$$
\begin{aligned}
&a \equiv a(t) \equiv \frac{dv}{dt} \\
&\Rightarrow \int_{t_i}^{t} \frac{dv}{dt} dt = \int_{t_i}^{t} a(t) dt \\
&\Rightarrow v(t) - v(t_i) = \int_{t_i}^{t} a(t) dt
\end{aligned}
$$

Para $a \equiv$ constante,

$$
v(t) = v_i + a(t)(t - t_i)
$$

$$
\begin{aligned}
&\frac{d^2x}{dt^2} \equiv a(t) \\
&\Rightarrow \int_{t_i}^{t} \frac{d^2x}{dt^2} dt = \int_{t_i}^{t} a(t) dt \\
&\Rightarrow \left[\frac{dx}{dt}\right]_t - \left[\frac{dx}{dt}\right]_{t_i} = \frac{dx(t)}{dt} - v_i = \int_{t_i}^{t} a(t) dt \\
&\Rightarrow \int_{t_i}^{t} \frac{dx(t')}{dt'} dt' = \int_{t_i}^{t} v_i + \int_{t_i}^{t} dt' \int_{t_i}^{t'} a(t'') dt'' \\
&\Rightarrow x(t) = x_i + v_i(t - t_i) + \int_{t_i}^{t} dt' \int_{t_i}^{t'} a(t'') dt''
\end{aligned}
$$

Para $a \equiv$ constante,

$$
\begin{aligned}
x(t) &= x_i + v_i(t - t_i) + a \int_{t_i}^{t} dt' \int_{t_i}^{t'} dt'' \\
&= x_i + v_i(t - t_i) + a \int_{t_i}^{t} dt'(t' - t_i) \\
&= x_i + v_i(t - t_i) + a[\frac{t'^2}{2} - t' t_i]_{t_i}^{t} \\
&= x_i + v_i(t - t_i) + a[\frac{1}{2}t^2 - t t_i - \frac{1}{2}t_i^2 + t_i^2] \\
&= x_i + v_i(t - t_i) + \frac{1}{2}a(t - t_i)^2
\end{aligned}
$$