Superconducting qubits

Fundamental requirements for good qubits

i) well-defined two state systems
ii) accuracy in preparing the initial state
iii) long phase coherence ; order of 10^4 coherent operations
iv) controllable effective fields
v) quantum measurement to read out the quantum information

Model hamiltonian $H = H_{qb} + H_{meas} + H_{env}$

$$H_{qb} = -\frac{1}{2}\sum_{i=1}^{N} \mathbf{B}^{(i)}(t) \cdot \vec{\sigma}^{(i)} + \sum_{i \neq j} \sum_{a,b} J_{ij}^{ab}(t) \sigma_a^{(i)} \sigma_b^{(j)}$$

$$a, b = x, y, z$$

We consider only 1 qubit coupled to its environment.

Flux qubit

This is the model we introduced before to study quantum coherent tunnelling of the flux

$$H_{fqb} = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x$$

When the external flux bias is about half flux quantum

$$B_z(\phi_x) = 4\pi\sqrt{6(\beta_L - 1)}E_J\left(\frac{\phi_x}{\phi_0} - \frac{1}{2}\right) \qquad \beta_L \equiv 2\pi L i_0/\phi_0$$

This represents the bistable potential at half flux quantum.

We have addressed this problem before.

Charge qubit

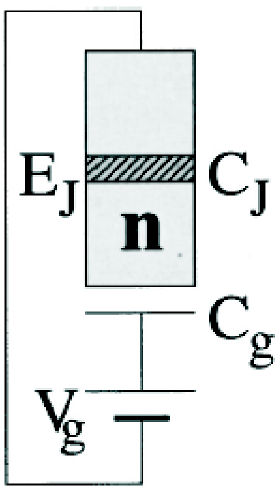


FIG. 1. A Josephson charge qubit in its simplest design formed by a superconducting single-charge box.

Charging energy

$$E_C = \frac{e^2}{2(C_J+C_g)}$$

Cooper pair box hamiltonian

$$H_{cqb} = 4E_C(n-n_g)^2 - E_J \cos\varphi$$

Canonical momentum

$$n = -i\hbar \frac{\mathrm{d}}{d(\hbar\varphi)}$$

if $n_g = \frac{1}{2}$ our hamiltonian becomes

$$H_{cqb} = \sum_n [4E_C(n - n_g)^2 |n\rangle\langle n| - \frac{1}{2} E_J(|n\rangle\langle n+1| + |n+1\rangle\langle n|)]$$

Within the two-dimensional subspace

$$|\uparrow\rangle \equiv |n=0\rangle \text{ and } |\downarrow\rangle \equiv |n=1\rangle$$

New two-state system hamiltonian $H_{cqb} = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x$

$$B_z = 4E_C(1 - 2n_g) \qquad \text{and} \qquad B_x = E_J$$

Band structure of the CPB

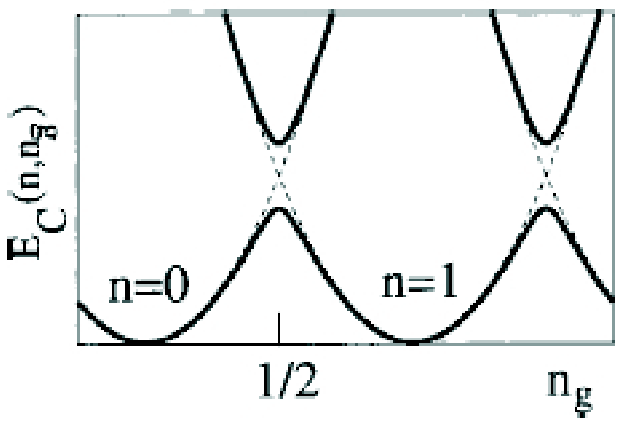


FIG. 2. The charging energy of a superconducting electron box is shown as a function of the gate charge n_g for different numbers of extra Cooper pairs n on the island (dashed parabolas). Near degeneracy points the weaker Josephson coupling mixes the charge states and modifies the energy of the eigenstates (solid lines). In the vicinity of these points the system effectively reduces to a two-state quantum system.

