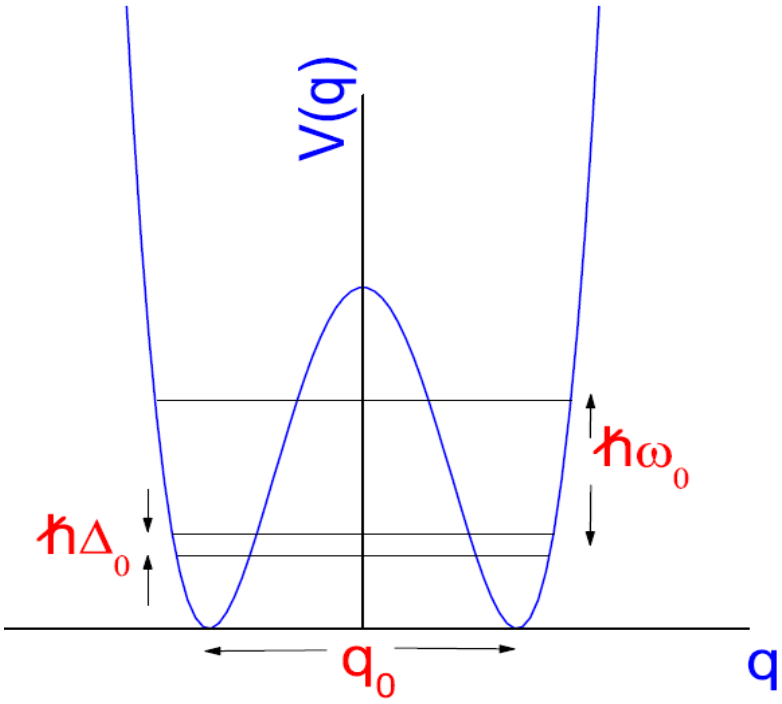
Dissipative quantum coherence

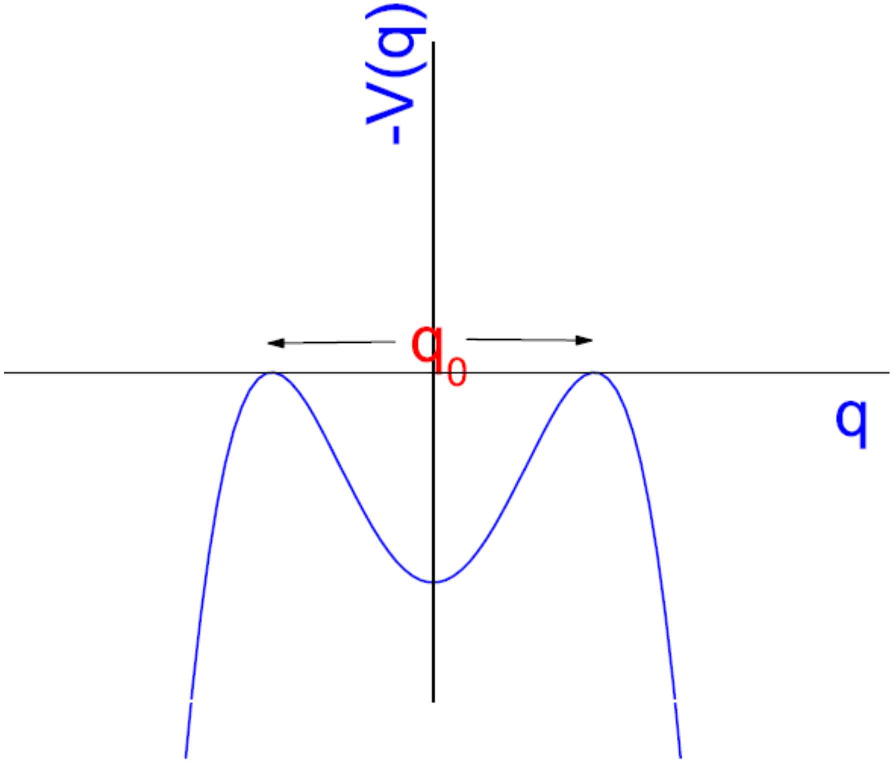


$$V(q) = \frac{M\omega_0^2 q_0^2}{32}\left[\left(\frac{q}{q_0/2}\right)^2 - 1\right]^2$$

$$\psi_E = \frac{1}{\sqrt{2}}(\psi_R + \psi_L) \quad \text{and} \quad \psi_O = \frac{1}{\sqrt{2}}(\psi_R - \psi_L)$$

$$\hbar\Delta_0 = E_O - E_E \propto \langle\psi_R|\mathcal{H}|\psi_L\rangle = \langle\psi_L|\mathcal{H}|\psi_R\rangle$$

Undamped case



$$\Delta_0 = A_0 \exp - \frac{B_0}{\hbar}$$

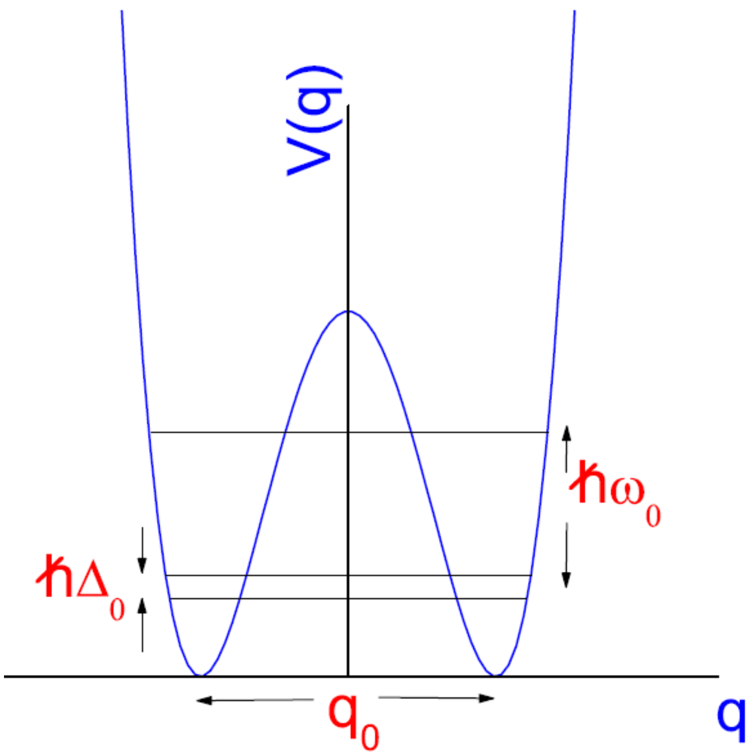
$$q_c^{(0)}(\tau) = \frac{q_0}{2} \tanh \frac{\omega_0 \tau}{2}$$

