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que por argumentos dimensionais:

$$G(\vec{r}) = \frac{1}{r^{D-2+\eta}}\, g\left(\frac{r}{\xi}\right)$$

$$g\left(\frac{r}{\xi}\right) \sim \exp -\frac{r}{\xi} \quad \text{se } r \gg a.$$

Expoentes críticos:

$C \sim |t|^{-\alpha}$

$M \sim |t|^{\beta}$ se $t<0$

$\chi \sim |t|^{-\gamma}$

$M \sim H^{1/\delta}$

$\xi \sim |t|^{-\nu}$

e η : $t \equiv \frac{T-T_c}{T_c}$

Se $T \neq T_c$ podemos estabelecer uma relação entre os expoentes = lei de escala.

$T \neq T_c$, $\tilde{G}(0)$ é finito $\Rightarrow$ para compensar $q^{\eta-2}$, $f(q\xi) \sim (q\xi)^{2-\eta}$ se $q \to 0$.

$$\Rightarrow \tilde{G}(0) \sim \xi^{2-\eta} \sim |T-T_c|^{-\nu(2-\eta)}$$

Divergência em χ define γ: $\chi \sim |T-T_c|^{-\gamma}$

$$\Rightarrow \gamma = \nu(2-\eta)$$

Assim se $\eta < 2$, o que é o caso usual, χ diverge em $T=T_c$.

<u>Teoria do campo médio</u>:

Sejam $\hat{\rho}$ e $\hat{\rho}_\lambda$ operadores densidade

$$\hat{\rho} = \frac{e^{-\beta H}}{Z} \quad \text{e} \quad \hat{\rho}_\lambda = \frac{e^{-\beta H_\lambda}}{Z_\lambda}$$

$$Z = \mathrm{tr}\, e^{-\beta H} \qquad Z_\lambda = \mathrm{tr}\, e^{-\beta H_\lambda}$$

$H_\lambda \to$ hamiltoniana variacional

Resultado conhecido: $\mathrm{tr}\, X \ln Y - \mathrm{tr}\, X \ln X \leq \mathrm{tr}\, Y - \mathrm{tr}\, X$

$X = \hat{\rho}_\lambda \quad Y = \hat{\rho} \quad \Rightarrow \mathrm{tr}\, \hat{\rho}_\lambda \ln \hat{\rho} - \mathrm{tr}\, \hat{\rho}_\lambda \ln \hat{\rho}_\lambda \leq \underbrace{\mathrm{tr}\, \hat{\rho} - \mathrm{tr}\, \hat{\rho}_\lambda}_{=0}$

$$\Rightarrow -\mathrm{tr}\, \hat{\rho}_\lambda \ln \hat{\rho}_\lambda \leq -\mathrm{tr}\, \hat{\rho}_\lambda \ln \hat{\rho}$$

$$\Rightarrow \mathrm{tr}\left[\hat{\rho}_\lambda (\beta H_\lambda + \ln Z_\lambda)\right] \leq \mathrm{tr}\left[\hat{\rho}_\lambda (\beta H + \ln Z)\right]$$

$$\Rightarrow F \leq F_\lambda + \langle H - H_\lambda \rangle_\lambda \equiv \phi(\lambda) \quad ; \quad \langle * \rangle_\lambda \equiv \mathrm{tr}(* \hat{\rho}_\lambda)$$

$$H = -\frac{1}{2} \sum_{i,j} J_{ij} S_i S_j - \mu \sum_i B_i S_i$$

$J_{ij} = J$ para $\langle ij \rangle$.

$$H_\lambda = -\mu \sum_i \lambda_i S_i \Rightarrow Z_\lambda = \prod_i \left[2 \cosh(\beta \mu \lambda_i)\right].$$

$$F_\lambda = -\frac{1}{\beta} \sum_i \ln\left[2 \cosh \beta \mu \lambda_i\right]$$

$$M_i = -\frac{1}{\mu} \frac{\partial F_\lambda}{\partial \lambda_i} = \tanh(\beta \mu \lambda_i)$$

