PROBLEM SET 4

Macroscopic Quantum Phenomena and Quantum Dissipation

1) Consider a complex function $F(z) = -V(z) = -\frac{1}{2}m\omega^2 z^2 - \frac{1}{4}\lambda z^4$, where $z = x + iy$ as usual.
i) Find the points on the complex plane where $\frac{dF}{dz} = 0$ for the case $\lambda > 0$ and $\lambda < 0$. (0.5 points)
ii) Expand $F(z)$ up to 2^{nd} order about each of the points and show that they are all saddle points of $F(z)$ for both cases $\lambda \gtrless 0$. (0.5 points)
iii) Study the path of the steepest descent of $F(z)$ at each of these points for $\lambda \gtrless 0$. (1 point)
iv) Compute the classical partition function

$$Z = \int\int \frac{dqdp}{h} e^{-\beta H}, \quad H = \frac{p^2}{2m} + V(q)$$

for $\lambda > 0$ and $\lambda < 0$ in the saddle point approximation. (1 point)
v) Using that the fact that the Helmholtz free energy $F = -\frac{1}{\beta}\ln Z$, compute it for $V(q)$ when $\lambda \gtrless 0$ and show that the imaginary part of the free energy when $\lambda < 0$ is proportional to $e^{-\Delta V/kT}$, where ΔV is the barrier height measured from $V(0) = 0$. (1 point)

2) Assuming that $q_c^{(0)}(\tau)$ in eq. (8.22) is the undamped ($\eta = 0$) bounce solution of (8.15), show that (8.23) holds and evaluate it to demonstrate (8.24). (2 points)

3) Write the equation (8.15) in terms of the Fourier transform $q(\omega)$ of $q(\tau')$ and obtain (8.28). Neglecting the ω^2 term in the latter show that (8.29) is indeed a solution of this equation for the values A and κ given in (8.31). (2 points)

4) Compute the overdamped action of $q_c(\omega) = Ae^{-\kappa|\omega|}$ in (8.29) and show that it is given by (8.33). (2 points)