Transverse field can be tuned with a new circuit design

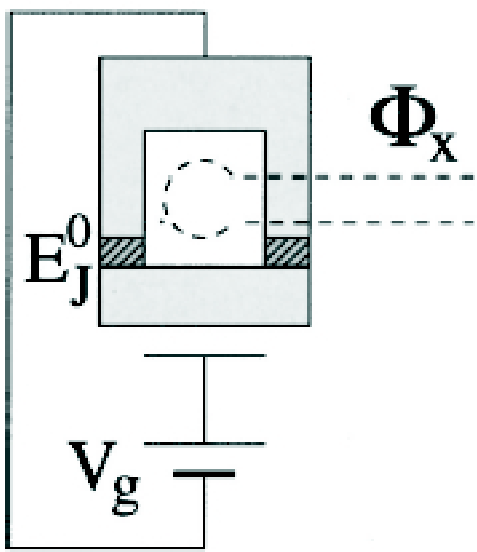
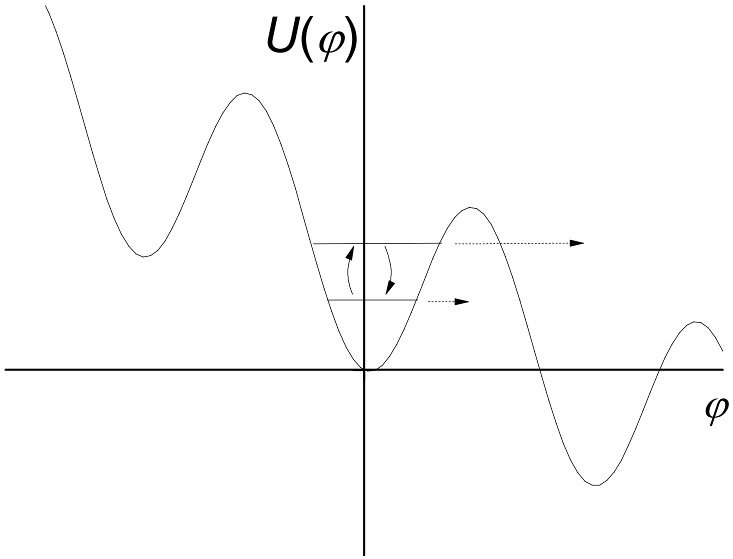


FIG. 3. A charge qubit with tunable effective Josephson coupling. The single Josephson junction is replaced by a flux-threaded SQUID. The flux in turn can be controlled by a current-carrying loop placed on top of the structure.

$$B_x = E_J(\phi_x) = 2E_J^0 \cos\left(\pi \frac{\phi_x}{\phi_0}\right)$$

Phase qubit and transmons

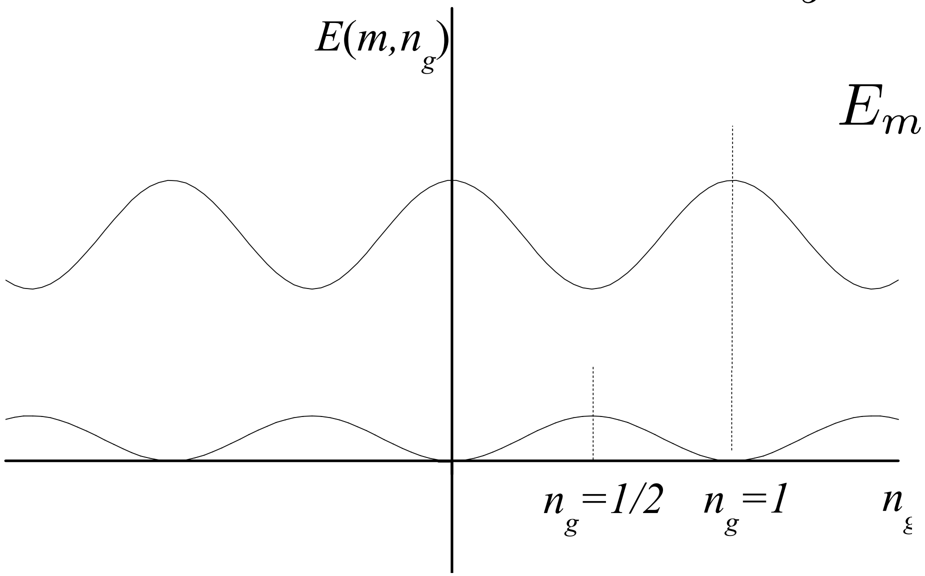


$$I(t) = I_{\rm dc} + \delta I_{\rm dc}(t)$$
$$+I_{\mu{\rm wc}}(t)\cos\omega_{10}t + I_{\mu{\rm ws}}(t)\sin\omega_{10}t$$