$$B_0 = \int_{-\infty}^{\infty} \left[\frac{1}{2} M \dot{q_c}^2 + V(q_c) \right] d\tau = \frac{4V_0}{\omega_0} \qquad V_0 = \frac{M\omega_0^2 q_0^2}{32}$$

$$A_0 = 2 \left(\frac{B_0}{2\pi\hbar} \right)^{1/2} \left| \frac{\det(-M\partial_t^2 + M\omega_0^2)}{\det'(-M\partial_t^2 + V''(q_c^{(0)}))} \right|^{1/2}$$

Undamped case

$$Z(\beta) = \text{const.} \int dq \int_{q(0)=q}^{q(\hbar\beta)=q} \mathcal{D}q(\tau) \exp - \frac{S_{eff}[q(\tau)]}{\hbar}$$



$$Z_0(\beta) = \cosh(\beta\hbar\Delta_0/2)$$

$$\hbar\Delta_0 = E_O - E_E$$

$$\propto \langle\psi_R|\mathcal{H}|\psi_L\rangle = \langle\psi_L|\mathcal{H}|\psi_R\rangle$$

$$H = -\frac{1}{2}\hbar\Delta\sigma_x + \frac{1}{2}\,\epsilon\,\sigma_z$$

$$\Delta,\ k_BT/\hbar,\ \epsilon/\hbar << \ \omega_0$$

Dissipative (ohmic) case

$$Z(\beta) = \text{const.} \int dq \int_{q(0)=q}^{q(\hbar\beta)=q} \mathcal{D}q(\tau) \exp - \frac{S_{eff}[q(\tau)]}{\hbar}$$

Slow oscillators $\omega \leq \Delta$

$$\omega_0 \to \omega_b$$

Fast oscillators $\omega \geq \omega_0$

$$\Delta,\ k_B T/\hbar,\ \epsilon/\hbar << \ \omega_c << \ \omega_b$$

$$J_0(\omega) = J(\omega) + J'(\omega) \begin{cases} J(\omega) \equiv J_0(\omega)\, e^{-\omega/\omega_c} \\ J'(\omega) \equiv J_0\,(\omega)(1 - e^{-\omega/\omega_c}) \end{cases}$$

$$\alpha'(\tau - \tau') = \frac{1}{2\pi} \int_0^\infty d\omega\, J'(\omega) e^{-\omega|\tau-\tau'|}$$

Spin-Boson Hamiltonian

$$Z(\beta) = \cosh(\beta\hbar\Delta/2) \qquad \Delta \equiv 2A \exp - S'_{eff}[q_c(\tau)]/\hbar$$

$$\Delta \sim \Delta_0 \exp\left[- \frac{q_0^2}{2\pi\hbar} \int_{\omega_c}^{\omega_b} d\omega \frac{J_0(\omega)}{\omega^2} \right] \sim \Delta_0 \left(\frac{\omega_c}{\omega_b} \right)^{\alpha}$$

$$\alpha \equiv \eta q_0^2 / 2\pi\hbar$$

Hilbert space $\quad \Psi_{\pm}(q, \{x_j\}) = \frac{1}{\sqrt{2}} [\Psi_R(q, \{x_j\}) \pm \Psi_L(q, \{x_j\})]$

$$\mathcal{H} = -\frac{1}{2}\hbar\Delta\sigma_x + \frac{1}{2}\epsilon\,\sigma_z + \frac{1}{2}\,q_0\,\sigma_z \sum_k C_k\,q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2}\,m_k\,\omega_k^2\,q_k^2$$

Spin-Boson dynamics: weak coupling

$$\frac{dS_x}{dt} = -\frac{S_x - S_x^{(eq)}}{T_1}$$

$$S_i = \hbar\langle\sigma_i\rangle/2$$

$$\frac{dS_y}{dt} = \Delta S_z - \frac{S_y}{T_2}$$

$$S_x^{(eq)} = \hbar\tanh(\hbar\beta\Delta)/2$$

$$\frac{dS_z}{dt} = -\Delta S_y$$

$$P(t) \equiv S_z(t)$$

$$\frac{d^2P}{dt^2} + \frac{1}{T_2}\frac{dP}{dt} + \Delta^2 P = 0$$

$$\frac{1}{T_1} = \frac{1}{T_2} = \frac{q_0^2}{2\hbar}J(\Delta)\coth\frac{\beta\hbar\Delta}{2}$$

Bloch equations in a field $\mathbf{B} = -\Delta\sigma_x\hat{\mathbf{x}} + \epsilon\sigma_z\hat{\mathbf{z}}$

Adiabatic renormalization

$$|\Psi_+^{(0)}\rangle = |+\rangle \prod_k |g_{k+}\rangle \quad \text{and} \quad |\Psi_-^{(0)}\rangle = |-\rangle \prod_k |g_{k-}\rangle$$

$$|g_{k\pm}\rangle = \exp\left(\pm \frac{1}{2} i\hat{\Omega}_k\right)|0\rangle_k \quad \text{where} \quad \hat{\Omega}_k \equiv \frac{q_0 C_k}{\hbar m_k \omega_k^2}\hat{p}_k$$

$$\Delta'(\omega_l) = \Delta \prod_k \langle g_{k+}|g_{k-}\rangle \qquad \Delta'(\omega_l) = \Delta \exp - \int_{\omega_l}^{\infty} \frac{q_0^2}{2\pi\hbar} \frac{J(\omega)}{\omega^2}\, d\omega$$

