PROBLEM SET 2

Macroscopic Quantum Phenomena and Quantum Dissipation

1

Show that the probability density P_1 for the Wiener process obeys the diffusion equation

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial y^2}$$

for $D = 1/2$. (1 point)

2

Show that the Ornstein-Uhlenbek process obeys

$$\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial y_2}(y_2 T) + \frac{\partial^2 T}{\partial y_2^2} \tag{1}$$

$$\frac{\partial T}{\partial \tau} = -y_1\frac{\partial T}{\partial y_1} + \frac{\partial^2 T}{\partial y_1^2} \tag{2}$$

These are the forward and backward Kolmogorov equations for the Ornstein-Uhlenbek process. (2 points)

3

Show that if $A_1 < 0$ in

$$\frac{\partial P}{\partial t}(y,t) = -\frac{\partial}{\partial y}[(A_0 + A_1 y)P] + \frac{1}{2}B_0\frac{\partial^2 P}{\partial y^2} \tag{3}$$

by shifting y and rescaling, one can reproduce Eq. 1 of the previous exercise. (1 point)

4

The velocity v of a charged particle in a constant magnetic field and random electric field obeys

$$\dot{\mathbf{v}} = \mathbf{v} \times \mathbf{B} - \beta\mathbf{v} + \mathbf{E}(t) \tag{4}$$

where $< E_i(t)E_j(t') >= C\delta_{ij}\delta(t-t')$. Find the mean square velocity of the particle in the case $\mathbf{B} = B\mathbf{e_z}$. Hint: Use that the solution of $\dot{v}_i - A_{ij}v_j = E_i(t)$ is $\mathbf{v}(t) = \exp(\hat{A}t)\mathbf{v}(0) + \exp(\hat{A}t)\int_0^t dt' \exp(-\hat{A}t')\mathbf{E}(t')$. (2 points)

5

A Markov process whose transition matrix factorizes, $W(y|y') = u(y)v(y')$ for $y \neq y'$, is called a "kangaroo process". Show that such Master-equations can be solved, i.e. P(y,t) can be expressed in $P(y,0)$ by means of integrals. (2 points)

6

Find the explicit solution of a kangaroo process with $v(y) = const.$ and interpret the result ("Kubo-Anderson process"; the dichotomic Markov process is a special case). (2 points)