Teoria de Landau:

Ansatz variacional do campo médio:
$\phi(x_i) \to \Psi(M_i)$ que pode ser expandido para $M_i \ll 1$ (perto de T_c) como

$$\Psi(M_i) \simeq -\frac{1}{2}\sum_{ij} J_{ij} M_i M_j - \mu \sum_i M_i B_i + \frac{1}{\beta}\sum_i \frac{M_i^2}{2} + \frac{M_i^4}{12}$$

$M_i \equiv M(\vec{r}_i)$ $\quad M_{i+\alpha} = M(\vec{r}_i + a\hat{e}_\alpha)$

$$-\frac{1}{2}\sum_{ij} J_{ij} M_i M_j = -J\sum_{i\alpha} M_i M_{i+\alpha} = \frac{1}{2}\sum_{i\alpha}(M_i - M_{i+\alpha})^2$$

$$-DJ\sum_i M_i^2$$

$$\Psi = \frac{J}{2}\sum_{i\alpha}(M_i - M_{i+\alpha})^2 + \left(\frac{1}{2\beta} - DJ\right)\sum_i M_i^2 + \frac{1}{12\beta}\sum_i M_i^4$$

($\frac{1}{12\beta} \to u_0$)

$$-\mu\sum_i B_i M_i \qquad \frac{k}{2}(T - T_0) = \frac{kT_0}{2}\left(\frac{T}{T_0} - 1\right)$$

$$kT_0\left(\frac{T}{T_0} - 1\right) \equiv r_0(T) \quad \text{ou} \quad \bar{r}_0 (T - T_0)$$

$$\sum_i M_i^2 \to \frac{1}{a^D}\int d^D r\, M^2(\vec{r}) \; ; \; \sum_i M_i^4 \to \frac{1}{a^D}\int d^D r\, M^4(\vec{r})$$

$$\sum_i \mu B_i M_i \to \frac{\mu}{a^D}\int d^D r\, B(\vec{r}) M(\vec{r})$$

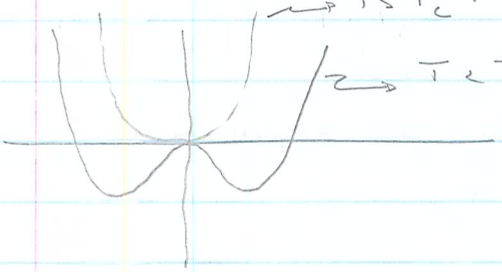
$$\sum_{i\alpha}(M_i - M_{i+\alpha})^2 \to \frac{1}{a^{D-2}}\int d^D r \left(\frac{M(\vec{r}) - M(\vec{r} + \vec{a})}{a^2}\right)^2$$

$$\frac{a^2}{a^D}\int d^D r\, (\vec{\nabla} M)^2$$

$\sqrt{Ja^2}\, M \to M$ $\qquad r_0$ e u_0

$$\mathcal{L}[M(\vec{r})] = \int d^D r \left\{ \frac{1}{2}(\vec{\nabla} M)^2 + \frac{1}{2} r_0(T) M^2(\vec{r}) + \frac{1}{4!} u_0 M^4(\vec{r}) - \mu B(\vec{r}) M(\vec{r}) \right\}$$

"densidade de energia potencial"



$\mathcal{L}[M(\vec r)] \Rightarrow$ se $M(\vec r) = M_0(\vec r) + \delta M(\vec r)$

$$\mathcal{L}[M_0(\vec r)+\delta M(\vec r)] = \mathcal{L}[M_0(\vec r)] + \int d^D r\, \delta M(\vec r)\, \frac{\delta \mathcal{L}}{\delta M(\vec r)}$$

$$+ \frac{1}{2}\int d^D r\, d^D r'\, \delta M(\vec r)\, \delta M(\vec r')\, \frac{\delta^2 \mathcal{L}}{\delta M(\vec r)\,\delta M(\vec r')}$$

$$f(\ldots x_i^{(0)} + dx_i \ldots) = f(x_i^{(0)} \ldots) + \sum_i \left.\frac{\partial f}{\partial x_i}\right|_{x_i^{(0)}} dx_i$$

