98

BECs

Voltando ao caso massivo,

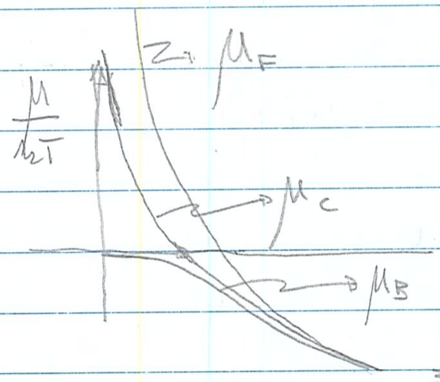
$$\frac{N}{V} = \frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty d\epsilon \frac{\sqrt{\epsilon}}{e^{\beta(\epsilon-\mu)}-1}$$

e $\epsilon - \mu \geq 0$ ou $\mu \leq \epsilon \ \forall \epsilon$

$\Rightarrow$ gás de Bósons: $\mu \leq 0$

Por outro lado, sabemos que $\frac{\mu_c}{kT} = \ln\left[\frac{n}{g}\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}\right]$

$\Rightarrow \frac{\mu}{kT} \to -\infty$ se $T \to \infty$



$\Rightarrow \exists\ T_0$ tq. abaixo deste valor, ao invés de μ se tornar > 0 ele se anula!

$$\Rightarrow \frac{N}{V} = \frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty d\epsilon \frac{\sqrt{\epsilon}}{e^{\epsilon/kT_0}-1}$$

$$x = \frac{\epsilon}{kT_0}$$

$$\Rightarrow \frac{N}{V} = \frac{g}{4\pi^2}\left(\frac{2mkT_0}{\hbar^2}\right)^{3/2}\underbrace{\int_0^\infty dx \frac{\sqrt{x}}{e^x-1}}_{\zeta(3/2)\Gamma(3/2)}$$

$$T_0 = \frac{\hbar^2}{2m}\left[\frac{4\pi^2}{g\,\Gamma\left(\frac{3}{2}\right)\zeta\left(\frac{3}{2}\right)}\right]^{3/2}\left(\frac{N}{V}\right)^{2/3} = \frac{3.31}{g^{2/3}}\frac{\hbar^2}{mk}n^{2/3}$$

$kT_0 \sim \frac{\hbar^2}{m} n^{2/3}$ energia de p.^{to} zero associada por localizar a partícula em $v = \frac{V}{N}$

Abaixo de T_0: muitos bósons passam a ocupar ϵ_0.

$$N = \sum_i \frac{1}{e^{\beta(\epsilon_i - \mu)} - 1} \quad \text{se } \mu \to 0 : \quad N = \sum_i \frac{1}{e^{\beta \epsilon_i} - 1}$$

se $\epsilon_i \neq 0$ $n_i \to$ finito se $\epsilon_i \to 0$ n_i diverge.

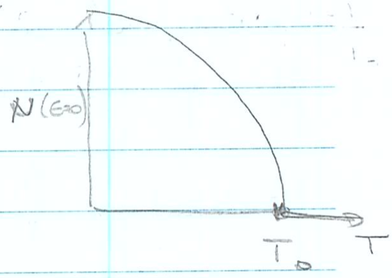
Se $T < T_0$ fazemos

$$\frac{dN(\epsilon)}{V} = \frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{\sqrt{\epsilon}\, d\epsilon}{e^{\beta\epsilon} - 1} \qquad (\therefore \mu = 0^-)$$

$$\Rightarrow \frac{N(\epsilon > 0)}{V} = \frac{g}{4\pi^2}\left(\frac{2mkT}{\hbar^2}\right)^{3/2} \int_0^\infty dx \frac{\sqrt{x}}{e^x - 1} = \frac{N}{V}\left(\frac{T}{T_0}\right)^{3/2}$$

$$\Rightarrow \frac{N(\epsilon = 0)}{V} = \frac{N}{V} - \frac{N}{V}\left(\frac{T}{T_0}\right)^{3/2} \quad \text{ou}$$

$$\frac{N(\epsilon = 0)}{V} = \frac{N}{V}\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right]$$



