Formulação no espaço dos momentos

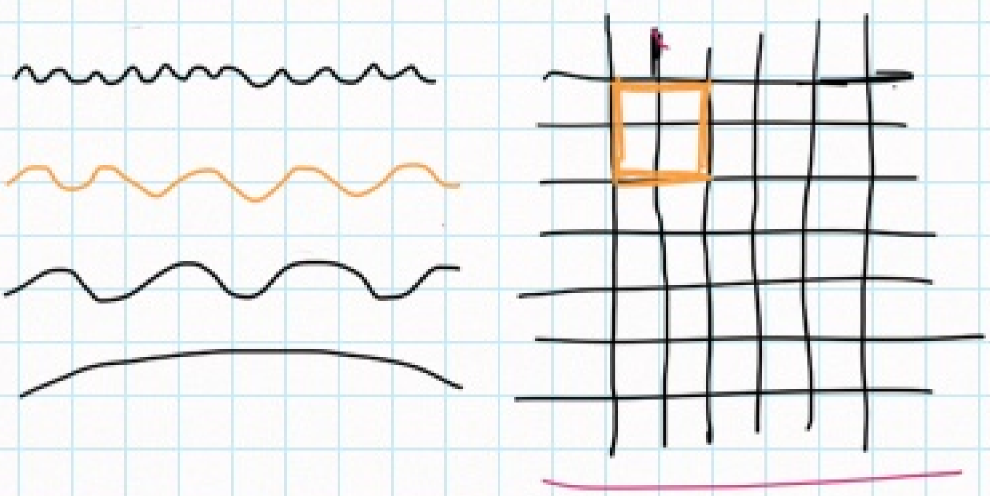
Teoria de Landau:

$$E[m] = \int (dx)\, \psi(m(x)) \quad ; \quad x \in \mathbb{R}^d \quad e \quad (dx) = d^d x$$

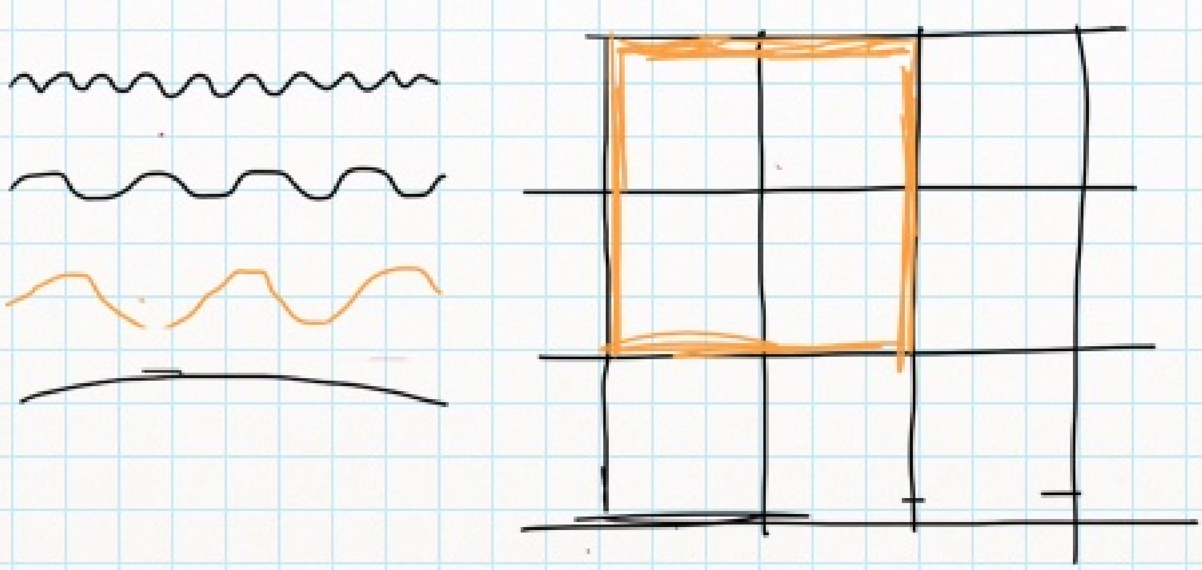
$$\psi(m(x)) = \frac{1}{2} |\vec{\nabla} m(x)|^2 + \sum_{n=1}^{\infty} K_n\, m^n(x) + \ldots$$