Renormalized "splitting" $\quad \Delta_r = \begin{cases} \Delta\left(\Delta/\omega_c\right)^{\frac{\alpha}{(1-\alpha)}} & \text{if} \quad \alpha < 1 \\ 0 & \text{if} \quad \alpha \geq 1 \end{cases}$

Feynman-Vernon approach

$$\rho(x_f, x_f, t) = \int_{x_i}^{x_f} \mathcal{D}x(\tau) \int_{x_i}^{x_f} \mathcal{D}y(\tau)\, \mathcal{A}[x(\tau)]\, \mathcal{A}^*[y(\tau)]\, \mathcal{F}[x(\tau), y(\tau)]$$

$$\mathcal{F}[x(\tau), y(\tau)] = \exp \frac{i}{\pi\hbar} \int_0^t \int_0^\tau d\tau d\sigma [x(\tau) - y(\tau)] L_1(\tau - \sigma)[x(\sigma) + y(\sigma)]$$

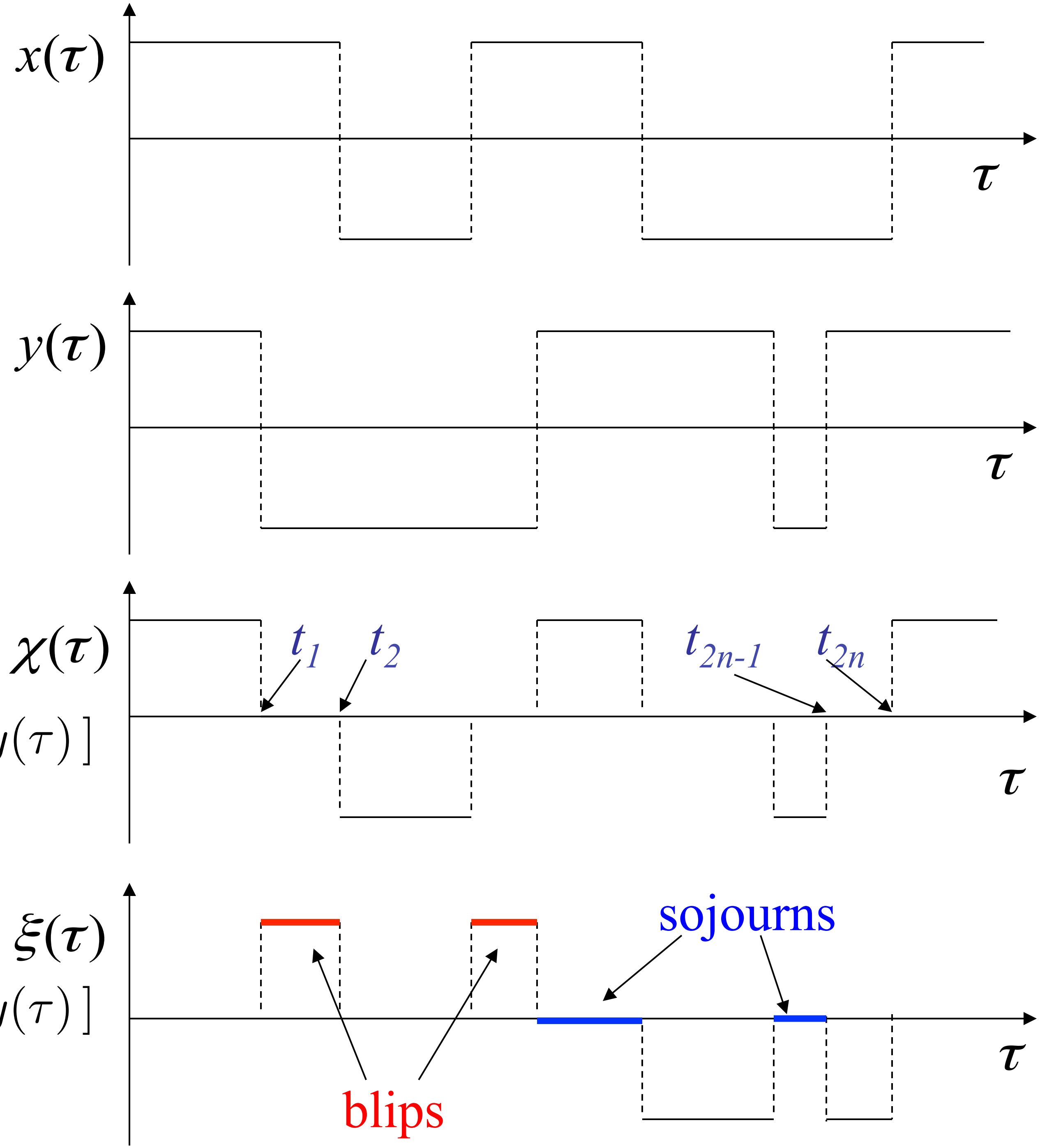
$$\times \exp - \frac{1}{\pi\hbar} \int_0^t \int_0^\tau d\tau d\sigma [x(\tau) - y(\tau)] L_2(\tau - \sigma)[x(\sigma) - y(\sigma)]$$

$$L_2(\tau - \sigma) \equiv \pi\alpha_R(\tau - \sigma) = \int_0^\infty d\omega\, J(\omega) \coth \frac{\hbar\omega}{2k_B T} \cos\omega(\tau - \sigma),$$

$$L_1(\tau - \sigma) \equiv -\pi\alpha_I(\tau - \sigma) = \int_0^\infty d\omega\, J(\omega) \sin\omega(\tau - \sigma)$$

$\{+,+\}$, $\{+,-\}$, $\{-,+\}$ and $\{-,-\}$ $\qquad (+) = q_0/2$ and $(-) = -q_0/2$

