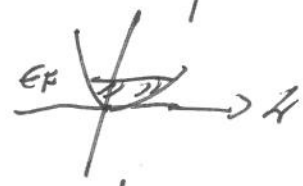
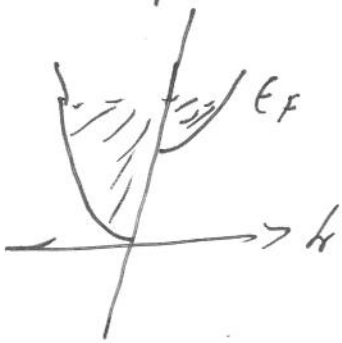
$$\delta \to 0 \Rightarrow \gamma \to \frac{4}{3}$$

$$\Rightarrow \frac{4}{3} < \gamma < 2^{2/3} \Rightarrow 0 < \delta < 1$$

$\delta = 1 \Rightarrow$ ferromagnetismo forte



$\delta = 0 \Rightarrow$ não-ferromagnético



$0 < \delta < 1$ ferromagn. fraco