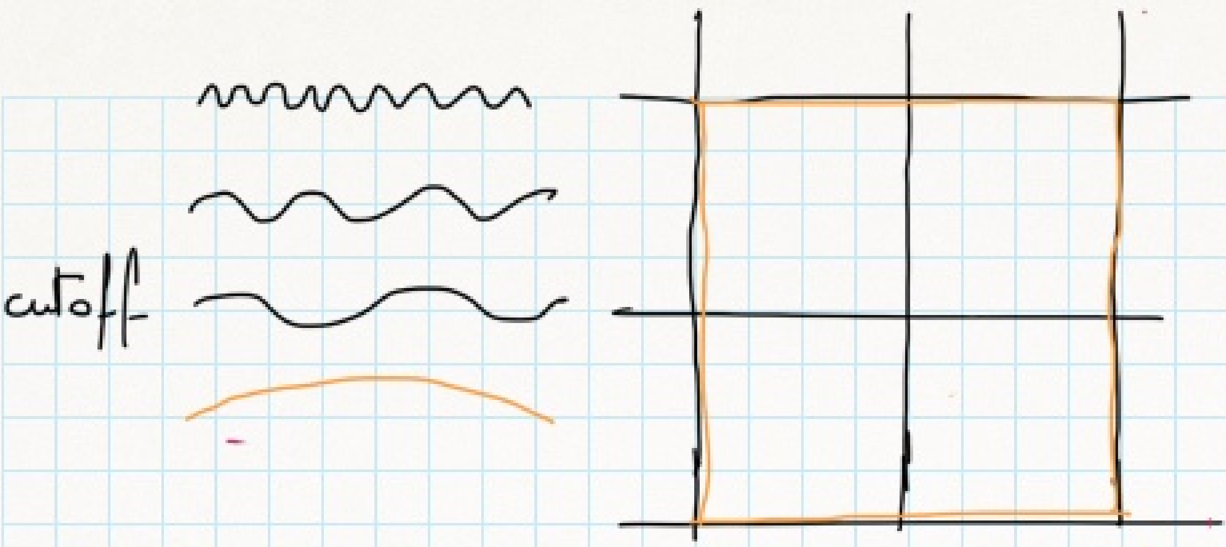
$m(x) \to$ "coarse graining", $K_n = K_n(\Lambda)$, $\Lambda \to \infty$

cutoff



cutoff





Em termos de $\tilde{m}(k) = \int_V (dx)\, e^{-ik.x}\, m(x)$

$k_i = \frac{2\pi n_i}{L}$; $m(x) = \frac{1}{V} \sum_k e^{ik.x}\, \tilde{m}(k) \xrightarrow[V\to\infty]{} \int (dk)\, e^{ik.x}\, \tilde{m}(k)$

$(dk) = \frac{d^d k}{(2\pi)^d}$. $\int_V (dx) e^{ik.x} = V \underbrace{\delta_k(k)}_{\text{Kronecker}} \xrightarrow[V\to\infty]{} (2\pi)^d \delta(k)$

$$\int (dx)\, |\vec{\nabla} m|^2 = \frac{1}{V} \sum_k k^2\, \tilde{m}^*(k)\, \tilde{m}(k)$$

$$\int (dx)\, m(x)\, H^*(x) = \frac{1}{V} \sum_k \tilde{m}(k)\, \tilde{H}^*(k)$$

$$\int (dx)\, |m(x)|^2 = \frac{1}{V} \sum_k \tilde{m}^*(k)\, \tilde{m}(k)$$

$$\int (dx)\, |m(x)|^4 = \frac{1}{V^3} \sum_{k_1 - k_4} \delta_K(k_1+k_2-k_3-k_4)\, \tilde{m}^*(k_1)\, \tilde{m}^*(k_2)\, \tilde{m}(k_3)\, \tilde{m}(k_4)$$