H_λ é paramagnética $\Rightarrow \langle S_i S_j \rangle_\lambda = \langle S_i \rangle_\lambda \langle S_j \rangle_\lambda = M_i M_j$

$$\Rightarrow \langle H - H_\lambda \rangle_\lambda = -\frac{1}{2} \sum_{i,j} J_{ij} M_i M_j - \mu \sum_i (B_i - \lambda_i) M_i$$

$$e \quad \phi(\lambda) = -\frac{1}{\beta} \sum_i \ln\left[2 \cosh(\beta \mu \lambda_i)\right] - \frac{1}{2} \sum_{ij} J_{ij} M_i M_j$$

$$- \mu \sum_i (B_i - \lambda_i) M_i$$

como M_i = função crescente de λ_i: $\frac{d}{d\lambda_i} \to \frac{d}{dM_i}$

$$\frac{\partial F}{\partial M_i} = \frac{\partial F}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial M} = -\mu M_i \frac{\partial \lambda_i}{\partial M_i}$$

Definindo $\psi(M) = \phi(\lambda(M))$

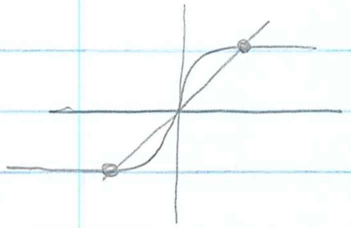
$$\Rightarrow \frac{\partial \psi}{\partial M_i} = -\mu M_i \frac{\partial \lambda_i}{\partial M_i} - \sum_j J_{ij} M_j - \mu B_i + \mu \lambda_i + \mu M_i \frac{\partial \lambda_i}{\partial M_i}$$

$\Rightarrow \dfrac{\partial \Psi}{\partial M_i} = 0 \qquad \Rightarrow \mu\lambda_i = \mu B_i + \sum_j J_{ij} M_j$

ou $\dfrac{1}{\beta} \tanh^{-1} M_i = \mu B_i + \sum_j J_{ij} M_j$

se $B_i = 0$ e $J_{ij} = J\delta_{ij} \Rightarrow \dfrac{1}{\beta} \tanh^{-1} M_i = J M_i$

$\Rightarrow M_i = \tanh \beta J M_i$



$\beta J \leq 1$

Derivando com respeito a B_k:

$$\frac{1}{1 - M_i^2} \langle S_i S_k \rangle_c = \delta_{ik} + \beta \sum_j J_{ij} \langle S_j S_k \rangle_c$$

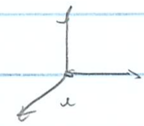
seja $B_i = B$ e $M_i = M$

$$\Rightarrow \sum_j \left(\frac{\delta_{ij}}{1 - M^2} - \beta J_{ij} \right) \langle S_j S_k \rangle_c = \delta_{ik}$$

Convolução discreta : T.F. de $f_i = f(\vec r_i)$

$$\Rightarrow \tilde f(\vec q) = \sum_i e^{i \vec q \cdot \vec r_i} f(\vec r_i)$$

Em D-dim



D vetores $\hat e_\mu \Rightarrow \pm a \hat e_\mu$.

$$J(\vec q) = \sum_i e^{i \vec q \cdot \vec r_i} J(\vec r_i, \vec r_j) = J \sum_{\pm\mu} e^{i a q_\mu} = 2J \sum_\mu \cos a q_\mu .$$

$$\tilde G(\vec q) = \sum_i e^{i \vec q \cdot \vec r_i} G(\vec r_i, \vec r_j)$$

$$\Rightarrow \tilde G(\vec q) = \frac{1 - M^2}{1 - 2\beta (1 - M^2) \sum_\mu \cos a q_\mu}$$

se $q_\mu a \ll 1$ $\quad \sum_\mu \cos a q_\mu \simeq 1 - \dfrac{1}{2} a^2 q^2 + \dfrac{1}{24} \sum_\mu a^4 q_\mu^4$

$qa \to 0$ Limite isotrópico:

$$\tilde{G}(\vec{q}) \simeq \frac{1-M^2}{1-2D\beta J(1-M^2)+\beta J(1-M^2)(aq)^2}$$

$$T>T_c \quad M=0 \Rightarrow \tilde{G}(q) \simeq \frac{1}{(1-2D\beta J)+\beta J q^2}$$