$$\delta I_{\rm dc}(t), I_{\mu{\rm wc}}(t),\ \text{and}\ I_{\mu{\rm ws}}(t)$$
$$\text{slow in comparison with } 2\pi/(\omega_{10}-\omega_{21})$$

$$H(t) = \frac{\sigma_z}{2}\,\delta I_{\rm dc}(t)\,\frac{\partial E_{10}}{\partial I_{\rm dc}} + \frac{\sigma_x}{2}\sqrt{\frac{\hbar}{2\omega_{10}C}}\,I_{\mu{\rm wc}}(t) + \frac{\sigma_y}{2}\sqrt{\frac{\hbar}{2\omega_{10}C}}\,I_{\mu{\rm ws}}(t)$$

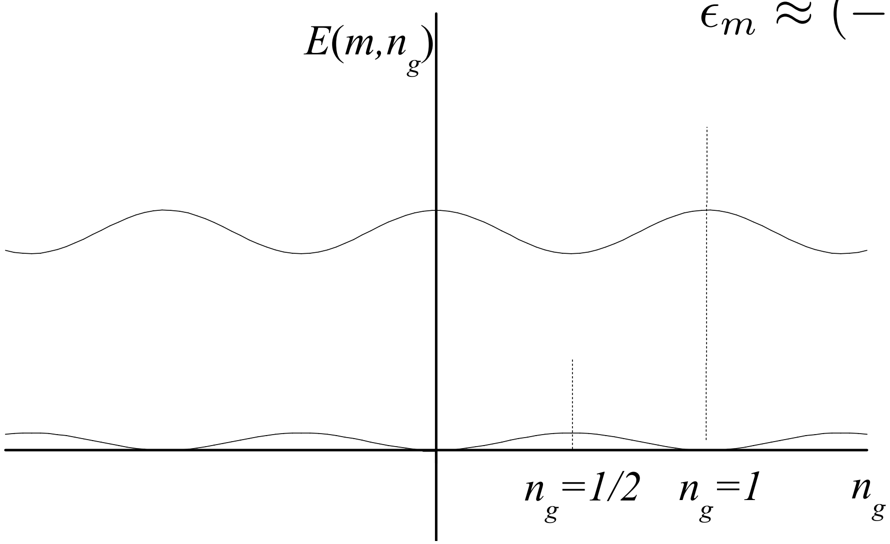
$$E_J \gg E_C$$



$$E_m(n_g) = E_m(n_g = 1/4) - \frac{\epsilon_m}{2}\cos(2\pi n_g)$$

$$\epsilon_m \equiv E_m(n_g = 1/2) - E_m(n_g = 0)$$

$$\epsilon_m \approx (-1)^m E_C \frac{2^{4m+5}}{m!}\sqrt{\frac{2}{\pi}}\left(\frac{E_J}{2E_C}\right)^{\frac{2m+3}{4}} \exp -\sqrt{\frac{8E_J}{E_C}}$$



$$E_m(n_g = 0) \approx -E_J + \sqrt{8E_J E_C}\left(m + \frac{1}{2}\right) - \frac{E_C}{12}(6m^2 + 6m + 3)$$

Decoherence

$$|0\rangle = \cos\frac{\lambda}{2}|\uparrow\rangle + \sin\frac{\lambda}{2}|\downarrow\rangle$$

$$|1\rangle = -\sin\frac{\lambda}{2}|\uparrow\rangle + \cos\frac{\lambda}{2}|\downarrow\rangle$$

$\rho_z(t)$ and $\rho^{(\pm)}(t) \equiv [\rho_x(t) \pm i\,\rho_y(t)]/2$

System dominated regime

$\lambda \equiv \tan^{-1}(B_x/B_z)$

$$\tau_{rel}^{-1} = \pi\,\alpha\,\sin^2\lambda\;\Delta\coth\frac{\hbar\Delta}{2kT}$$

$$\tau_{\varphi}^{-1} = \frac{1}{2}\,\tau_{rel}^{-1} + \pi\,\alpha\,\cos^2\lambda\;\frac{2kT}{\hbar},$$