$m(x) \in \mathbb{R} \to \tilde{m}^*(k) = \tilde{m}(-k)$

Temos:

$$E[\tilde{m}] = \frac{1}{2}\int (dk)\, (k^2 + r_0)\, |\tilde{m}(k)|^2 + \ldots \quad \left(K_2 = \frac{r_0}{2}\right)$$

$$Q (= Z) \equiv e^{-G} = \mathcal{N} \prod_{k<|\Lambda|} \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E[\tilde{m}]}$$

Transformação do GR:

1) Integrar k's entre Λ e Λ/b $(b>1)$ p/ definir $E'[\tilde{m}]$

$$e^{-E'[\tilde{m}]} = e^{\Omega} \prod'_{\frac{\Lambda}{b}<|k|<\Lambda} \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E[\tilde{m}]}$$

$\Omega = \Omega(\Lambda, \{K\})$; $E'[m] \Leftrightarrow m(k)$ com $|k| < \Lambda/b$

$$E'[m] = \frac{1}{2}\int (dk)\, (Ak^2 + \tilde{r}_0)\, |\tilde{m}(k)|^2 + \ldots$$

$$Q \equiv e^{-G} = e^{\Omega} \mathcal{N} \prod'_{|k|<\frac{\Lambda}{b}} \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E'[\tilde{m}]}$$

2) Reescalar: restaurar Λ fazendo $k' = bk$

$$\Rightarrow E'[\tilde{m}] = \frac{b^{-d}}{2}\int (dk')\left(\frac{A}{b^2}k'^2 + \tilde{r}_0\right)\left|\tilde{m}\left(\frac{k'}{b}\right)\right|^2 + \dots$$

3) Normalização: coeficiente de $k'^2 = 1/2$

$$\tilde{m}'(k') = \sqrt{\frac{A}{b^{d+2}}}\,\tilde{m}\left(\frac{k'}{b}\right)$$; análogo à Transformação do spin do bloco (plaqueta)

$$\Rightarrow E'[m'] = \frac{1}{2}\int (dk')(k'^2 + r_0')|\tilde{m}'(k')|^2 + \dots \quad \left(r_0' = \frac{b^2}{A}\tilde{r}_0\right)$$

e $Q = e^{\Omega} N' \prod_{|k'|<\Lambda}' \int d\tilde{m}'(k')\, d\tilde{m}'^{*}(k')\, e^{-E'[\tilde{m}']}$

Resultado líquido: é equivalente a $K' = RK$ e

$$g(K) = \mu(K) + b^{-d} g(K')$$

Modelo gaussiano:

$$E[\tilde{m}] = \frac{1}{2}\int (dk)(k^2 + r_0)|\tilde{m}(k)|^2 - h\,\tilde{m}(0)$$

$h = \frac{H}{k_B T}$

Passos do GR:

1) Integração em cada $\underline{k}$ separadamente

$$Q = N' \prod'_{|k|<\frac{\Lambda}{b}} \int d\tilde{m}(k)\, d\tilde{m}^*(k)\, e^{-E'[m]} \quad ; \; E' = E$$

2) Restauração do velho Λ através de $k' = bk$

$$E'[m] = \frac{b^{-d}}{2}\int (dk')\left(\frac{k'^2}{b^2} + r_0\right)\left|\tilde{m}\left(\frac{k'}{b}\right)\right|^2 - h\,\tilde{m}(0)$$

3) Restauração da normalização de $\tilde{m}(k)$

$$\tilde{m}'(k') = b^{-(d+2)/2}\, \tilde{m}\left(\frac{k'}{b}\right)$$

$$E'[m'] = \frac{1}{2}\int (dk')\left(k'^2 + r_0'\right)\left|\tilde{m}'(k')\right|^2 - h'\tilde{m}'(0)$$

onde $\begin{cases} r_0' = b^2 r_0 & \Rightarrow D_t = 2 \\ h' = b^{(d+2)/2} h & \Rightarrow D_h = d+2/2 \end{cases}$

p^{to} fixo instável $(r_0, h) = (0,0)$. r_0 e h relevantes

$\xi^2 r_0 = \text{const} \Rightarrow \xi \sim r_0^{-1/2} \sim t^{-1/2}$

Renormalização de Q:

$$Q = e^{-G/k_BT} = \mathcal{N}\int (\mathcal{D}m)\, e^{-E[m,h]}$$

$$E[m,h] = \int (dx)\left[-\frac{1}{2} m(x)\nabla^2 m(x) + \frac{r_0}{2} m^2(x) - m(x)h(x)\right]$$

se $K \equiv -\nabla^2 + r_0$ $\quad Q = \mathcal{N}(\det K)^{-1/2} e^{\frac{1}{2}(h, K^{-1}h)}$

onde $(h, K^{-1}h) = \int (dx)\, h(x)(-\nabla^2 + r_0)^{-1} h(x) = \frac{1}{V}\sum_k \frac{\tilde{h}(k)\tilde{h}(-k)}{k^2 + r_0^2}$

Autovetores de K: $e^{ik.x}$ com autovalor $k^2 + r_0$

$$\Rightarrow \det K = \prod_k (k^2 + r_0)$$

$$\Rightarrow \ln Q = -\frac{1}{2}\sum_k \ln(k^2 + r_0) + \frac{1}{2V}\sum_k \frac{\hat{h}^*(k)\tilde{h}(k)}{k^2 + r_0}$$

Se $h = 0$ e $V \to \infty$: $\frac{G}{V} = -\frac{k_BT}{2}\int_0^\Lambda (dk)\ln(k^2 + a_0 t)$

$$\frac{C}{V} \sim \frac{1}{V}\frac{\partial^2 G}{\partial t^2}\Big|_{T=T_c} = \int_0^\Lambda dk\, \frac{k^{d-1}}{(k^2 + a_0 t)} \sim t^{(d-4)/2}\int_0^{\Lambda'(t)} \frac{ds\, s^{d-1}}{(s^2+1)^2}$$

$$\Lambda'(t) = \frac{\Lambda}{\sqrt{a_0 t}}$$

Q pode $\to \infty$ se $\Lambda \to \infty$ c/ a_0 - fixo. Mas

a_0 pode depender de Λ $\to$ const. acoplamento

ñ renormalizada. GR $\Rightarrow$ Q finito.

$$Q(a_0 t, \Lambda) = \mathcal{N} \left\{ \prod_{|k|<\frac{\Lambda}{b}} (k^2 + a_0 t)^{-1/2} \right\} \left\{ \prod_{\frac{\Lambda}{b}<|k|<\Lambda} (k^2 + a_0 t)^{-1/2} \right\}$$

$t=0$ no 2º termo porque sua contribuição é regular $\Rightarrow$ $Q(a_0 t, \Lambda) = \mathcal{N}' \prod_{|k|<\frac{\Lambda}{b}} (k^2 + a_0 t)^{-1/2}$

$k' = k/b$ $\to$ $Q(a_0 t, \Lambda) = \mathcal{N}' \, b^{N(\Lambda)} \prod_{|k'|<\Lambda} (k'^2 + a_0 b^2 t)^{-1/2}$

onde $N(\Lambda) = \#$ k's $\subset$ esfera em d dimensões e raio Λ

$$\Rightarrow Q\left(a_0 t, \frac{\Lambda}{b}\right) = b^{N(\Lambda)} \, Q\left(a_0 b^2 t, \Lambda\right)$$

$$\Rightarrow Q(a_0 t, \Lambda) = b^{-N(\Lambda)} \, Q\left(\frac{a_0 t}{b^2}, \frac{\Lambda}{b}\right)$$; escolhendo $b = \Lambda/\lambda$

$$Q(a_0 t, \Lambda) = Z(\Lambda, \lambda) \, Q(a t, \lambda) \; ; \; a = (\lambda/\Lambda)^2 a_0$$

a = const. acoplamento renormalizada. $Z = (\Lambda/\lambda)^{-N(\Lambda)}$ é fisicamente irrelevante.

A menos de uma constante aditiva

$$\frac{G(t)}{k_B T V} = -\frac{1}{V}\ln Q(at,\lambda) = \frac{1}{2}\int_0^{\lambda} dk\, S_d(k)\ln(k^2+at)$$

$$S_d(k) = \frac{2\pi^{d/2}\,k^{d-1}}{\Gamma\left(\frac{d}{2}+1\right)}$$; área da superfície da d-esfera de raio $= k$!

