Lista de exercícios 3:

Problemas 1-8 : Questões 1, 2, 3, 4, 6, 7, 8, 12 e 14 do Cap. III do Cohen-Tannoudji.

Problema 9:

Consider a system whose state is given in terms of a complete and orthonormal set of ve vectors $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle$ as follows:

$$|\psi\rangle = \frac{1}{\sqrt{19}}|\phi_1\rangle + \frac{2}{\sqrt{19}}|\phi_2\rangle + \sqrt{\frac{2}{19}}|\phi_3\rangle + \sqrt{\frac{3}{19}}|\phi_4\rangle + \sqrt{\frac{5}{19}}|\phi_5\rangle,$$

where $|\phi_n\rangle$ are eigenstates to the system's Hamiltonian, $\hat{H}|\phi_n\rangle = n\varepsilon_0|\phi_n\rangle$ with $n = 1, 2, 3, 4, 5$, and where ε_0 has the dimensions of energy.

(a) If the energy is measured on a large number of identical systems that are all initially in the same state $|\psi\rangle$, what values would one obtain and with what probabilities?

(b) Find the average energy of one such system.

Problema 10:

A particle of mass m, which moves freely inside an in nite potential well of length a, has the following initial wave function at $t = 0$:

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} \sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right),$$

where A is a real constant.

(a) Find A so that $\psi(x, 0)$ is normalized.

(b) If measurements of the energy are carried out, what are the values that will be found and what are the corresponding probabilities? Calculate the average energy.

(c) Find the wave function $\psi(x, t)$ at any later time t.

(d) Determine the probability of nding the system at a time t in the state $\varphi(x, t) = \sqrt{2/a}\sin(5\pi x/a)\exp(-iE_5 t/\hbar)$; then determine the probability of nding it in the state $\chi(x, t) = \sqrt{2/a}\sin(2\pi x/a)\exp(-iE_2 t/\hbar)$.

Problema 11:

A system is initially in the state $|\psi_0\rangle = [\sqrt{2}|\phi_1\rangle + \sqrt{3}|\phi_2\rangle + |\phi_3\rangle + |\phi_4\rangle]/\sqrt{7}$, where $|\phi_n\rangle$ are eigenstates of the system's Hamiltonian such that $\hat{H}|\phi_n\rangle = n^2\mathcal{E}_0|\phi_n\rangle$.

(a) If energy is measured, what values will be obtained and with what probabilities?

(b) Consider an operator $\hat{A}$ whose action on $|\phi_n\rangle$ is de ned by $\hat{A}|\phi_n\rangle = (n+1)a_0|\phi_n\rangle$. If A is measured, what values will be obtained and with what probabilities?

(c) Suppose that a measurement of the energy yields $4\mathcal{E}_0$. If we measure A immediately afterwards, what value will be obtained?