Entropia Estatística e a distribuição de Boltzmann.

Objetivo: obtenção da entropia através de argumentos microscópicos.
Caminho diferente do usual: ponto de partida será a entropia de informação de uma distribuição de probabilidade.

Descrição de um sistema quântico
informação máxima $\rightarrow |\psi\rangle = |\psi(0)\rangle$
$\hat{A}_1, \hat{A}_2 \ldots \hat{A}_N \longleftrightarrow$ CCOC
$\hat{A}_n |\psi\rangle = a^{(n)}|\psi\rangle \Rightarrow \hat{A}_n |\psi_i^{(n)}\rangle = a_i^{(n)} |\psi_i^{(n)}\rangle$; $|\psi_i^{(n)}\rangle \equiv |a_i^{(n)}\rangle$
$\Rightarrow$ N operadores: $|a^{(1)}, a^{(2)} \ldots a^{(N)}\rangle$

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle \Rightarrow i\hbar \frac{dU(t)}{dt} = H(t)\,U(t)$$

$U(t)$ - operador de evolução temporal.
U é unitário pois $\langle\psi(t)|\psi(t)\rangle = 1 \Rightarrow U^+ = U^{-1}$
e satisfaz $U(t,t_0) = U(t,t_1)\,U(t_1,t_0)$
Se $H \neq H(t)$: $U(t) = e^{-iHt/\hbar}$
Essa é a versão de Schrödinger.

Versão de Heisenberg : $|\psi_H\rangle \equiv U^{-1}(t)|\psi(t)\rangle \quad (= |\psi(0)\rangle)$

e $\hat{A}_H(t) = U^{-1}(t)\,\hat{A}(t)\,U(t) \Rightarrow \langle\psi(t)|\hat{A}(t)|\psi(t)\rangle =$
$= \langle\psi_H|\hat{A}_H(t)|\psi_H\rangle$

$\Rightarrow$ se $\hat{A} = \hat{A}(t)$ (dependência explícita) $\left(\frac{\partial \hat{A}}{\partial t}\right)_H = U^{-1}\frac{\partial \hat{A}}{\partial t}U$

$$\Rightarrow \frac{d\hat{A}_H}{dt} = \left(\frac{\partial \hat{A}}{\partial t}\right)_H + \frac{1}{i\hbar}[\hat{A}, H]$$

Definindo $\hat{\rho} = |\psi\rangle\langle\psi|$ Temos $\langle\psi|A|\psi\rangle =$

$= \sum_n \langle\psi|b_n\rangle\langle b_n|A|\psi\rangle = \sum_n \langle b_n|A|\psi\rangle\langle\psi|b_n\rangle$

$= \mathrm{tr}\,\hat{A}\hat{\rho} = \mathrm{tr}\,\hat{\rho}\hat{A}$.

<u>Operadores densidade</u>:

$\hat{\rho} = |\psi\rangle\langle\psi|$ se $|\psi\rangle = \sum_n c_n |n\rangle$ estado puro

$\Rightarrow \hat{\rho} = \sum_{mn} c_n c_m^* |n\rangle\langle m| \Rightarrow \hat{\rho}_{mn} = c_m c_n^*$.

Vamos rotular $|\psi\rangle \to |\psi^{(n)}\rangle$ n ñ é necessariamente discreto.

$\Rightarrow |\psi^{(n)}\rangle = \sum c_i^{(n)} |i\rangle$

$\langle\psi^{(n)}|\psi^{(n)}\rangle = 1$ mas $|\psi^{(m)}\rangle$ e $|\psi^{(n)}\rangle$ não são necessariamente ortogonais

Distribuição de probabilidades $\{p_n\} \Rightarrow \sum p_n = 1$

$p_n \geq 0$

$\Rightarrow$ mistura estatística: $\hat{\rho} = \sum_n p_n |\psi^{(n)}\rangle\langle\psi^{(n)}|$

$= \sum_{ijn} p_n c_i^{(n)} c_j^{*(n)} |i\rangle\langle j|$

$\langle\hat{A}\rangle = \sum_n p_n \langle\psi^{(n)}|\hat{A}|\psi^{(n)}\rangle = \sum_n p_n \mathrm{tr}\,\hat{\rho}_n \hat{A}$

$\Rightarrow \hat{\rho} = \sum_n p_n \hat{\rho}_n$ Temos $\langle\hat{A}\rangle = \mathrm{tr}\,\hat{\rho}\hat{A}$

Ex: Stern-Gerlach $|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle = |+\rangle_x$

$= \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \neq \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \frac{1}{2}|\uparrow\rangle\langle\uparrow| + \frac{1}{2}|\downarrow\rangle\langle\downarrow|$