<u>Quebra de simetria e expoentes críticos.</u>

B uniforme:

$$\frac{1}{\beta}\tanh^{-1} M = \mu B + 2DJM$$

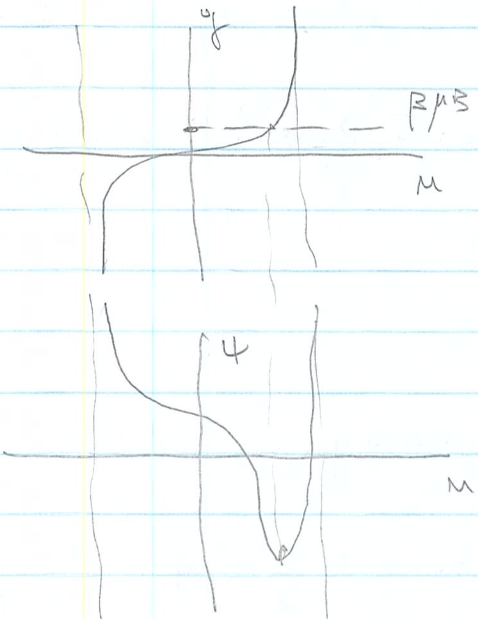
Definindo $g(M) \equiv \tanh^{-1} M - \beta K J M$

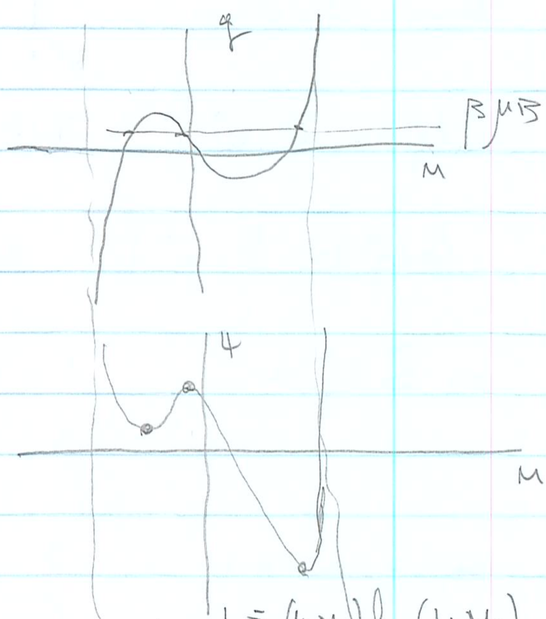
$K \equiv 2D$ # de vizinhos próximos

Campo médio: $g(M) = \beta \mu B$

$$M \to 0; \quad g(M) \simeq M - \beta K J M \Rightarrow \begin{cases} g'(0) > 0 & \text{se } T > KJ/k \\ g'(0) < 0 & \text{se } T < KJ/k \end{cases}$$

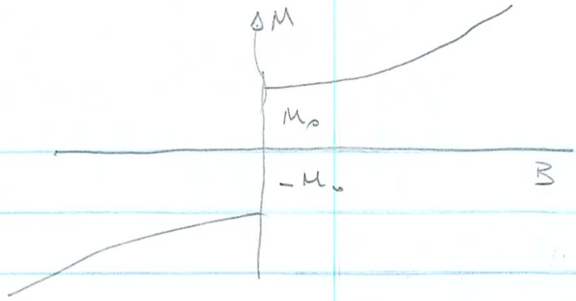
$T_c \equiv KJ/k$





$$\psi = -\sum_{ij} J_{ij} M_i M_j - \mu \sum_i B_i M_i + \frac{1}{\beta}\sum_i \left[\frac{1+M_i}{2}\ln\left(\frac{1+M_i}{2}\right) + \frac{1-M_i}{2}\ln\left(\frac{1-M_i}{2}\right)\right]$$

se $T < T_c$



$$M + \frac{M^3}{3} = \frac{\mu B}{kT} + \frac{kJM}{kT}$$

$$M + \frac{M^3}{3} = \frac{\mu B}{kT} + \frac{T_c}{T} M = \frac{T_c}{T}\left(\frac{\mu B}{kT_c} + M\right)$$