Se $T < T_0$

$$\frac{E}{V} = \frac{g}{4\pi^2}\left(\frac{2mkT}{\hbar^2}\right)^{3/2} kT \int_0^\infty dx \frac{x^{3/2}}{e^x - 1}$$

$$\frac{E}{V} = \frac{g}{4\pi^2}\left(\frac{2mkT}{\hbar^2}\right)^{3/2} kT \; \zeta\left(\frac{5}{2}\right)\Gamma\left(\frac{5}{2}\right)$$

⇒ em Termos de T_0

$$E = \frac{5\,\zeta(5/2)\,\Gamma(5/2)}{\zeta(3/2)\,\Gamma(3/2)}\, NkT \left(\frac{T}{T_0}\right)^{3/2} = 0.77\, NkT \left(\frac{T}{T_0}\right)^{3/2} ; \quad T \le T_0$$

$$\Rightarrow C_V = \frac{5}{2}\left[0.77\, Nk \left(\frac{T}{T_0}\right)^{3/2}\right] \qquad T \le T_0$$

Usando $\frac{E}{V}$ podemos ainda escrever

$$P = \frac{2E}{3V} = 0.08 \left(\frac{m}{\hbar^2}\right)^{3/2} g \,(kT)^{5/2}$$

que se anula a $T=0$ porque as partículas estão em ϵ_0 e $\langle p \rangle = 0$.

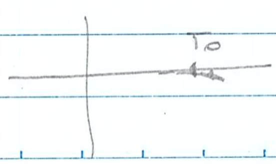
Vamos estudar o comportamento ñ-analítico de μ do $T \to T_0$. $\quad \mu = \frac{\partial F}{\partial N} = \frac{\partial G}{\partial N}$

Grandeza fictícia $\quad N_0(T) = \frac{gV}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \int d\epsilon \, \frac{\sqrt{\epsilon}}{e^{\epsilon/kT} - 1}$

$$\Rightarrow \frac{N_0(T)}{N_0(T_0)} = \frac{N_0(T)}{N} = \left(\frac{T}{T_0}\right)^{3/2} \quad \text{se } T > T_0$$

$$N - N_0(T) = \frac{gV}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^\infty d\epsilon \sqrt{\epsilon} \left\{ \frac{1}{e^{(\epsilon-\mu)/kT} - 1} - \frac{1}{e^{\epsilon/kT} - 1} \right\}$$

$\frac{\mu}{kT} \to 0^-$ se $0 < T - T_0 \ll T_0$



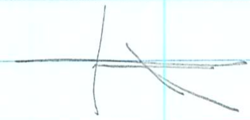
$\epsilon \approx 0$ domina a integral

$$\Rightarrow N - N_0(T) \simeq \frac{gV}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \mu k T \int_0^\infty \frac{d\epsilon}{\sqrt{\epsilon}\,(\epsilon + |\mu|)}$$

$$\approx -\frac{gV}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \pi k T_0 \sqrt{|\mu|}$$

$$\Rightarrow \mu = -\left[\frac{\zeta(3/2)\,\Gamma(3/2)}{\pi}\right]^2 k T_0 \left[\left(\frac{T}{T_0}\right)^{3/2} - 1\right]^2 \qquad T \geq T_0$$

$$\left.\frac{\partial \mu}{\partial T}\right|_{T \to T_0^+}$$ é descontínua!



Calor específico:

$$\left.\frac{\partial E}{\partial \mu}\right|_{TV} = \frac{3}{2}\left.\frac{\partial (PV)}{\partial \mu}\right|_{TV} = -\frac{3}{2}\left.\frac{\partial \Omega}{\partial \mu}\right|_{TV} = \frac{3N}{2}$$

$$\Rightarrow E = \begin{cases} E(T,V) & T < T_0 \quad \text{(calculado antes)} \\ E(T,V) + \frac{3N}{2}\mu & \text{se } T > T_0 \end{cases}$$

$$\Rightarrow \Delta\left[\frac{\partial C_v}{\partial T}\right]_{T_0} = -\frac{3}{2} N k T_0 \left[\frac{\zeta(\frac{3}{2})\,\Gamma(\frac{3}{2})}{\pi}\right]^2 \left\{\frac{\partial^2}{\partial T^2}\left[\left(\frac{T}{T_0}\right)^{3/2} - 1\right]^2\right\}_{T_0}$$