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Stern-Gerlach:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

$$\Rightarrow \hat{\rho}_1 = \frac{1}{2}|\uparrow\rangle\langle\uparrow| + \frac{1}{2}|\downarrow\rangle\langle\downarrow| + \frac{1}{2}|\uparrow\rangle\langle\downarrow| + \frac{1}{2}|\downarrow\rangle\langle\uparrow|$$

$$= \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$\hat{\rho}_2 = \frac{1}{2}|\uparrow\rangle\langle\uparrow| + \frac{1}{2}|\downarrow\rangle\langle\downarrow| = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

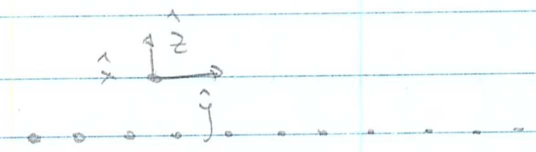
$$\langle\sigma_z\rangle_1 = \mathrm{tr}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \mathrm{tr}\begin{pmatrix} 1/2 & -1/2 \\ 1/2 & -1/2 \end{pmatrix} = 0$$

$$\langle\sigma_z\rangle_2 = \mathrm{tr}\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \mathrm{tr}\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} = 0$$

$$\langle\sigma_x\rangle_1 = \mathrm{tr}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \mathrm{tr}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} = 1$$

$$\langle\sigma_x\rangle_2 = \mathrm{tr}\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \mathrm{tr}\begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} = 0$$

$\hat{\rho}_1 \rightarrow$



$$\hat{\rho}_2 \rightarrow \underbrace{\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow}_{50\%\ |\uparrow\rangle\ \text{e}\ 50\%\ |\downarrow\rangle}$$

(20)

Representação contínua:

$$\hat{\rho} = \sum_{nij} p_n \, c_i^{(n)} c_j^{(n)*} \, |i\rangle\langle j|$$

$n \to x_0 \qquad \sum_n \to \int dx_0 \qquad |i\rangle \leftrightarrow |x\rangle \qquad p_n \to p(x_0)$

$|\psi^{(n)}\rangle \to |\psi^{(x_0)}\rangle$ tal que $|\psi^{(x_0)}\rangle = \int dx \, |x\rangle\langle x|\psi^{(x_0)}\rangle$

funções centradas em $x_0 \Rightarrow \langle x|\psi^{(x_0)}\rangle = \psi(x - x_0)$

$$\Rightarrow \hat{\rho} = \iiint_{x_0 x' y'} p(x_0) \, \psi(x' - x_0) \, \psi^*(y' - x_0) \, |x'\rangle\langle y'|$$

$$\Rightarrow \hat{\rho}(x,y) = \langle x|\hat{\rho}|y\rangle = \int_{x_0} p(x_0) \, \psi(x - x_0) \, \psi^*(y - x_0)$$

$$\int dx_0 \, p(x_0) = 1$$

$$\Rightarrow \int \hat{\rho}(x,x) \, dx = \iint_{x \; x_0} p(x_0) \, \psi(x - x_0) \, \psi^*(x - x_0) \, dx$$

$$= \int p(x_0) \, dx_0 = 1 \qquad \Rightarrow \operatorname{tr} \hat{\rho} = 1.$$

Se $p(x_0) = \delta(x_0)$, por exemplo,

$\Rightarrow \hat{\rho}(x,y) = \psi(x) \, \psi^*(y) = \langle x|\psi\rangle\langle\psi|y\rangle \to$ estado puro.

Propriedades de $\hat{\rho}$:

i) $\hat{\rho} = \hat{\rho}^+$

ii) $\mathrm{tr}\,\hat{\rho} = \sum_n p_n = 1$

iii) $\langle \varphi | \hat{\rho} | \varphi \rangle \geq 0$

iv) estado puro: $\hat{\rho}^2 = \hat{\rho}$

i) $\hat{\rho}^+ = \sum_n p_n \left(|\psi^{(n)}\rangle \langle \psi^{(n)}| \right)^+ = \sum_n p_n \; |\psi^{(n)}\rangle \langle \psi^{(n)}|$

ii) $\mathrm{tr}\,\hat{\rho} = \sum_{ni} p_n \; \langle i | \psi^{(n)} \rangle \langle \psi^{(n)} | i \rangle = \sum p_n = 1$

iii) $\langle \varphi | \hat{\rho} | \hat{\varphi} \rangle = \sum_n p_n \; \langle \varphi | \psi^{(n)} \rangle \langle \psi^{(n)} | \varphi \rangle = \sum_n \underset{0}{\underset{\vee\!|}{p_n}} \; \underset{0}{\underset{\vee\!|}{|\langle \varphi | \psi^{(n)} \rangle|^2}}$