In general

$\cos\lambda_i \equiv \mathbf{B}\cdot\mathbf{n}_i/|\mathbf{B}|$

$$H = H_{qb} + \sum_i \vec{\sigma}\cdot\mathbf{n}_i(\sum_a C_a^{(i)} q_a^{(i)}) + \sum_i H_{env}^{(i)}$$

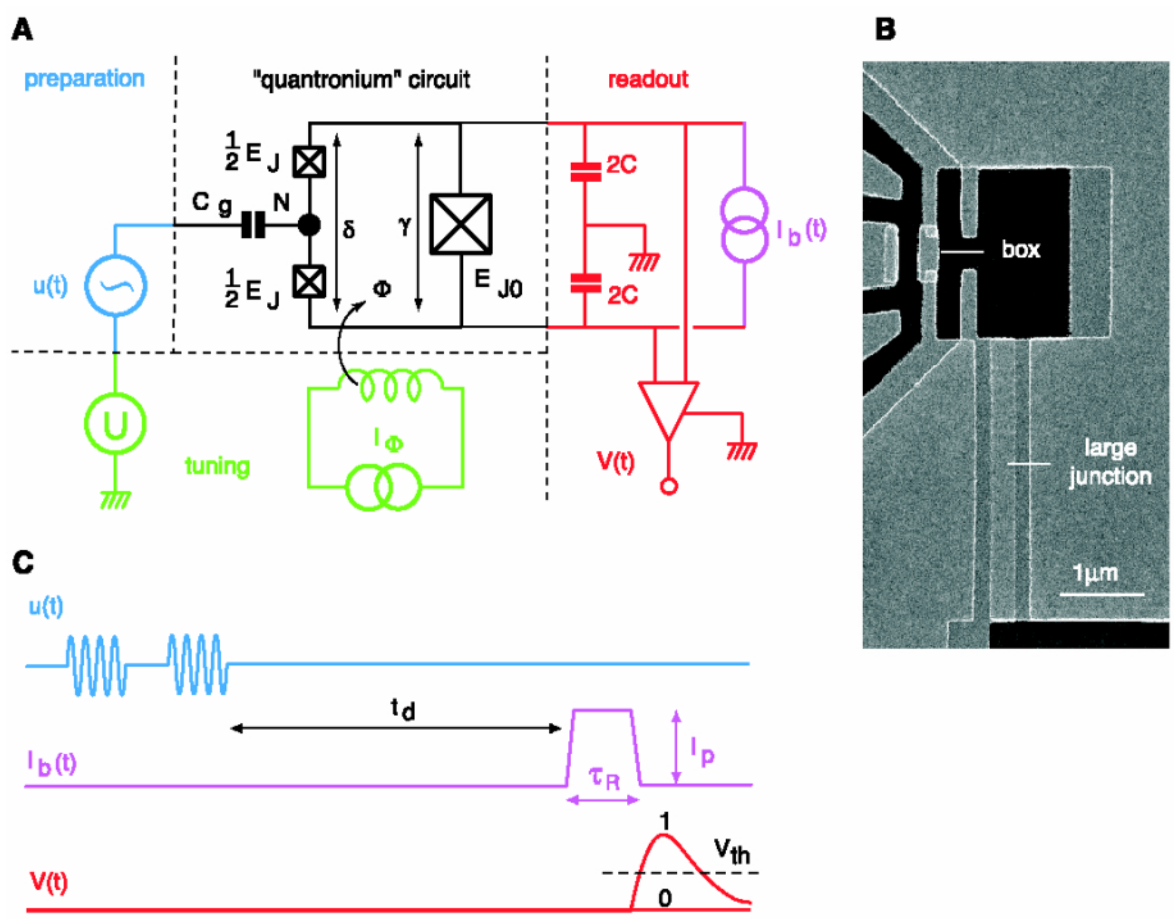
$\hbar\Delta \gg \sum_i \alpha_i kT$

$$\tau_{rel}^{-1} = \sum_i \pi\,\alpha_i\,\sin^2\lambda_i\;\Delta\coth\frac{\hbar\Delta}{2kT}$$

$$\tau_{\varphi}^{-1} = \frac{1}{2}\,\tau_{rel}^{-1} + \sum_i \pi\,\alpha_i\,\cos^2\lambda_i\;\frac{2kT}{\hbar},$$

B. Experimental results

Cooper Pair Box



Rabi oscillation and Ramsey fringe experiments to measure the Switching probability