λ é arbitrário. $\lambda \to \lambda'$ apenas acrescenta uma constante a G sem alterar as singularidades do infravermelho da integral quando $t \to 0$.

<u>Modelo de Landau - Wilson</u>

$$E[m] = \int (dx)\left\{ \frac{|\vec{\nabla} m(x)|^2}{2} + \frac{r_0}{2} m^2(x) + u_0\, m^4(x) \right\}$$

Eq^s do G.R.

Integrar k's $\subset$ casca esférica reduzindo $\Lambda \to \Lambda/b$ $(b \gtrsim 1)$

$$N = \frac{1}{2} V S_d(\Lambda)\Delta k \Rightarrow \frac{\Delta k}{\Lambda} = \delta(\ln\Lambda) = \ln b$$

$m(x) = \bar{m}(x) + \delta m(x)$. Componentes de $\delta m(x)$ serão integrados e $\bar{m}(x)$ extra "coarse graining".

$$\delta m(x) = \sum_{i=1}^{N} c_i \phi_i(x) \quad \text{onde} \quad \phi_i(x) = \sum_{\frac{\Lambda}{b} < k < \Lambda} f(k)\, e^{i\vec{k}(x - x_i)}$$

x_i cobre todo o volume V. $\phi_i(x)$ objetos estendidos de dimensão $\delta x \sim 1/\delta k$ contendo ondas de $\lambda \sim 1/\Lambda$ e gozando das seguintes propriedades.

$$\int (dx)\, \phi_i^2(x) = 1 \quad ; \quad \int (dx) \left[\phi_i(x)\right]^n = 0 \quad (n \text{ ímpar})$$

$$\text{e} \int (dx)\, \phi_i(x)\, \phi_j(x) \simeq 0 \quad \text{se } i \neq j$$

$$m(x) = \overline{m}(x) + \delta m(x) \; ; \quad \delta m(x) = \sum_i c_i \phi_i(x)$$

Usar decomposição em $E[m]$ e:

a) $\overline{m}(x) \approx$ const no pacote

b) $\int (dx)\, |\vec{\nabla} \phi_i|^2 \approx \Lambda^2$

c) $(\delta m)^4 \to 0$

Então:

$$\int (dx)\frac{1}{2}\left|\vec{\nabla}(m(x)+\delta m)\right|^2 \simeq \int (dx)\,|\vec{\nabla}\bar{m}|^2 + \Lambda^2\sum_{i=1}^{N} c_i^2$$

$$\int (dx)\left[\bar{m}(x)+\delta m(x)\right]^2 \simeq \int (dx)\,\bar{m}^2(x) + \sum_{i=1}^{N} c_i^2$$

$$\int (dx)\left[\bar{m}(x)+\delta m(x)\right]^4 \simeq \int (dx)\,\bar{m}^4(x) + 6\sum_{i=1}^{N} c_i^2\,\bar{m}(x_i)$$

$$\bar{m}(x_i) \to \bar{m}(x\simeq x_i)$$

$$E[m] = E[\bar{m}] + \sum_{i=1}^{N}\left[\frac{1}{2}(\Lambda^2+r_0) + 6u_0\bar{m}^2(x_i)\right]c_i^2$$

$$GR \Rightarrow e^{-E[\bar{m}]} = \prod_{i=1}^{N}\int_{-\infty}^{\infty} dc_i\, e^{-E[m]}$$

$$= e^{-E[\bar{m}]}\prod_{i=1}^{N}\sqrt{\frac{\pi}{2}}\left[\Lambda^2+r_0+12u_0\bar{m}^2(x_i)\right]^{-1/2}$$

$$\Rightarrow E'[\bar{m}] = E[\bar{m}] + \frac{1}{2}\sum_{i=1}^{N}\ln\left\{\left(\frac{2\Lambda^2}{\pi}\right)\left[1+\frac{r_0}{\Lambda^2}+\frac{12u_0}{\Lambda^2}\bar{m}^2(x_i)\right]\right\}$$

$$\sum_i \to S_d(\Lambda)\,\delta k\int (dx) = C_d\Lambda^d(\ln b)\int (dx)\ ;\ C_d = 2\pi^{-d/2}/\Gamma\left(\frac{d}{2}+1\right)$$