$$+ \frac{1}{2}\sum_i\sum_j \left.\frac{\partial^2 f}{\partial x_i \partial x_j}\right|_{x_i^{(0)}} dx_i\, dx_j + \ldots$$

$$f(M_0(\vec r) + \delta M(\vec r)) = \left.\frac{\partial f}{\partial M}\right|_{M_0} \delta M(\vec r) + \ldots$$

$$\frac{1}{2}\int d^D r\, (\vec\nabla(M_0 + \delta M))^2\, d^D r = \frac{1}{2}\int d^D r\, (\vec\nabla M_0 + \vec\nabla \delta M)^2$$

$$= \frac{1}{2}\int d^D r\, \{(\vec\nabla M_0)^2 + 2\vec\nabla M_0 \cdot \vec\nabla \delta M + (\vec\nabla \delta M)^2\}$$

$$= \int d^D r\, \vec\nabla\cdot(\vec\nabla M_0\, \delta M) - \int d^D r\, \nabla^2 M_0\, \delta M$$

Campo médio: $\partial f / \partial M_i = 0$

$$\Rightarrow \frac{\delta \mathcal{L}}{\delta M} = -\nabla^2 M + r_0 M(\vec r) + \frac{u_0}{3!} M^3(\vec r) - \mu B(\vec r) = 0$$

B e M uniformes:

$$r_0 (T - T_0) M + \frac{u_0}{3!} M^3 = \mu B$$

Voltemos à eq. p/ M.

$$-\nabla^2 M + r_0(T) M + \frac{u_0}{3!} M^3 = \mu B(\vec{r})$$

Tomando $\delta / \delta B(\vec{r}')$

$$-\nabla^2 \frac{\delta M(\vec{r})}{\delta B(\vec{r}')} + r_0(T) \frac{\delta M(\vec{r})}{\delta B(\vec{r}')} + \frac{u_0}{2} M^2(\vec{r}) \frac{\delta M(\vec{r})}{\delta B(\vec{r}')} = \mu\, \delta(\vec{r}-\vec{r}')$$

mas $$\frac{\delta M(\vec{r})}{\delta B(\vec{r}')} = \frac{1}{\beta\mu} \frac{\partial^2 \ln Z}{\partial B(\vec{r}) \partial B(\vec{r}')} = \beta\mu\, G(\vec{r}-\vec{r}')$$

$$-\nabla^2 G + r_0(T) G + \frac{u_0}{2} M^2(\vec{r})\, G(\vec{r}-\vec{r}') = \delta(\vec{r}-\vec{r}')$$

$\Rightarrow$ F.T. p/ $M(\vec{r}) = M$

$$\left(q^2 + r_0(T) + \frac{u_0}{2} M^2\right) G(q) = 1$$

$\Rightarrow T > T_0$ $$G(q) = \frac{1}{q^2 + r_0(T)} = \frac{1}{q^2\left[1 + \frac{\bar{r}_0 (T-T_0)}{q^2}\right]}$$

$\Rightarrow \xi = [\bar{r}_0 (T-T_0)]^{-1/2} \Rightarrow \eta = 0$ e $\nu = 1/2$ $\quad\left\{ G(q) = \frac{f(q\xi)}{q^{2-\eta}} \right.$

$T < T_0$ e $B = 0$; $M_0^2 = -\frac{6 r_0}{u_0}$

$$\Rightarrow G(q) = \frac{1}{q^2\left(1 + \frac{2\bar{r}_0 (T_0 - T)}{q^2}\right)} \Rightarrow \xi = [2\bar{r}_0 (T_0 - T)]^{-1/2}$$

$\Rightarrow$ TFI: $$G(r) \simeq \frac{1}{2 r^{D-2}} \left(\frac{1}{2\pi}\right)^{\frac{D-1}{2}} \left(\frac{\xi}{r}\right)^{\frac{3-D}{2}} e^{-r/\xi}.$$

que é exato para $D = 3$.