Expoentes críticos: i) $\tilde{\beta}$

Expandir $\tanh^{-1} M$ em série de potências de M:

$$\tanh^{-1} M \simeq M + \frac{M^3}{3} + \dots$$

$$\Rightarrow M_0 + \frac{1}{3} M_0^3 = \beta k J M_0 = \frac{T_c}{T} M_0$$

$$\Rightarrow M_0(T) = \sqrt{\frac{3}{T_c}}\,(T_c - T)^{1/2}$$ mas $M_0(T) \propto |T_c - T|^{\tilde{\beta}}$ se $T \to T_c$

$$\Rightarrow \boxed{\tilde{\beta} = 1/2}$$

ii) γ: calcular a susceptibilidade χ se $T > T_c$ e $B = 0$;

$$M + \frac{M^3}{3} = \frac{T_c}{T}\left(\frac{\mu B}{kT_c} + M\right)$$

se $M \ll 1$: $$M = \frac{T_c}{T}\frac{\mu B}{kT_c} + \frac{T_c}{T} M \Rightarrow M\left(1 - \frac{T_c}{T}\right) = \frac{\mu B}{kT}$$

$$M = \frac{\mu B}{k(T - T_c)} = \frac{\mu B}{kT_c}\frac{1}{(T/T_c - 1)} \Rightarrow \chi \sim \frac{T_c}{(T - T_c)} \Rightarrow \boxed{\gamma = 1}$$

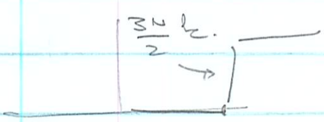
iii) δ: $T = T_c$ $\quad M \propto B^{1/\delta}$ $\quad M + \frac{M^3}{3} = \beta \mu B + kJM$

$$\Rightarrow \underbrace{(1 - kJ)}_{=0} M + \frac{M^3}{3} = \beta \mu B \Rightarrow M \sim B^{1/3}$$ ou $\boxed{\delta = 3}$

iv) α: $C \propto (T - T_c)^{-\alpha}$ se $B = 0$

Mas $$E = \begin{cases} -\frac{1}{2} kJN M_0^2 = \frac{3}{2} Nk (T - T_c) & T < T_c \\ 0 & T > 0 \end{cases}$$

$\Rightarrow C$ é descontínuo em T_c se $B = 0$



v) ν e η: $\tilde{G}(q) = \dfrac{1}{q^{2-\eta}} f(q\xi)$

$\tilde{G}(q) \cong \dfrac{1}{J\beta_c q^2 \left[1 + \dfrac{T-T_c}{J q^2}\right]}$; $f(q\xi) = \dfrac{1}{1 + \dfrac{1}{(q\xi)^2}}$

$\xi^2 \equiv J/|T-T_c| \Rightarrow \xi = \sqrt{J}\,(T-T_c)^{-1/2} \Rightarrow \boxed{\eta = 0}$ e $\boxed{\nu = 1/2}$

	C Médio	D = 2 (Exato)	D = 3 (Numérico)
$T_c/\kappa J$	1	0.57	0.75
α	disc.	$\ln\lvert T-T_c\rvert$	0.11 ± ...
β	1/2	0.125	0.312
γ	1	1.75	1.23
δ	3	15	5
η	0	0.25	0.03
ν	1/2	1	0.63

i) C. Médio prevê Transição que $\nexists$ a D=1

ii) C. Médio melhora à medida que D aumenta

iii) Lei de escala $\gamma = \nu(2-\eta)$ satisfeita p/ campo médio

iv) Expoentes de C.M. corretos se $D \geq 4$

v) T_c só quando $D \to \infty$

Cap XVI - Huang:

Fisher: $\gamma = \nu(2-\eta)$

Rushbrooke $\alpha + 2\beta + \gamma = 2$

Widom $\gamma = \beta(\delta - 1)$

Josephson $\nu D = 2 - \alpha$.