PROBLEM SET 1

Macroscopic Quantum Phenomena and Quantum Dissipation

1 Problem

Consider the microscopic fields $\vec{e}$ and $\vec{h}$. Using the London equations, the Maxwell equations (Gaussian units) and the two fluid model $\vec{j} = \vec{j_s} + \vec{j_n}$, where $\vec{j_n} = \sigma\vec{e}$, show that

a) $\vec{h}$, $\vec{e}$, $\vec{j}$ satisfy equations of the form

$$c^2\ \vec{\nabla} \times (\vec{\nabla} \times \vec{f}) + \frac{4\pi}{\Lambda}\vec{f} + 4\pi\sigma\dot{\vec{f}} + \ddot{\vec{f}} = 0, \tag{1}$$

whereas the charge density ρ satisfies

$$\frac{4\pi}{\Lambda}\rho + 4\pi\sigma\dot{\rho} + \ddot{\rho} = 0. \tag{2}$$

b) Solve the equation for ρ and show that it relaxes within $\tau \approx 10^{-12}sec.$

c) Using the continuity equation, argue that $\vec{h}$, $\vec{e}$, and $\vec{j}$ now obey

$$c^2\ \nabla^2\vec{f} = \frac{4\pi}{\Lambda}\vec{f} + 4\pi\sigma\dot{\vec{f}} + \ddot{\vec{f}}. \tag{3}$$

d) Relate the 3 terms on the RHS of the above equations to the contributions of the supercurrent $\vec{j_s}$, the normal current $\vec{j_n}$, and the displacement current $\vec{j_d} = \dot{\vec{e}}/4\pi$.

e) Show that, for an alternating field of frequency $\omega/2\pi$, the share of each of the 3 currents is given by the proportion

$$|\vec{j_s}| : |\vec{j_n}| : |\vec{j_d}| = 1 : \Lambda\sigma\omega : \frac{\Lambda\omega^2}{4\pi}. \tag{4}$$

f) Then, for quasistationary conditions $\omega \ll 1/\sigma\Lambda$ ($\approx 10^{12}sec^{-1}$) the field equations of (c) become of the form

$$-\nabla^2\vec{f} + \frac{4\pi}{\Lambda c^2}\vec{f} = 0. \tag{5}$$

g) Establish the following boundary conditions for the fields: $h_{||}$, $e_{||}$ and $h_\perp$, $j_\perp$ must be continuous at the boundaries. Furthermore $(\Lambda j_s)_{||}$ must be continuous at the boundary between two different superconductors.

2 Problem

Determine the fields and currents of a superconducting sphere of radius R in a magnetic field $\vec{H}_0$. (3 points)

3 Problem

Consider a cylindrical wire of radius a whose axis points along $\hat{z}$. Between $z = -b$ and $z = b$ the material is a superconductor. Find the current distribution within the superconducting region once the current is uniformly fed in the normal region. (4 points)

4 Problem

Find the equation for the flux ϕ inside a superconducting ring with two equal Josephson junctions when it is subject to an external flux ϕ_x. (3 points)