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Paredes:

$$-\nabla^2 M + r_0 M + \frac{u_0}{6} M^3 = \mu B$$

$B = 0$ e $T < T_0 \Rightarrow r_0 = +\bar{r}_0 (T_0 - T) < 0$

1-D.

$$-\frac{d^2 M}{dx^2} - |r_0| M + \frac{u_0}{6} M^3 = 0.$$

$$\frac{d^2 M}{dx^2} + |r_0| M - \frac{u_0}{6} M^3 = 0$$

$M = \pm M_0$ $\qquad y \equiv M / M_0$

$$\frac{d^2 y}{dx^2} + |r_0| M_0 y - \frac{u_0 M_0^3}{6} y^3 = 0$$

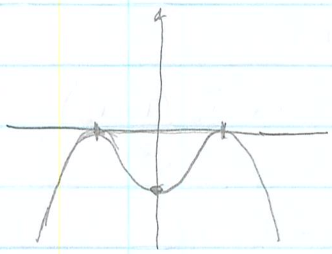
$$\frac{d^2 y}{dx^2} + ay - by^3 = 0.$$

y - posição

x - Tempo

$V'(y) = ay - by^3$

$V(y) \equiv \frac{1}{2} ay^2 - \frac{1}{4} by^4 + V_0$



$V'(y) = 0 \Rightarrow y = 0$ ou $y^2 = \frac{a}{b}$

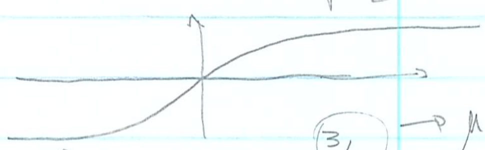
$$\frac{dy}{dx} \frac{d^2 y}{dx^2} + \frac{dy}{dx} V'(y) = 0$$

$$\frac{d}{dx}\left[\frac{1}{2}\left(\frac{dy}{dx}\right)^2 + V(y)\right] = 0 \Rightarrow \frac{1}{2}\left(\frac{dy}{dx}\right)^2 = -V(y)$$

$$\frac{1}{2}\left(\frac{dy}{dx}\right)^2 = \frac{b}{4} y^4 - \frac{1}{2} a y^2 \qquad \frac{dy}{dx} = \sqrt{\frac{by^4}{2} - ay^2}$$

$$\Rightarrow dx = \frac{dy}{\sqrt{\frac{by^4}{2} - ay^2}} \Rightarrow \int_0^x = \int_0^y \frac{dy}{\sqrt{\frac{by^4}{2} - ay^2}}$$

$M(x) = M_0 \tanh \frac{x}{2\xi}$



Energia na interface: $\sigma = \frac{2}{3} \frac{M_0^2}{\xi} \propto (T_0 - T)^{3/2} \to \mu$.

Quebra de simetria contínua:

Ising $\Leftrightarrow$ magneto c/ eixo fácil

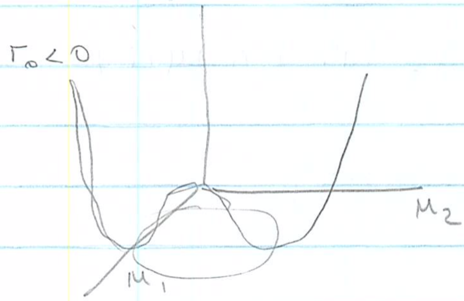
mas, em geral, $\vec{M} \in \mathbb{R}^3 \Rightarrow \dim \vec{M} = n = 3$.

