PROBLEM SET 5

Macroscopic Quantum Phenomena and Quantum Dissipation

1) Consider a particle in the potential

$$V(q) = \frac{M\omega_0^2 q_0^2}{32}[(\frac{q}{(q_0/2)})^2 - 1]^2$$

Project the full Hamiltonian $H = \frac{p^2}{2M} + V(q)$ onto the space spanned by the states $\psi_L(q)$ and $\psi_R(q)$, where $\psi_L(q) = \psi_0(q+q_0/2)$ and $\psi_R(q) = \psi_0(q-q_0/2)$ and $\psi_0(q)$ is the ground state of a harmonic oscillator of frequency ω_0. Show that the projected operator has the form

$$H_2 = -\frac{\hbar}{2}\Delta\sigma_x$$

and compute Δ. (3 points)

2) Considering that in the spin-boson model the coupling to the reservoir can be treated perturbatively show that, for $\vec{S} \equiv \frac{\hbar<\vec{\sigma}>}{2}$,

$$\begin{aligned}
\frac{dS_x}{dt} &= -\frac{S_x - S_x^{(eq)}}{T_1} \\
\frac{dS_y}{dt} &= \Delta S_z - \frac{S_y}{T_2} \\
\frac{dS_z}{dt} &= -\Delta S_y
\end{aligned}$$

where $S_x^{(eq)} = \frac{\hbar}{2}\tanh(\frac{\beta\hbar\Delta}{2})$ is the thermal equilibrium value of S_x. Show that to 2^{nd} order in the system - environment coupling

$$T_1^{-1} = T_2^{-1} = \frac{q_0^2}{2\hbar}J(\Delta)\coth(\frac{\beta\hbar\Delta}{2})$$

Notice that $J(\omega)$ isn't necessarily Ohmic. (5 points)

3) Perform the unitary transformation $\hat{H}' = \hat{U}H\hat{U}^{-1}$ of the spin-boson Hamiltonian,

$$H = -\frac{1}{2}\ \hbar\Delta\sigma_x + \frac{1}{2}\ \epsilon\ \sigma_z + \frac{1}{2}\ q_0\sigma_z\sum_k C_k q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2}\ m_k\omega_k^2 q_k^2$$

to the basis of harmonic-oscillators displaced by $\delta_k \sigma_z$, where

$$\delta_k = -\frac{1}{2} q_0 C_k / m_k \omega_k^2.$$

which yields

$$\hat{H}' = -\frac{1}{2}\,\hbar\Delta(\sigma_+ e^{-i\hat{\Omega}} + H.c.) + \frac{1}{2}\,\epsilon\,\sigma_z + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2}\,m_k \omega_k^2 q_k^2.$$

Write down the corresponding unitary operator $\hat{U}$ and $\hat{\Omega}$. (2 points)