Paths of integration



$$\chi(\tau) \equiv q_0^{-1}[\,x(\tau) + y(\tau)\,]$$

$$\xi(\tau) \equiv q_0^{-1}[\,x(\tau) - y(\tau)\,]$$

Feynman-Vernon approach

$$\chi(\tau) \equiv q_0^{-1}[\,x(\tau) + y(\tau)\,] \qquad \xi(\tau) \equiv q_0^{-1}[\,x(\tau) - y(\tau)\,]$$

$$\mathcal{F}[x(\tau), y(\tau)] = \exp \frac{iq_0^2}{\pi\hbar} \int_0^t \int_0^\tau d\tau d\sigma \xi(\tau) L_1(\tau - \sigma)\chi(\sigma)$$

$$\times \exp - \frac{q_0^2}{\pi\hbar} \int_0^t \int_0^\tau d\tau d\sigma \xi(\tau) L_2(\tau - \sigma)\xi(\sigma)$$

$$A \equiv \{+,+\},\ B \equiv \{+,-\},\ C \equiv \{-,+\} \text{ and } D \equiv \{-,-\}$$

Amplitude to stay in the same state in *dt* is $\exp -i\epsilon\xi(t)dt$

Amplitude to flip in *dt* is $i\lambda \frac{\Delta}{2} dt \begin{cases} \lambda = 0 \text{ for } A \rightleftharpoons D \text{ and } B \rightleftharpoons C \\ \lambda = -1 \text{ for } A \rightleftharpoons B \text{ and } D \rightleftharpoons C \\ \lambda = 1 \text{ for } A \rightleftharpoons C \text{ and } B \rightleftharpoons D \end{cases}$

NIBA- Non-Interacting Blip Approximation

$$P(t) = \sum_{n=0}^{\infty}(-1)^n \underbrace{\Delta^{2n}}_{} \underbrace{K_n(t)}_{}$$

from the hops

from the influence functional

$$K_n(t) \equiv 2^{-n}\sum_{\{\zeta_j\}}\int_0^t dt_{2n}\int_0^{t_{2n}} dt_{2n-1}\ldots\int_0^{t_2} dt_1\, F_n(t_1, t_2, \ldots, t_{2n}; \zeta_1, \zeta_2, \ldots, \zeta_n; \epsilon),$$

$$F_n(t_1 \ldots t_{2n}) = \prod_{j=1}^{n}\cos\left(\frac{q_0^2}{\pi\hbar}Q_1(t_{2j}-t_{2j-1})\right)\exp-\frac{q_0^2}{\pi\hbar}Q_2(t_{2j}-t_{2j-1})$$

$$Q_1(t) = \int_0^{\infty}\frac{J(\omega)}{\omega^2}\sin\omega t\; d\omega$$

$$Q_2(t) = \int_0^{\infty}\frac{J(\omega)}{\omega^2}(1-\cos\omega t)\coth\left(\frac{\beta\hbar\omega}{2}\right)d\omega$$

NIBA- Non-Interacting Blip Approximation

$$P(t) = \sum_{n=0}^{\infty} (-1)^n \int_0^t dt_{2n} \int_0^{t_{2n}} dt_{2n-1} \ldots \int_0^{t_2} dt_1 \prod_{j=1}^{n} f(t_{2j} - t_{2j-1})$$

$$f(t) = \Delta^2 \cos\left(\frac{q_0^2}{\pi\hbar} Q_1(t)\right) \exp - \frac{q_0^2}{\pi\hbar} Q_2(t)$$

$$P(t) \; = \; \frac{1}{2\pi i} \int_C d\lambda \, e^{\lambda t} \tilde{P}(\lambda)$$

$$\tilde{P}(\lambda) \; = \; \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} dt \int_0^{\infty} dt_1 \ldots \int_0^{\infty} dt_{2n} e^{-\lambda(t_1 + t_2 + \ldots + t_{2n})} \prod_{j=1}^{n} f(t_{2j})$$

$$= \; \sum_{n=0}^{\infty} (-1)^n \frac{[f(\lambda)]^n}{\lambda^{n+1}} = \frac{1}{\lambda + f(\lambda)}$$

NIBA- Non-Interacting Blip Approximation

$$P(t) = \frac{1}{2\pi i}\int_C d\lambda\, e^{\lambda t}\tilde{P}(\lambda) \equiv \frac{1}{2\pi i}\int_C d\lambda\, e^{\lambda t}\left[\lambda + f(\lambda)\right]^{-1}$$

$$f(\lambda) \equiv \Delta^2 \int_0^\infty dt\, \cos\left[\frac{q_0^2}{\pi\hbar}Q_1(t)\right] \exp - \left[\lambda t + \frac{q_0^2}{\pi\hbar}Q_2(t)\right]$$