Vamos estudar $n = 2 \Rightarrow \vec{M} = (M_1, M_2)$

$\frac{u_0}{3!} = w_0$: lembrar que $M_0 = \pm \frac{6 r_0}{u_0}$

$$\mathcal{L}(\vec{M}) = \int d^D r \left\{ \frac{1}{2}(\vec{\nabla} M_1)^2 + \frac{1}{2}(\vec{\nabla} M_2)^2 + \frac{w_0}{4}\left[M^2 + \frac{r_0}{w_0}\right]^2 \right\}$$

Potencial: $V(\vec{M}) = \frac{w_0}{4}\left[(M_1^2 + M_2^2)^2 + \frac{r_0}{w_0}\right]^2$



$$= \frac{1}{2} r_0 (M_1^2 + M_2^2) + \frac{w_0}{4}(M_1^2 + M_2^2)^2 + \text{cont.}$$

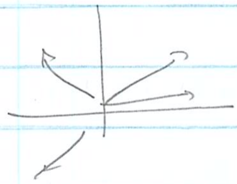
simetria $O(2)$

$M_1' = M_1 \cos\theta + M_2 \sin\theta$

$M_2' = -M_1 \sin\theta + M_2 \cos\theta$

$r_0 < 0 \Rightarrow$ mínimo de M: $M^2 = \frac{-r_0}{w_0} = \frac{|r_0|}{w_0} = v^2$

$\Rightarrow$



para direcionar $\vec{M}$, $\Rightarrow -\mu \vec{B} \cdot \int \vec{M}(\vec{r}') \, d^D r'$

se $\vec{B} = (B, 0) \Rightarrow M_1 = \sqrt{\frac{|r_0|}{w_0}}$ e $M_2 = 0$

$\Rightarrow \tilde{M} = M_1 - v$ $\quad V(\tilde{M}) = |r_0| \tilde{M}^2 + v w_0 \tilde{M}(\tilde{M}^2 + M_2^2) + \frac{w_0}{4}(\tilde{M}^2 + M_2^2)^2$

$$G^{-1}_{ij}(\vec{r} - \vec{r}') = \left. \frac{\delta^2 \mathcal{L}}{\delta M_i(\vec{r}) \, \delta M_j(\vec{r}')} \right|_{M_1 = v, M_2 = 0}$$

$$\left. \frac{\delta}{\delta M_1} \right|_{M_1 = v} = \left. \frac{\delta}{\delta \tilde{M}} \right|_{\tilde{M} = 0}$$

$\Rightarrow \tilde{G}_{11}(q) = \frac{1}{q^2 + 2|r_0|}$ $\quad \tilde{G}_{12} = \tilde{G}_{21} = 0$ $\quad \tilde{G}_{22} = \frac{1}{q^2}$

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$$\xi(T) = |2r_0(T)|^{-1/2}$$

$$\xi_{\parallel} = |2r_0(T)|^{-1/2} = [2\bar{r}_0(T_0 - T)]^{-1/2}$$

e $\xi_{\perp} = \infty$ ← boson de Goldstone → $m = 0$.

Em TQC:

$$\mathcal{L} = \frac{1}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}|\vec{\nabla}\phi|^2 - \frac{1}{2}m\phi^2 + \frac{1}{4}\lambda\phi^4$$

Hamiltoniana de GL:

Podemos obter GL através de uma 1ª aproximação para Z.

$$Z[B(\vec{r})] = \int \mathcal{D}\varphi(\vec{r})\, e^{-H[\varphi(\vec{r})] - \int d^D r\, B(\vec{r})\varphi(\vec{r})}$$

$$= \int \mathcal{D}\varphi(\vec{r})\, e^{-H_1[\varphi(\vec{r}), B(\vec{r})]}$$

$$H[\varphi] = \int d^D r \left\{ \frac{1}{2}|\nabla\varphi|^2 + \frac{1}{2}r_0\varphi^2(\vec{r}) + \frac{1}{4!}u_0\varphi^4(\vec{r}) \right\}$$

i) $\varphi(\vec{r})$ é uma variável estocástica $\varphi(\vec{r}) \in [-\infty, \infty]$ para cada $\vec{r}$.

ii) distribuição de probabilidade $e^{-H_1[\varphi, B]}$

iii) $\beta = 1$ porque $T \to T_c$ $\beta \to \beta_c$ e é ≈ cust.

iv) $\int \mathcal{D}\varphi(\vec{r}) = \mathcal{N}\int \prod_i d\varphi(\vec{r}_i)$

↳ $\Lambda = \frac{1}{a}$ é o cutoff.