Expandindo $\ln\{\ \}$ até 2ª ordem em r_0

$$E'[\bar{m}] = E[\bar{m}] + C_d \Lambda^d \ln b \int (dx) \left[6\left(\frac{u_0}{\Lambda^2} - \frac{r_0 u_0}{\Lambda^4}\right) \bar{m}^2(x) - \frac{36 u_0^2}{\Lambda^4} \bar{m}^4(x) \right]$$

$$\Rightarrow E'[\bar{m}] = \int (dx) \left[\frac{1}{2} |\vec{\nabla} \bar{m}(x)|^2 + \frac{1}{2} \tilde{r}_0 \bar{m}(x) + \tilde{u}_0 \bar{m}^4(x) \right]$$

$$\tilde{r}_0 = r_0 + 12(\ln b) C_d \left(\Lambda^{d-2} u_0 - \Lambda^{d-4} r_0 u_0 \right)$$

$$\tilde{u}_0 = u_0 - 36 (\ln b) C_d \Lambda^{d-4} u_0^2$$

GR 2 $x \to x'/b$ e GR3 $\to$ restaurar normalização de $m(x)$.

$\Rightarrow r_0' = b^2 \tilde{r}_0$ e $u' = b^{4-d} \tilde{u}_0$ Como $b \approx 1$ $b^n \approx 1 + n \ln b$

$\Rightarrow$ em 1ª ordem em $\ln b$

$$r_0' - r_0 = \left[2 r_0 + 12 C_d \left(\Lambda^{d-2} u_0 - \Lambda^{d-4} r_0 u_0 \right) \right] \ln b$$

$$u_0' - u_0 = \left[(4-d) u_0 - 36 C_d \Lambda^{d-4} u_0^2 \right] \ln b$$

1ºs termos, gaussianos. Λ e r_0 definidos por um fator finito. Escolha apropriada absorve C_d

Etapas do GR infinitesimais $\Rightarrow$ acoplamentos transformados são funções de $\ln b$: $r_0(\tau) = r_0'$ e $u(\tau) = u_0'$ com $r(0) = r_0$ e $u(0) = u_0$

$\tau = \ln b$

$$\frac{dr}{d\tau} = 2r + 12\Lambda^{d-2}u - 12\Lambda^{d-4}ur$$

$$\frac{du}{d\tau} = (4-d)u - 36\Lambda^{d-4}u^2$$

São as eq.s do GR.

<u>P.tos fixos e Trajetórias</u>

$$x \equiv \frac{r}{\Lambda^2} \quad e \quad y \equiv \frac{u}{\Lambda^{4-d}}$$

$$\frac{dx}{d\tau} = 2x - 12xy + 12y$$

$$= \quad \frac{dy}{d\tau} = \epsilon y - 36y^2 \quad \text{onde } \epsilon \equiv 4-d \ (\ll 1)$$

P.tos fixos: x^* e y^* t.q. $\frac{dx}{d\tau} = \frac{dy}{d\tau} = 0$

Gaussiano: $x^* = 0$ e $y^* = 0$

Ñ trivial: $x^* = -\frac{\epsilon}{6}$ e $y^* = \frac{\epsilon}{36}$

Na vizinhança de (x^*, y^*); $x = x^* + \delta x$ e $y = y^* + \delta y$

$$\Rightarrow \frac{d}{d\tau}\begin{pmatrix}\delta x\\ \delta y\end{pmatrix} = \begin{pmatrix} 2(1-6y^*) & 12(1-x^*) \\ 0 & \epsilon - 72y^*\end{pmatrix}\begin{pmatrix}\delta x\\ \delta y\end{pmatrix}$$