≥ 0

iv) $\hat{\rho} = |\psi\rangle\langle\psi| \quad \Rightarrow \quad \hat{\rho}^2 = |\psi\rangle \underbrace{\langle\psi|\psi\rangle}_{1} \langle\psi| = |\psi\rangle\langle\psi| = \hat{\rho}$

Dinâmica de $\hat{\rho}$:

$$\hat{\rho}(t) = \sum_n p_n \; U(t) \, |\psi^{(n)}\rangle \langle \psi^{(n)}| \, U^+(t)$$

$$\frac{d\hat{\rho}}{dt} = \sum_n p_n \; \frac{dU}{dt} |\psi^{(n)}\rangle \; \langle \psi^{(n)} | U^+ + \sum_n p_n \; U |\psi^{(n)}\rangle \langle \psi^{(n)} | \frac{dU^+}{dt}$$

$$= \frac{1}{i\hbar} \sum_n p_n H U |\psi^{(n)}\rangle \langle \psi^{(n)} | U^+ - \frac{1}{i\hbar} \sum_n p_n U |\psi^{(n)}\rangle \langle \psi^{(n)} | U^+ H$$

$$= \frac{1}{i\hbar} [H, \hat{\rho}] \quad \Rightarrow \quad \boxed{i\hbar \frac{d\hat{\rho}}{dt} = [H, \hat{\rho}]}.$$

$\hat{\rho}$ na versão de Schrödinger.

Versão de Heisenberg: $\hat{\rho}_H \equiv U^{-1}(t) \, \hat{\rho}(t) \, U(t)$

$$\Rightarrow \langle \hat{A}(t) \rangle = \mathrm{tr}\,[\hat{\rho}_H \hat{A}_H(t)] = \mathrm{tr}\,[\hat{\rho}(t) \hat{A}(t)].$$

Espaço de fase quântico:

Necessidade de se obter macroestados ⇒ contagem do # de estados: quando tratamos do estado de partículas numa caixa podemos sempre expandir o estado geral numa base de ondas planas

$$\psi_{\vec{k}}(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$$

com condições periódicas de contorno $\psi_{\vec{k}}(\vec{r}) = \psi_{\vec{k}}(\vec{r}+\vec{L})$
$\vec{L} = (L_x, L_y, L_z)$

$$\Rightarrow e^{i\vec{k}\cdot\vec{r}} = e^{i\vec{k}\cdot\vec{r}} e^{i\vec{k}\cdot\vec{L}} \Rightarrow \vec{k}_n = (k_x^{(n)}, k_y^{(n)}, k_z^{(n)})$$

$$k_i^{(n)} = \frac{2\pi n_i}{L_i} \; ; \; \frac{2\pi}{L_i} \ll 1.$$

$$\Rightarrow \sum_n f(\vec{k}_n) = \sum_{n_x n_y n_z} f(k_x^{(n)}, k_y^{(n)}, k_z^{(n)})$$

$$\rightarrow \sum_{n_x n_y n_z} f(n_x, n_y, n_z)\, \Delta n_x \Delta n_y \Delta n_z \qquad \Delta n_i = (n+1)_i - n_i$$

$$= \frac{V}{(2\pi)^3} \sum_{n_x n_y n_z} f(k_x, k_y, k_z)\, \Delta k_x \Delta k_y \Delta k_z$$

$$\Delta k_i \ll 1 \Rightarrow \sum \rightarrow \frac{V}{(2\pi)^3}\int \Rightarrow \boxed{\rho(\vec{k})\, d^3k = \frac{V d^3k}{(2\pi)^3}}$$

Densidade de níveis de energia:

$$E = \frac{\hbar^2 k^2}{2m} \qquad dN(E) = \rho(E)\, dE = 4\pi k^2 dk \frac{V}{(2\pi)^3}$$

$$k = \frac{\sqrt{2mE}}{\hbar} \Rightarrow dE = \frac{\hbar^2}{m} k\, dk \Rightarrow dk = \frac{m\, dE}{\hbar\sqrt{2mE}}$$

$$\Rightarrow \rho(E)\, dE = 4\pi \frac{2mE}{\hbar^2} \frac{V}{(2\pi)^3} \frac{m\, dE}{\hbar\sqrt{2mE}} = \frac{mV}{2\pi^2\hbar^3}\sqrt{2mE}\, dE$$

$$\rho(E) = \frac{mV\sqrt{2mE}}{2\pi^2\hbar^3}$$

Gas ideal monoatômico:

$$E_r = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$$

$$\Rightarrow \langle E_r \rangle = \frac{\hbar^2 \pi^2}{2m}\left(\frac{\langle n_x^2\rangle}{L_x^2} + \frac{\langle n_y^2\rangle}{L_y^2} + \frac{\langle n_z^2\rangle}{L_z^2}\right)$$

isotropia: $\dfrac{\langle n_x^2\rangle}{L_x^2} = \dfrac{\langle n_y^2\rangle}{L_y^2} = \dfrac{\langle n_z^2\rangle}{L_z^2} = \dfrac{1}{3}\dfrac{2m\langle E\rangle}{\hbar^2\pi^2}$

$$\frac{\partial E_r}{\partial L_x} = -\frac{\hbar^2\pi^2}{m}\frac{n_x^2}{L_x^3}$$

$$\Rightarrow \frac{\partial\langle E_r\rangle}{\partial L_x} = -\frac{\hbar^2\pi^2}{m L_x}\frac{\langle n_x^2\rangle}{L_x^2} = -\frac{\hbar^2\pi^2}{m L_x}\frac{1}{3}\frac{2m\langle E_r\rangle}{\hbar^2\pi^2} = -\frac{2}{3L_x}\langle E_r\rangle$$

Mudança adiabática de volume:

$$dE = đW = -P L_y L_z \, dL_x \qquad E = N\langle E_r\rangle$$

$$= \frac{\partial E}{\partial L_x} dL_x \quad \Rightarrow \quad -P L_y L_z = N\frac{\partial\langle E_r\rangle}{\partial L_x} = -\frac{2}{3L_x}E$$

$$\Rightarrow \boxed{PV = \frac{2}{3}E}$$ p/ gás ñ-relativístico (no outro caso $PV = \frac{E}{3}$)

Descrição clássica: q_i e p_i

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i U(q_i)$$

$$\frac{\partial H}{\partial p_i} = \dot{q}_i \qquad \frac{\partial H}{\partial q_i} = -\dot{p}_i$$

microestado: $(q_i(t), p_i(t))$

$$\Rightarrow q' = q + \dot{q}\,dt = q + \partial H/\partial p \, dt$$
$$p' = p + \dot{p}\,dt = p - \partial H/\partial q \, dt$$

$$\frac{\partial(q', p')}{\partial(q, p)} = \begin{vmatrix} 1 + \partial^2 H/\partial q\partial p \, dt & \partial^2 H/\partial p^2 \, dt \\ -\partial^2 H/\partial q^2 \, dt & 1 - \partial^2 H/\partial p \partial q \, dt \end{vmatrix}$$

$= 1 + \mathcal{O}(dt)^2 \Rightarrow J = \dfrac{\partial(q', p')}{\partial(q, p)}$

$\dfrac{dJ}{dt} = 0 \Rightarrow dq(0)\, dp(0) = dq(t)\, dp(t)$

$\Rightarrow d\Gamma = C \prod_{i=1}^{N} d^3r_i\, d^3p_i$ é invariante. (Teo. Liouville)

Densidade no espaço de fase:

Probabilidade de se observar microestado em $t=0$.

$\rho^{(N)}(\vec{r}_i, \vec{p}_i, 0) \equiv \rho_0^{(N)}(\vec{r}_i, \vec{p}_i) \Rightarrow \mathcal{N} = \int \rho_0^{(N)}(\vec{r}_i, \vec{p}_i)\, d^{3N}r_i\, d^{3N}p_i$

Se $\rho_0 \equiv \rho_0^{(N)} / \mathcal{N}$

$d\mathbb{P}(\vec{r}_i, \vec{p}_i) = \rho_0(\vec{r}_i, \vec{p}_i)\, d\Gamma \Rightarrow \underbrace{\int d\mathbb{P}(\vec{r}_i, p_i)}_{=1} = \int \rho_0(\vec{r}_i, \vec{p}_i)\, d\Gamma$

Análogo a $\hat{\rho}$ e $\mathrm{tr}\, \hat{\rho} = 1$

No Tempo: $\vec{r}_i \to \vec{r}_i(t)$ e $\vec{p}_i \to \vec{p}_i(t)$ $\quad (x \to y = \varphi_t(x))$

$\Rightarrow x = \varphi_t^{-1}(y) = \varphi_{-t}(y)$ e Liouville: $dx = dy$.