Pt. de sela: $\left.\frac{\delta H}{\delta\varphi}\right|_{\varphi_0} = B$ $\left(\left.\frac{\delta H_1}{\delta\varphi}\right|_{\varphi_0} = 0 \right)$

$$\Rightarrow Z[B] \propto e^{-H[\varphi_0]}\, e^{\int d^D r\, B(\vec{r})\varphi_0(\vec{r})}$$

$$\ln Z[B] = -H[\varphi_0] + \int d^D r\, B(\vec{r})\varphi_0(\vec{r})$$

$$M(\vec{r}) = \frac{\delta \ln Z}{\delta B(\vec{r})} = -\int d^D r' \left.\frac{\partial H}{\partial \varphi(\vec{r}')}\right|_{\varphi_0} \frac{\delta \varphi(\vec{r}')}{\delta B(\vec{r})} + \varphi_0(\vec{r})$$

$$+ \int d^D r' \frac{\delta \varphi_0(\vec{r}')}{\delta B(\vec{r})} B(\vec{r}') = \varphi_0(\vec{r})$$

$$\Rightarrow \Gamma[M(\vec{r})] = \int d^D r \left[M(\vec{r}) B(\vec{r}) - \ln Z \right]$$ → substituir por p.to de sela $\Rightarrow \varphi_0 \to M$

$$\Rightarrow H[M(\vec{r})] = \int d^D r \left\{ \frac{1}{2} |\vec{\nabla} M|^2 + \frac{1}{2} r_0 M^2(\vec{r}) + \frac{1}{4!} u_0 M^4(\vec{r}) \right\}$$

Além da aproximação de Landau:

$$Z[B(\vec{r})] = \int \mathcal{D}\varphi(\vec{r})\, e^{-\frac{1}{\hbar} H_1[\varphi]}$$

$\hbar$ é artificial e $\to 0$; se $\psi(\vec{r}) = \varphi - \varphi_0$

$$\Rightarrow H_1[\varphi] \simeq H[\varphi_0] - \int d^D r\, B(\vec{r}) \varphi_0(\vec{r}) + \frac{1}{2} \int d^D r\, d^D r'\, \psi(\vec{r}) \mathcal{H}(\vec{r}, \vec{r}') \psi(\vec{r}')$$

$$\mathcal{H}(\vec{r}, \vec{r}') = \left.\frac{\delta^2 H[\varphi]}{\delta \varphi(\vec{r}) \delta \varphi(\vec{r}')}\right|_{\varphi = \varphi_0}$$

$$\Rightarrow \int \mathcal{D}\psi\, e^{-\frac{1}{2} \iint d^D r\, d^D r'\, \psi(\vec{r}) \mathcal{H}(\vec{r}, \vec{r}') \psi(\vec{r}')} \propto (\det \mathcal{H})^{-1/2}$$

$$= e^{-\frac{1}{2} \operatorname{Tr} \ln \mathcal{H}}$$

mas $\mathcal{H}(\vec{r}, \vec{r}') = \left\{ -\nabla^2 + r_0(T) + \frac{1}{2} u_0 \varphi_0^2 \right\} \delta(\vec{r} - \vec{r}')$

$$\operatorname{Tr} f = \int d^D r\, f(\vec{r}, \vec{r}) = \int d^D r'\, d^D r\, \delta^{(D)}(\vec{r} - \vec{r}')\, f(\vec{r} - \vec{r}')$$

$$= \int d^D r \int d^D r' \int \frac{d^D q}{(2\pi)^D} e^{i\vec{q}\cdot(\vec{r} - \vec{r}')} f(\vec{r} - \vec{r}') = V \int \frac{d^D q}{(2\pi)^D} \tilde{f}(\vec{q})$$