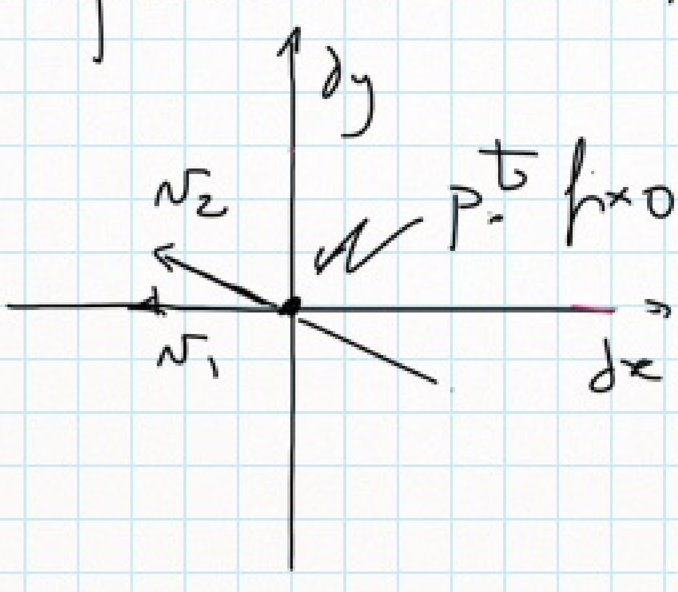
autovetores à esquerda e autovalores:

$$\phi_1 = \begin{pmatrix} 1 \\ \dfrac{12(1-x^*)}{2+60y^*-\epsilon}\end{pmatrix} \; ; \; \lambda_1 = 2(1-6y^*)$$

$$\phi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; ; \; \lambda_2 = (\epsilon - 72y^*)$$

Campos de escala $v_i = (\phi_i, q)$ $\quad q = \begin{pmatrix}\delta x\\ \delta y\end{pmatrix}$; $v_i(\tau) = v_0 e^{\lambda_i \tau} = v_0 b^{\lambda_i}$

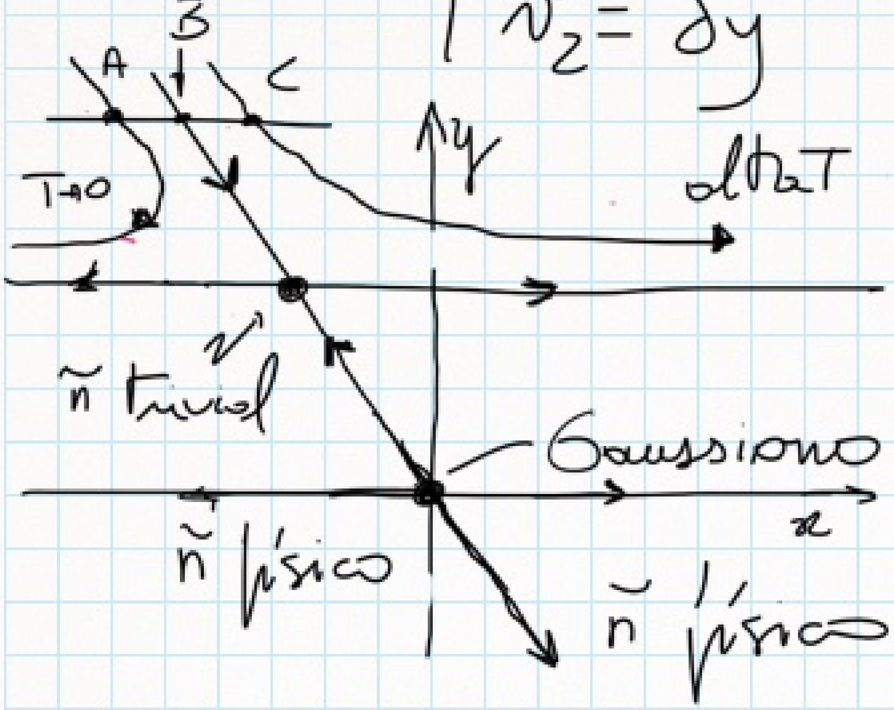
Explicitamente $v_1 = \delta x + \dfrac{12(1-x^*)}{2+60y^*-\epsilon}\,\delta y$ e $v_2 = \delta y$



p^{tos} fixos:

Gaussiano $\begin{cases} v_1 = \delta x + 6\,\delta y & \lambda_1 = 2 \\ v_2 = \delta y & \lambda_2 = \epsilon \end{cases}$

$\tilde{N}$ trivial: $\begin{cases} v_1 = \delta x + (6-\epsilon)\delta y \ ; & \lambda_1 = 2 - \epsilon/3 \\ v_2 = \delta y & ; \ \lambda_2 = -\epsilon \end{cases}$



Landau-Wilson $d < 4$

ñ-Trivial v_2 é irrelevante

e v_1 é relevante $\Rightarrow v_1 = 0$

é a superfície crítica.