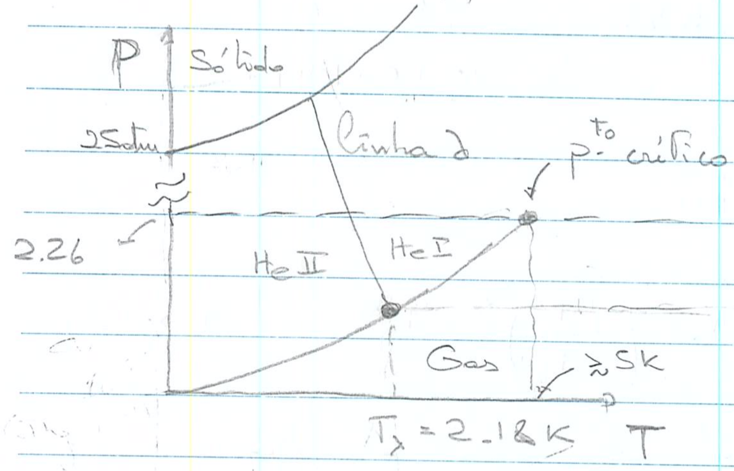
$$= -3.66\,\frac{Nk}{T_0}$$

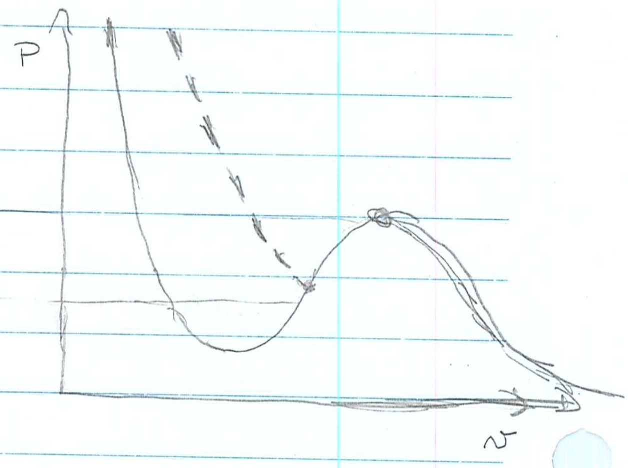
$T_0 \rightarrow$ condensação de B-E

Estado ordenado em momentum!

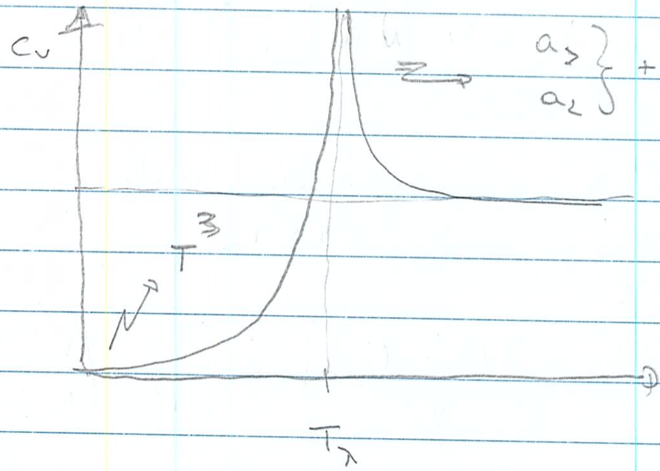
$\rho_4 = 0.145\ g\,cm^{-3} \Rightarrow T_0 = 3.14\ K$ (?)