$x \equiv (\vec{r}_1, \ldots \vec{r}_N, \vec{p}_1, \ldots \vec{p}_N)$

Em $t=0$ $\quad A(0) = A(\vec{r}_i, \vec{p}_i) = A(x)$

$A(t) = A(\vec{r}_i(t), \vec{p}_i(t)) = A(\varphi_t(x))$

$$\langle A(t=0) \rangle = \int dx\, \rho_0(x)\, A(x)$$

$\downarrow$

$$\langle A(t) \rangle = \int dx\, \rho_0(x)\, A(\varphi_t(x))$$

análogo à Heisenberg em MQ.

Analogia c/ Schrödinger.

$$\langle A(t) \rangle = \int dy\, \rho_0(\varphi_{-t}(y))\, A(y) = \int dx\, \rho(x(t), t)\, A(x)$$

$$\rho(x(t), t) = \rho_0(\varphi_{-t}(x))$$

$$\rho(q(t+dt), p(t+dt), t+dt)\, dq'dp' = \rho(q(t), p(t), t)\, dq\, dp$$

$$dq'dp' = dq\, dp \Rightarrow \frac{\partial \rho}{\partial q}\dot{q} + \frac{\partial \rho}{\partial p}\dot{p} + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial H}{\partial p}\frac{\partial \rho}{\partial q} - \frac{\partial H}{\partial q}\frac{\partial \rho}{\partial p} = 0$$

$$\frac{\partial \rho}{\partial t} - \{H, \rho\} = 0 \quad \text{ou} \quad \frac{\partial \rho}{\partial t} = \{H, \rho\}$$

$$\{A, B\} = \sum_i \frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i}$$

eq. quântico-clássico: $\frac{1}{i\hbar}[\,,\,] \leftrightarrow \{\,,\,\}$

$$\Rightarrow \frac{dA}{dt} = \{A, H\} + \frac{\partial A}{\partial t}$$

Nota: $\vec{r} \to (q_i, p_i)$

Movimento no espaço de fase: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

$\to$ conservação do # de cópias

$\vec{v} = (\dot{q}_i, \dot{p}_i)$

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \vec{\nabla})\rho + \rho \vec{\nabla} \cdot \vec{v} = 0$$

mas $\vec{\nabla} \cdot \vec{v} = \sum_i \frac{\partial \dot{q}_i}{\partial q_i} + \sum_i \frac{\partial \dot{p}_i}{\partial p_i} = \sum_i \frac{\partial^2 H}{\partial q_i \partial p_i} - \sum_i \frac{\partial^2 H}{\partial p_i \partial q_i} = 0$

$\Rightarrow \frac{\partial \rho}{\partial t} + (\vec{v} \cdot \vec{\nabla})\rho = 0$ ou $\frac{d\rho}{dt} = 0$ (fluido incompressível)

$$\Rightarrow \frac{\partial \rho}{\partial t} + \sum_i \dot{q}_i \frac{\partial \rho}{\partial q_i} + \sum_i \dot{p}_i \frac{\partial \rho}{\partial p_i} = \frac{\partial \rho}{\partial t} + \sum_i \frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial q_i} - \sum_i \frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial p_i} = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = \{H, \rho\}, \quad \text{Caso estacionário } \frac{\partial \rho}{\partial t} = 0 \Rightarrow \begin{cases} \rho = \text{const} \\ \rho = \rho(H) \end{cases}$$

Mais sobre a correspondência quântico-clássico:

$\hat{\rho} \to \hat{\rho}(x, y) \to$ elementos de matriz do operador densidade.

Definindo $q = \frac{x+y}{2}$ e $\xi = x - y$

Temos $\hat{\rho}(x, y) = \hat{\rho}\left(q + \frac{\xi}{2}, q - \frac{\xi}{2}, t\right)$

Transformada de Wigner:

$$\hat{W}(q, p, t) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} e^{ip\xi/\hbar} \hat{\rho}\left(q + \frac{\xi}{2}, q - \frac{\xi}{2}, t\right) d\xi.$$

$\hat{W}$ é um operador que depende dos parâmetros q e p e é tal que

$$\lim_{\hbar \to 0} \hat{W}(q, p, t) = \rho(q, p, t)$$

Ex:

$$\int_{-\infty}^{\infty} dp\, \hat{W}(q, p, t) = \int_{-\infty}^{\infty} d\xi\, \hat{\rho}(q, \xi, t) \underbrace{\int_{-\infty} \frac{1}{2\pi\hbar} dp\, e^{-ip\xi/\hbar}}_{\delta(\xi)}$$

$$\Rightarrow \int_{-\infty}^{\infty} dp\, \hat{W}(q, p, t) = \hat{\rho}(q, 0, t) = \hat{\rho}(x, x) = P(x) = \rho(q)$$