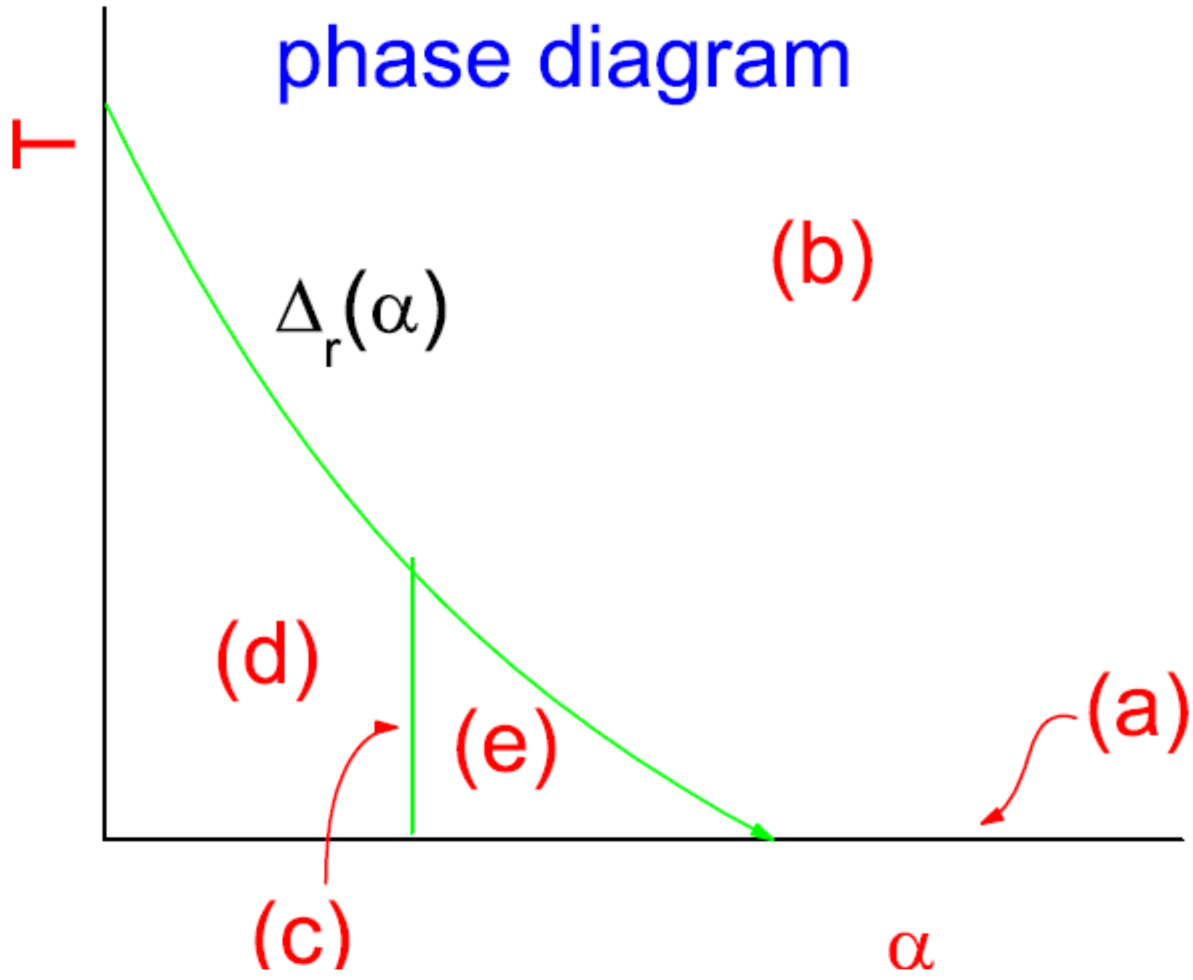
$$Q_1(t) = \int_0^\infty \frac{J(\omega)}{\omega^2}\, \sin\omega t\; d\omega$$

$$Q_2(t) = \int_0^\infty \frac{J(\omega)}{\omega^2}\,(1-\cos\omega t)\; \coth\left(\frac{\beta\hbar\omega}{2}\right)\; d\omega$$

For Ohmic dissipation

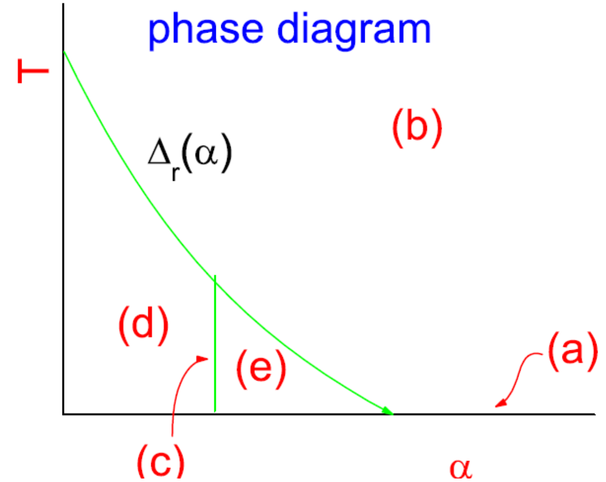
$$\begin{cases} Q_1(t) = \eta \tan^{-1}\omega_c t \\ Q_2(t) = \eta \ln(1+\omega_c^2 t^2) + \eta \ln\left[\frac{\beta\hbar}{\pi t}\sinh\frac{\pi t}{\beta\hbar}\right] \end{cases}$$