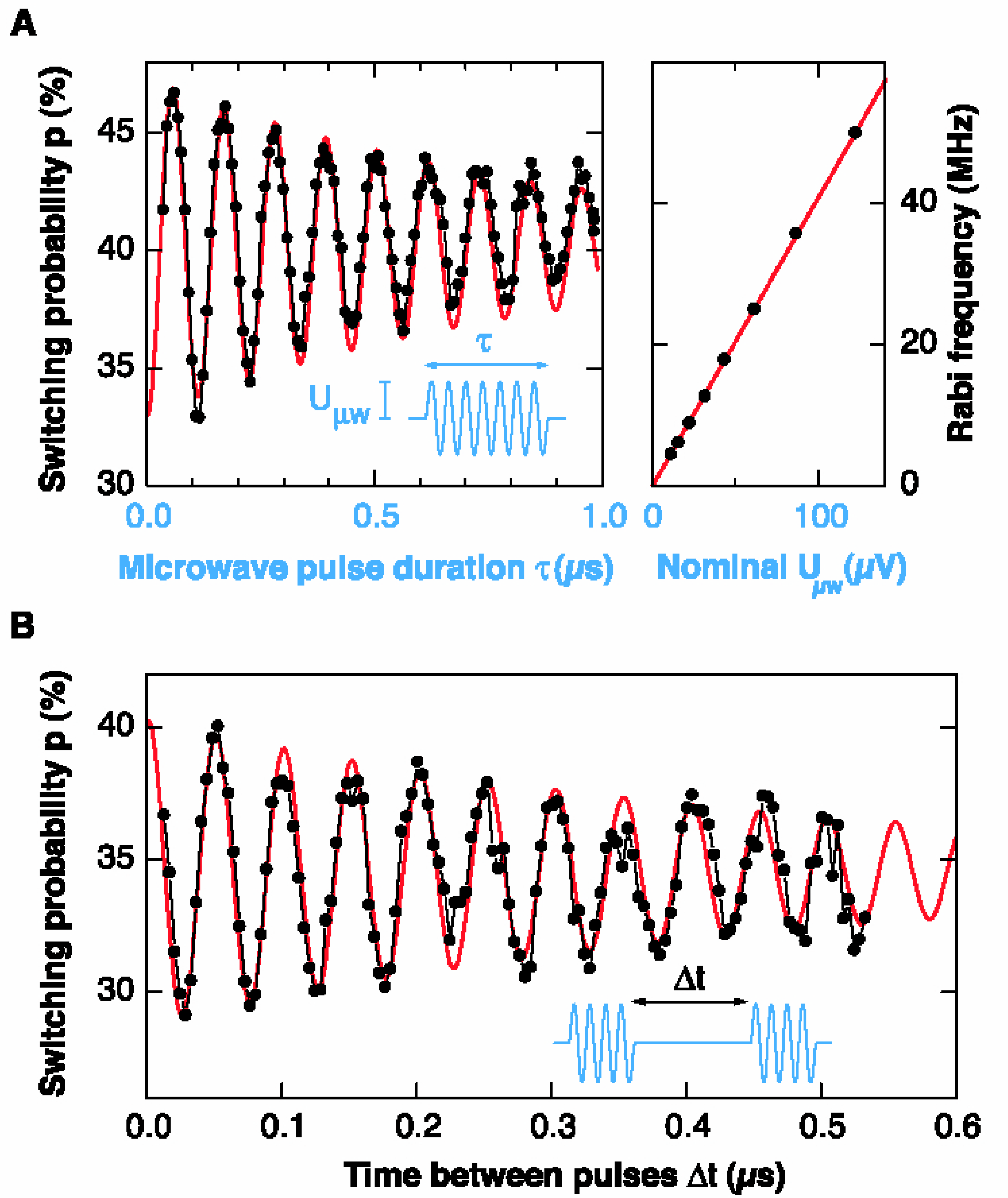
$$E_J = 0.865 k_B K$$

$$E_J / E_C = 1.27$$

$$T = 15 mK$$

$$\tau_\varphi \approx 0.50 \mu s$$

$$1/\nu_{01} \approx 60 ps$$



Excited CBJJ

Measurement of the decay rate of the excited state of a CBJJ via rabi oscillations

$T = 8mK$ $\tau_\varphi \gtrsim 4.9\mu s$