$\varphi_0(\vec{r}) = M = \text{const}$

$$\text{Tr} \ln \mathcal{H} = V \int \frac{d^D q}{(2\pi)^D} \ln \left(q^2 + r_0(T) + \frac{1}{2} u_0 M^2 \right)$$

$$\Gamma = \hbar \ln Z = H_1[\varphi_0] - \frac{\hbar}{2} \text{Tr} \ln \mathcal{H}$$

$$M = \frac{\delta(\hbar \ln Z)}{\delta B(\vec{r})} = \varphi_0 + \mathcal{O}(\hbar)$$

$$\Gamma[M] = H[M] + \frac{\hbar V}{2} \int \frac{d^D q}{(2\pi)^D} \ln \left(q^2 + r_0(T) + \frac{1}{2} u_0 M^2 \right) + \mathcal{O}(\hbar^2)$$

Critério de Ginzburg:

$$\frac{1}{V} \frac{\partial \Gamma}{\partial M} = B = r_0(T) M + \frac{u_0}{6} M^3 + \frac{u_0 \hbar}{2} M \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 + r_0(T) + \frac{1}{2} u_0 M^2}$$

Façamos $T > T_c$ $\quad B = M = 0$

$$\frac{\partial B}{\partial M} = \frac{1}{\chi} = \rho = r_0(T) + \frac{u_0 \hbar}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 + r_0(T)} + \mathcal{O}(\hbar^2)$$

$\frac{H}{\hbar}$ é adimensional $\qquad d^D r = [L]^D \qquad \vec{\nabla} = [L]^{-1}$

$\Rightarrow (\nabla \varphi)^2 [L]^D = [\hbar] \Rightarrow [L]^{-2} \varphi^2 [L]^D = [\hbar]$

$\varphi = [\hbar]^{1/2} [L]^{1 - D/2}$

$r_0 = [L]^{-2}$

$u_0 \varphi^4 = [L]^{-D} [\hbar] \Rightarrow u_0 [\hbar]^2 [L]^{4-2D} = [L]^{-D} [\hbar]$

$u_0 = [\hbar]^{-1} [L]^{D-4}$

como apenas u_0 depende dimensionalmente de $\hbar$

$\Rightarrow$ $\hbar = 1$ e expansão em u_0

$\rho = c_0 + c_1 u_0 + c_2 u_0^2 + \ldots$

$$\rho(T_c) \Rightarrow 0 \quad \Rightarrow \quad r_{0c} + \frac{u_0}{2}\int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2 + r_{0c}} = 0$$

$$r_{0c} \equiv r_0(T_c) = T_c - T_0$$

$$eq(\rho) - eq\ acima \Rightarrow \quad \rho = (r_0 - r_{0c})\left(1 + \frac{u_0}{2}\int \frac{d^Dq}{(2\pi)^D} \frac{1}{(q^2+r_0)(q^2+r_{0c})}\right)$$

$r_0 \to \rho \quad r_{0c} \to 0$

$$\rho = (r_0 - r_{0c})\left[1 + \frac{u_0}{2}\int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2(q^2+\rho)}\right]$$

se $D > 4$ $\quad \int < \infty$ em $q = 0$ mesmo p/ $\rho = 0$

$\Rightarrow \rho = (r_0 - r_{0c})(1 + u_0 A) = B(T - T_c)$

onde A, B constantes e $\gamma = 1$ como em GL.

$D < 4 \qquad q = \sqrt{\rho}\, q'$

$$\rho = (r_0 - r_{0c})\left(1 + \frac{u_0}{2}\rho^{\frac{D-4}{2}} \int \frac{d^Dq'}{(2\pi)^D} \frac{1}{q'^2(q'^2+1)}\right)$$

mesmo $u_0 \to 0$ $\quad \rho^{D-4/2} \to \infty$ se $T \to T_c$

Eq. p/ T_c

$$r_{0c} = r_0(T_c - T_0) = -\frac{u_0}{2}\int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2} = -\frac{u_0}{2} \frac{S_D \Lambda^{D-2}}{(2\pi)^D (D-2)}$$

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convergente no infra-vermelho se $D > 2$

$\Rightarrow$ GL válida se $D > 2$ e $D = 2$ é a dimensão crítica baixa do modelo,

se $q \to \infty$ não há divergência porque $\Lambda \sim \frac{1}{a}$

é o cutoff real do problema

$T_c < T_0$! e à medida que a dimensão diminui assim também o faz T_c