Desprezando campo irrelevante $\Rightarrow \delta y = 0$ e $v_1 = \delta x$ ou

$v_1 \sim t \ \Rightarrow D_t = \lambda_1 = 2 - \epsilon/3$

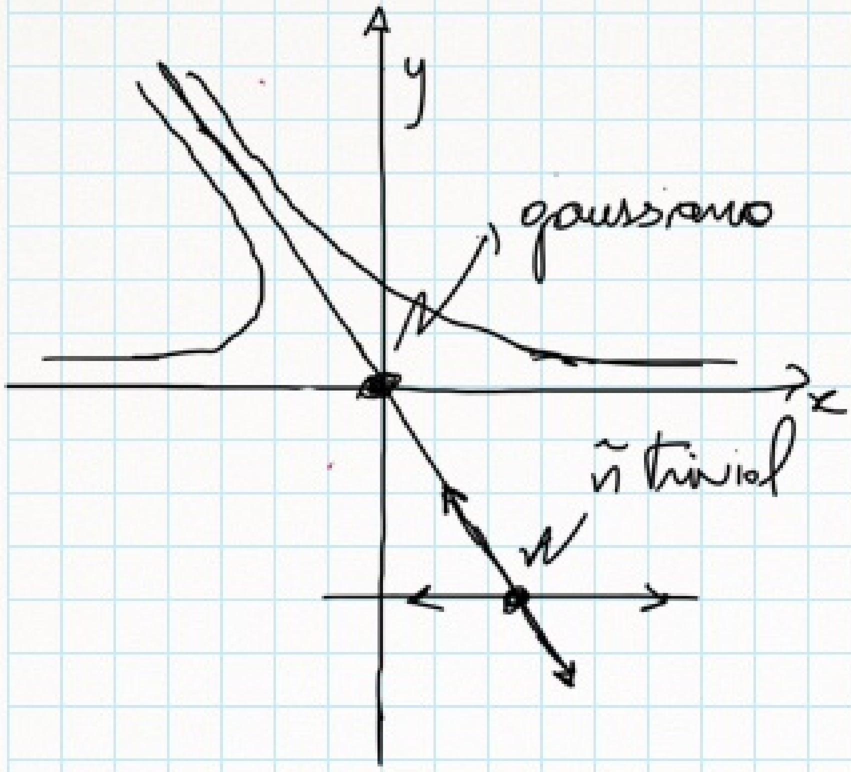
A, B, C → 3 Temperaturas diferentes $\begin{cases} T < T_c \rightarrow \text{flui p/ } T \rightarrow 0 \\ T = T_c \rightarrow \text{flui p/ pto fixo} \\ T > T_c \rightarrow \text{flui p/ } T \rightarrow \infty \end{cases}$

$\xi \sim b$ quando $b^{\lambda_1} t \sim 1 \Rightarrow \xi \sim t^{-1/D_t} \Rightarrow \nu = 1/D_t$

Demais podem ser obtidos usando D_h e D_t

$\Rightarrow \alpha = 2 - \dfrac{d}{2}\left(1 + \dfrac{\epsilon}{12}\right) \qquad \beta = \dfrac{d-2}{4}\left(1 + \dfrac{\epsilon}{6}\right)$

$\gamma = 1 + \frac{\epsilon}{6}$, $\delta = \frac{d+2}{d-2}$, $\nu = \frac{1}{2} + \frac{\epsilon}{12}$, $\gamma = 1 + \frac{\epsilon}{6}$ e $\eta = 0$



Landau-Wilson $d>4$ $(\epsilon<0)$

ñ trivial é instável

ν_1 e ν_2 relevantes

mas p.to na região ñ-física

$(u_0<0)$

Aqui não se pode desprezar m^6 para analisar a estabilidade do sistema. Baseados na TCM espera-se que as trajetórias que se originam do p.to ñ trivial fluam p/ região de transição de 1.ª ordem.