Condensação: ^4He





e

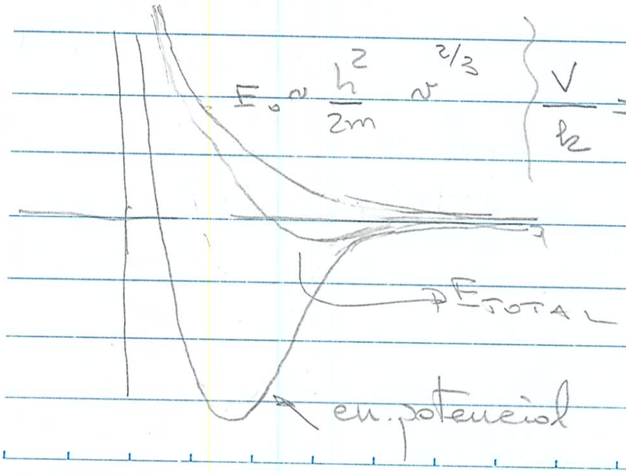


$\Rightarrow \left.\begin{matrix} a_> \\ a_< \end{matrix}\right\} + b \ln c|T-T_\lambda|$

$a_>$ p/ $T>T_\lambda$

$a_<$ p/ $T<T_\lambda$

Propriedades do He II:



$E_0 \sim \frac{h^2}{2m} v^{2/3}$

$\frac{V}{k} = v(r) = (5.67\times10^6)\, e^{-21.5\frac{r}{\sigma}} - 1.08\left(\frac{\sigma}{r}\right)^6$

$\sigma = 4.64$ Å e $v(r)$ está em K

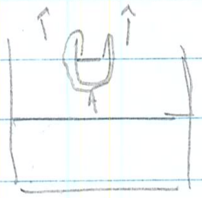
$\Leftarrow$ ^4He líquido!

Sulamericana

$T < T_\lambda \Rightarrow$ He II

Superfluidez do $^4He \Rightarrow$ He II

i) Flui ao longo de capilares com diâmetro $\sim 10^{-4}$ cm sem viscosidade.

ii)  efeito sifão.

$\Rightarrow$ ocorre devido à formação de filme fino de He II (100 camadas atômicas)

dentre outros.

Propriedades que caracterizam a superfluidez:

i) Inércia Rotacional ñ-clássica (ω muito pequeno)

$\vec{\omega}$ $\quad T > T_\lambda$ $\qquad\qquad T < T_\lambda$

$\vec{L} = MR^2\vec{\omega} = I_{cl}\,\vec{\omega}$ $\qquad\qquad \vec{L}(T) = I(T)\,\vec{\omega}$

$L(T) < MR^2 \Rightarrow \rho = \rho_n + \rho_s$ $\quad \rho_n \to$ componente viscosa continua o seu movimento e ρ_s cessa o movimento.

ii) Correntes persistentes

$\vec{\omega} \neq 0$ $\qquad\qquad \vec{\omega} = 0$

$T > T_\lambda$ $\qquad\qquad T < T_\lambda$

$\vec{L} = I_{cl}\,\vec{\omega}$ $\qquad\qquad \vec{L}(T) = \frac{\rho_s(T)}{\rho}\, I_{cl}\,\vec{\omega}$

Sulamericana

ρ_n para o ρ_s segue o movimento o mais próximo possível do clássico.

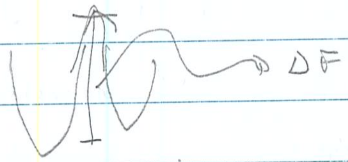
Abordagem fenomenológica:

i) Dois fluidos: $\rho = \rho_n + \rho_s$ e $\vec{J} = \rho_n \vec{v}_n + \rho_s \vec{v}_s$

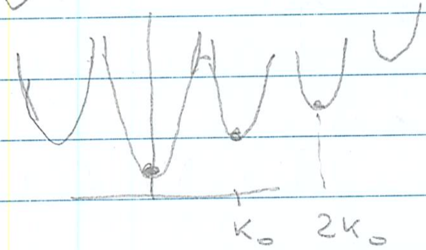
ii) Quantização da circulação:

$$\oint \vec{v} \cdot d\vec{l} = n \kappa_0 \qquad \kappa_0 = \frac{h}{m}$$

iii) Barreiras de energia livre.



$$\Gamma \propto e^{-\Delta F / k_B T} \to 0$$



Fenomenologia de supercondutores segue de forma análoga se substituirmos $\vec{\omega} \to \frac{-e}{2m} \vec{B}$

Movimento dos elétrons num supercondutor $\Leftrightarrow$ Movimento do superfluido visto do referencial girante!