Solutions

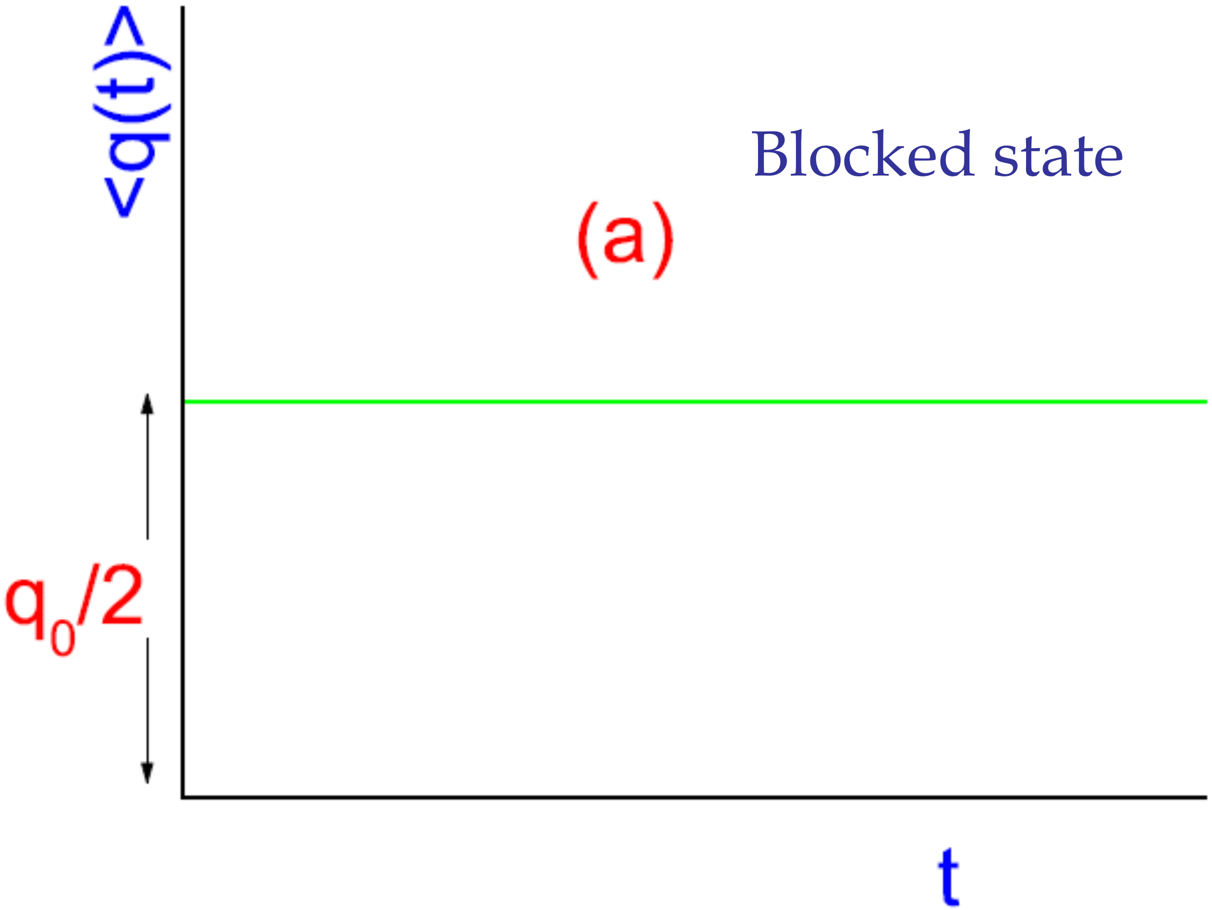


Solutions

$$\Delta_r = \begin{cases} \Delta\left(\Delta/\omega_c\right)^{\frac{\alpha}{(1-\alpha)}} & \text{if} \quad \alpha < 1 \\ 0 & \text{if} \quad \alpha \geq 1 \end{cases}$$

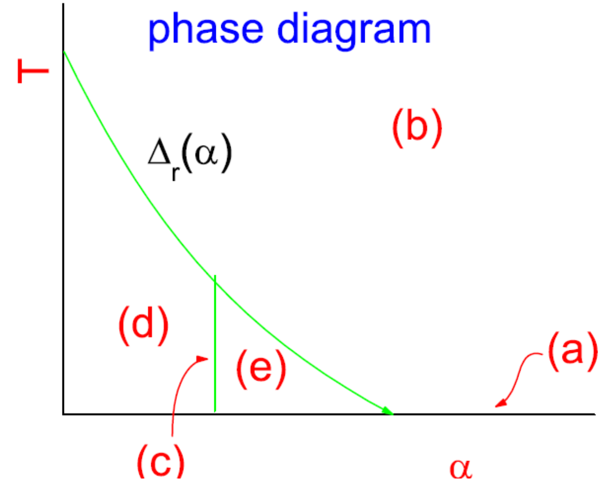


Blocked state



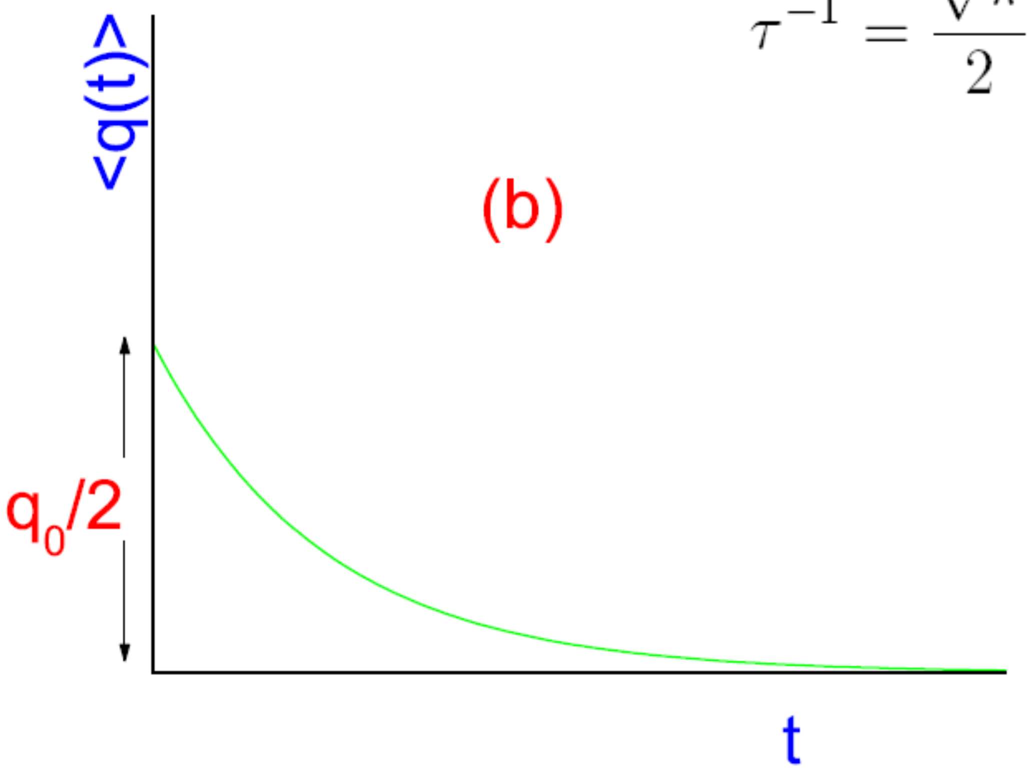
Solutions

Exponential relaxation



$$P(t) = \exp - \frac{t}{\tau}$$

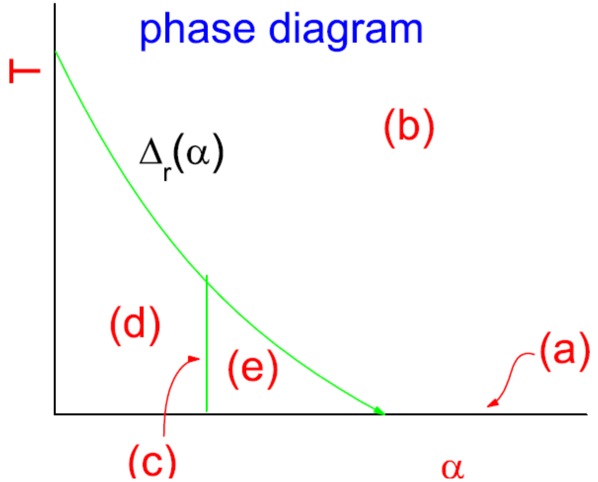
$$\tau^{-1} = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\alpha)}{\Gamma(\alpha + \frac{1}{2})} \frac{\Delta_r^2}{k_B T/\hbar} \left[\frac{\pi k_B T}{\hbar \Delta_r} \right]^{2\alpha}$$



