PROBLEM SET 3

Macroscopic Quantum Phenomena and Quantum Dissipation

1) Show that the spectral function $J(\omega)$ introduced in (5.14) is indeed the imaginary part of the Fourier transform of the retarded dynamical susceptibility defined in (5.16). (1.5 points)

2) Using (5.27), show that the force (5.26) obeys $< f(t) >= 0$ and $< f(t)f(t') >= 2\eta k_B T \delta(t-t')$. Evaluating (5.27) for quantum oscillators, obtain the quantum mechanical form of the latter expression which is given by (5.36). (1.5 point)

3) Use the quantum Langevin equation,

$$m\ddot{q} + 2m\gamma\dot{q} = f(t),$$

for the position operator q of a free Brownian particle to evaluate

$$\sigma^2(t) \equiv \langle q^2(t)\rangle - \langle q(t)\rangle^2$$

at $T = 0$. Remember that in this case $f(t)$ satisfies (5.36). (2.0 points)

4) Show that equation (6.24) is the coordinate representation of the operator equation (6.25). Once $\tilde{\rho}(x,y)$ satisfies (6.24) show that its Wigner transform (6.27) satisfies the Fokker-Planck equation (6.29). (2.0 points)

5) Using (7.26), (7.44), and (7.47) show that the attenuation factor in (7.51) is given by

$$f(t) = \frac{q_0^2 \alpha I_R(\theta)}{8\sigma^2 Q(\theta)}.$$

where α, $I_R(\theta)$, and $Q(\theta)$ are all defined in (7.53). (3.0 points)