Função de onda do condensado (com interação)

$\psi_0(\vec{r}_1, \ldots \vec{r}_N)$

Escoamento sem viscosidade → Transformação de Galileu: baixas velocidades

$\psi_{\vec{k}}(\vec{r}_1, \ldots \vec{r}_N)$; $\hbar\vec{k}$ momentum da partícula no condensado

$$\psi_{\vec{k}}(\vec{r}_1, \ldots \vec{r}_N) = e^{i\sum_k \vec{k}\cdot\vec{r}_k}\, \psi_0(\vec{r}_1, \ldots \vec{r}_N) \qquad \vec{v} = \frac{\hbar\vec{k}}{m}$$

↓ generalizando:

$$\psi(\vec{r}_1, \ldots \vec{r}_N) = e^{i\sum_k \theta(\vec{r}_k)}\, \psi_0(\vec{r}_1, \ldots \vec{r}_N)$$

$$n(\vec{r}) = \sum_k \int d^3r_1 \ldots d^3r_N\, \delta(\vec{r} - \vec{r}_k)\, \psi^*(\vec{r}_1, \ldots \vec{r}_N)\, \psi(\vec{r}_1, \ldots \vec{r}_N)$$

$$\vec{J}(\vec{r}) = \sum_k \int d^3r_1 \ldots \int d^3r_N \frac{e\hbar}{2mi}\left[\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right]\delta(\vec{r} - \vec{r}_k)$$

$$\vec{J}(\vec{r}) = \frac{e\hbar}{m}\, n(\vec{r})\, \vec{\nabla}\theta \qquad \text{onde}$$

$$n(\vec{r}) = N \int d^3r_2 \ldots d^3r_N\, \psi_0^*(\vec{r}_1, \ldots \vec{r}_N)\, \psi_0(\vec{r}_1, \ldots \vec{r}_N)$$

$n(\vec{r}) = n_1(\vec{r}; \vec{r})$ onde

$$n_1(\vec{r}; \vec{r}\,') = N \int d^3r_2 \ldots d^3r_N\, \psi_0^*(\vec{r}\, \vec{r}_2 \ldots \vec{r}_N)\, \psi_0(\vec{r}\,' \vec{r}_2 \ldots \vec{r}_N)$$

no caso de um sistema não interagente:

$\varphi_0(\vec{r}_1)\, \varphi_0(\vec{r}_2) \ldots \varphi_0(\vec{r}_N)$

$n(\vec{r}) = N\, \varphi_0^*(\vec{r})\, \varphi_0(\vec{r})$

Estado superfluido é tal que $n_1(\vec{x}; \vec{y}) = f^*(\vec{x})\, f(\vec{y})$

$f(\vec{x}) = \sqrt{n(\vec{r})}\, e^{i\varphi(\vec{r})} \rightarrow \psi(\vec{r}) = \sqrt{n(\vec{r})}\, e^{i\varphi(\vec{r})}$

função de onda macroscópica.

$$\vec{J} = \frac{e\hbar}{2mi}\left(\psi^* \vec{\nabla}\psi - \psi \vec{\nabla}\psi^*\right)$$

$$= \frac{e\hbar}{m} n(\vec{r}) \vec{\nabla}\varphi \Rightarrow \frac{\hbar}{m}\vec{\nabla}\varphi = \vec{v}_s.$$

$$K = \oint \vec{v}_s \cdot d\vec{l} = \frac{\hbar}{m}\Delta\varphi = \frac{2n\pi\hbar}{m} = \boxed{n\frac{h}{m}}$$

Função de onda macroscópica $\Leftrightarrow$ parâmetro de ordem.

Ginzberg-Landau p/ supercondutores (exercício)

Gross-Pitaevsky ~ Ginzberg-Landau

$$F\{\psi\} = F_0(T) + \int d^3r \left\{ \alpha(T)|\psi|^2 + \beta(T)|\psi|^4 + \gamma(T)|\vec{\nabla}\psi|^2 \right\}$$

$$\alpha(T) = \alpha_0(T - T_c) \qquad \beta(T) = \text{const} \qquad \gamma(T) = \text{const}.$$

e novamente nos vemos com GL para estudar o comportamento crítico do sistema.