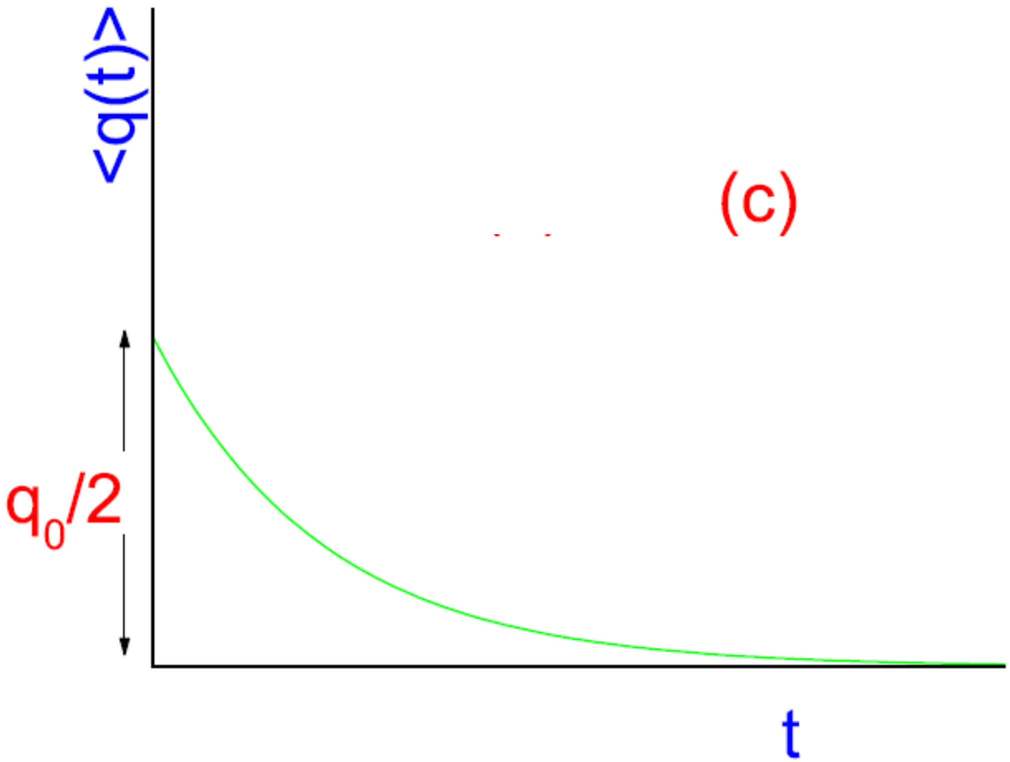
$$\alpha k_B T/\hbar \gg \Delta_r$$

Solutions

Exponential relaxation: exact solution



$$P(t) = \exp\left[-\frac{\pi}{2}\frac{\Delta^2}{\omega_c}t\right]$$

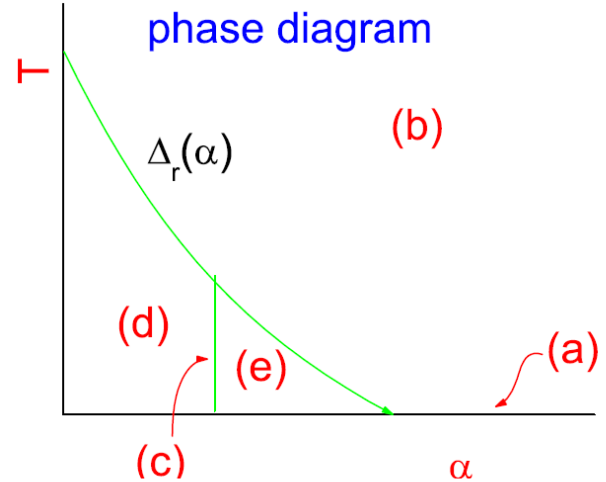


$$\Delta \sim (\omega_c/\omega_b)^\alpha$$

$$\alpha = 1/2$$

Solutions

$$y \equiv \Delta_{eff} t$$



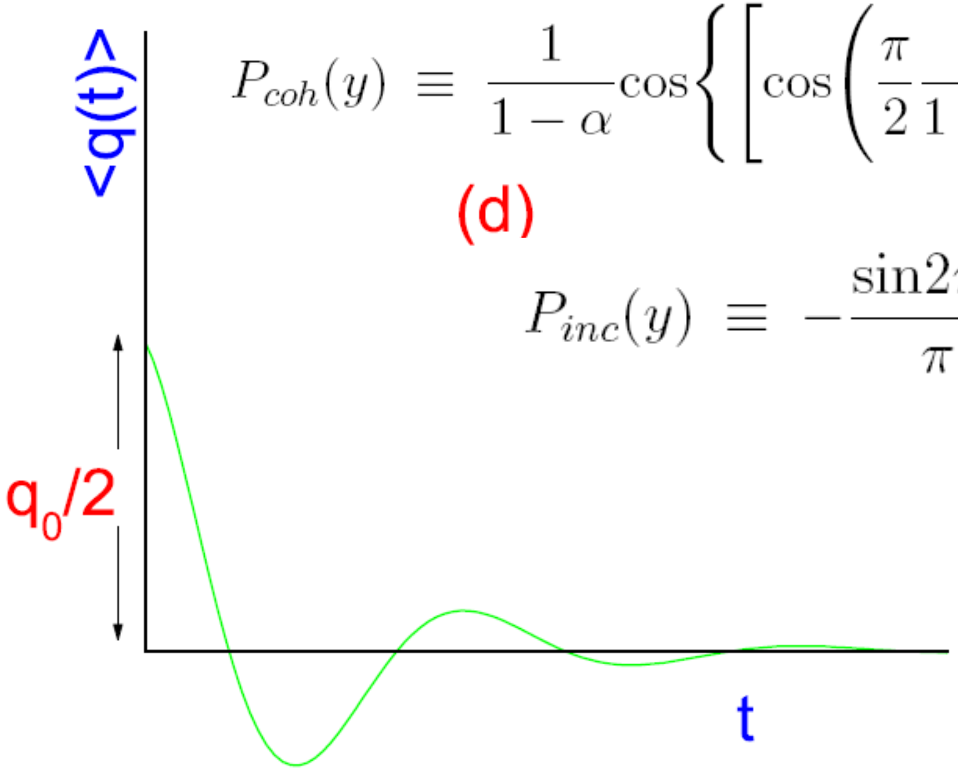
$$\Delta_{eff} \equiv [\Gamma(1-2\alpha)\cos\pi\alpha]^{\frac{1}{2(1-\alpha)}} \Delta_r$$

$$P(y) = P_{coh}(y) + P_{inc}(y)$$

$$P_{coh}(y) \equiv \frac{1}{1-\alpha}\cos\left\{\left[\cos\left(\frac{\pi}{2}\frac{\alpha}{1-\alpha}\right)\right]y\right\}\exp - \left\{\left[\sin\left(\frac{\pi}{2}\frac{\alpha}{1-\alpha}\right)\right]y\right\}$$

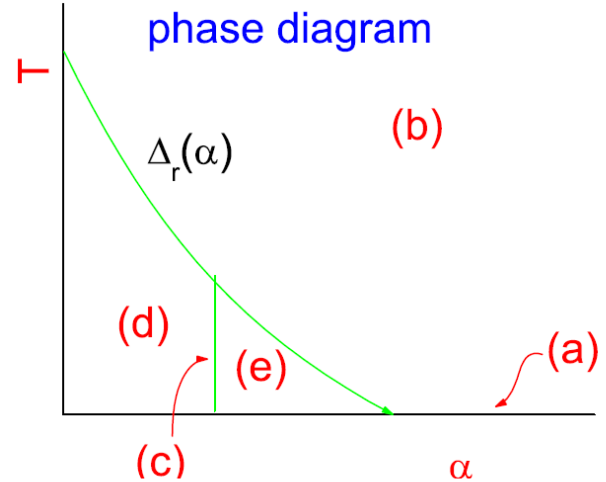
(d)

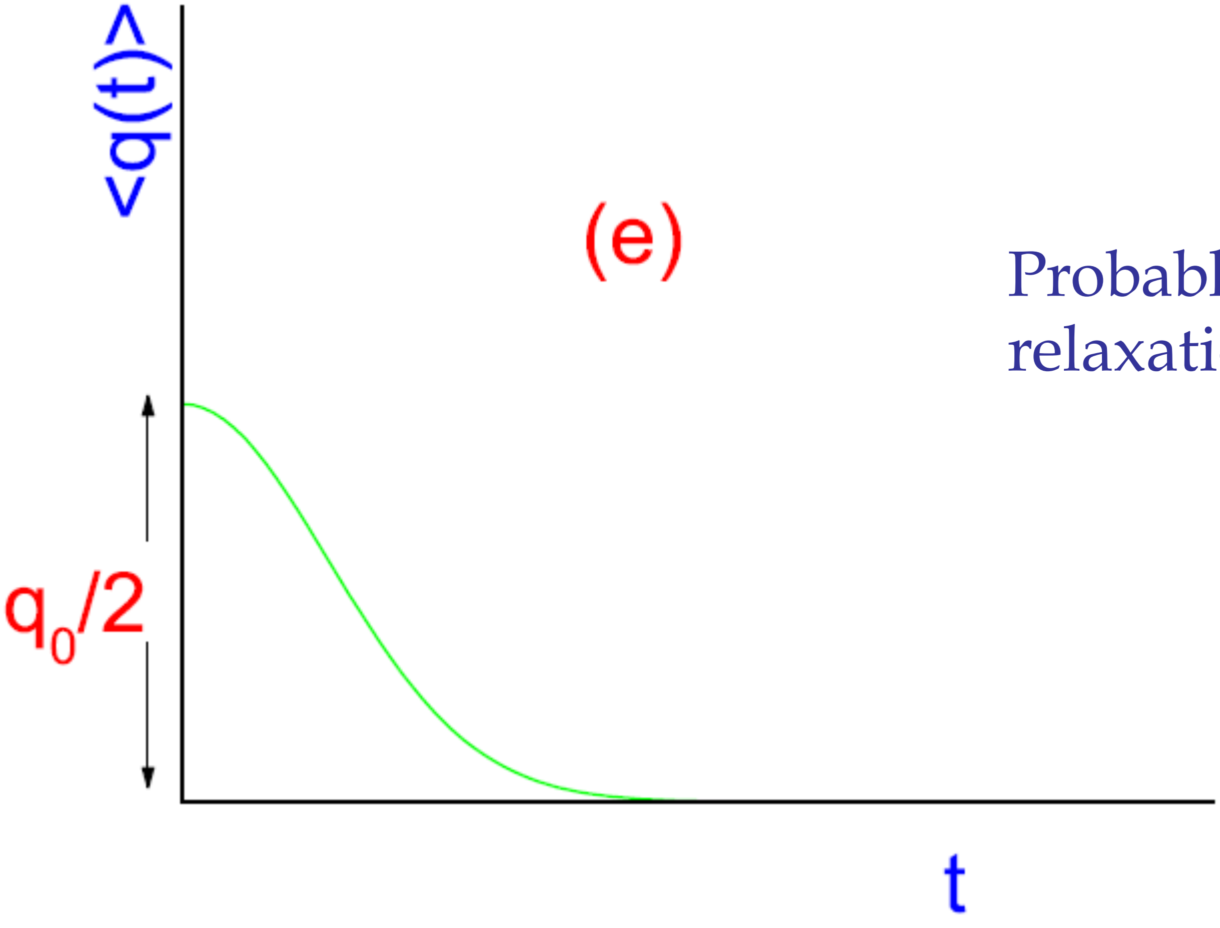
$$P_{inc}(y) \equiv -\frac{\sin 2\pi\alpha}{\pi}\int_0^\infty dz \frac{z^{2\alpha-1}e^{-zy}}{z^2 + 2z^{2\alpha}\cos 2\pi\alpha + z^{4\alpha-2}}$$



Oscillatory decay

Solutions





Probably incoherent relaxation