F 502 – Eletromagnetismo I – Problemas adicionais

Cap. 2: Eletrostática

Reitz and Milford, 4. ed ou (c1982, português):
Cap. 2: 04, 05, 06, 09, 10, 15, 16, 18, 19, 20
Cap. 6: 03, 05, 06, 10

P 2.21, Reitz. A long straight line of charge λ per unit length extends along the z-axis from a to ∞. Another line of charge with $-\lambda$ per unit length extends along the z-axis from $-a$ to $-\infty$.
a) By direct integration, determine the electric potential in cylindrical coordinates.
b) Find the ρ component of the electric field.

P 3.23, Zangwill. Let d and s be two unequal lengths. Assume that charge is distributed on the $z = 0$ plane with a surface density (cylindrical coordinates)

$$\sigma(\rho) = \frac{-qd}{2\pi(\rho^2 + s^2)^{3/2}}.$$

a) Integrate σ to find the total charge Q on the plane.
b) Show that the potential $V(z)$ produced by σ on the z-axis is identical to the potential produced by a point with charge Q on the axis at $z = -s$.

P 5.4, Zangwill. Two infinite conducting planes are held at zero potential at $z = -d$ and $z = d$. An infinite sheet with uniform charge per unit area σ is interposed between them at an arbitrary point z_0.
a) Find the charge density induced on each grounded plane and the potential at the position of the sheet of charge.
b) Find the force per unit area which acts on the sheet of charge.

Cap. 3: Eqs. de Poisson e Laplace

Reitz and Milford, 4. ed ou (c1982, português):
Cap. 2: 24 (22), 25 (23), 26 (24)
Cap. 3: 01, 04, 08, 09, 10, 14, 17, 21, 23
Cap. 6: 14

P 4.1, Zangwill. Find the electric dipole moment of:
a) a ring with charge per unit length $\lambda = \lambda_0 \cos\phi$ where ϕ is the angular variable in cylindrical coordinates;
b) a sphere with charge per unit area $\sigma = \sigma_0 \cos\theta$ where θ is the polar angle measured from the positive z-axis.

P 4.14, Zangwill. Find the primitive, Cartesian monopole, dipole, and quadrupole moments for each of the following charge distributions. Use the geometrical center of each as the origin.
a) Two charges $+q$ at two diagonal corners of a square $(\pm a, \pm a, 0)$ and two minus charges $-q$ at the two other diagonals of the square $(\pm a, \mp a, 0)$.
b) A line segment with uniform charge per unit length λ which occupies the interval $-l \leq z \leq +l$.
c) An origin-centered ring in the $x - y$ plane with uniform charge per unit length λ and radius R.

P 7.1, Zangwill. Use the orthogonality properties of the spherical harmonics to prove the following identities for a function $V(\mathbf{r})$ which satisfies Laplace's equation in and on an origin-centered spherical surface S of radius R:

$$\text{(a)} \quad \int_S dS\, V(\mathbf{r}) = 4\pi R^2 V(0).$$

$$\text{(b)} \quad \int_S dS\, zV(\mathbf{r}) = \frac{4\pi}{3} R^4 \left.\frac{\partial V}{\partial z}\right|_{\mathbf{r}=0}.$$

P 7.9, Zangwill. Two flat conductor plates (infinite in the x- and y-directions) occupy the planes $z = \pm d$. The $x > 0$ portion of both plates is held at $\phi = +\phi_0$. The $x < 0$ portion of both plates is held at $\phi = -\phi_0$. Derive an expression for the potential between the plates using a Fourier integral to represent the x variation of $\phi(x, z)$.

Cap. 4: Campos elétricos na matéria

Reitz and Milford, 4. ed ou (c1982, português):
Cap. 4: 01, 02, 03, 04, 06, 08, 11, 12, 18, 19
Cap. 6: 02, 13, 17, 18, 19

P 4.9, Reitz, A spherical conductor of radius R_1 is surrounded by a solid dielectric spherical shell of outer radius R_2, inner radius R_1 and dielectric constant k_1. This composite entity is immersed in a fluid of dielectric constant k_2 and is subjected to an initially uniform electric field $\mathbf{E}_0$. Determine the electric field in the two dielectric media.

P 6.14, Zangwill. A spherical conductor of radius R_1 is surrounded by a polarizable medium which extends from R_1 to R_2 with dielectric constant κ.
a) The conductor has charge Q. Find $\mathbf{E}$ everywhere and confirm that the total polarization charge is zero.
b) The conductor is grounded and the entire system is placed in a uniform electric field $\mathbf{E}_0$. Find the electrostatic potential everywhere and determine how much charge is drawn up from ground to the conductor.

P 7.14, Zangwill. A conducting sphere with radius R and charge Q sits at the origin of coordinates. The space outside the sphere above the $z = 0$ plane has dielectric constant k_1. The space outside the sphere below the $z = 0$ plane has dielectric constant k_2.
a) Find the potential every where outside the conductor.
b) Find the distributions of free charge and polarization charge wherever they maybe.

Cap. 5: Magnetostática

Reitz and Milford, 4. ed ou (c1982, português) :
Cap. 7: 03, 04, 05, 06, 07, 09, 10
Cap. 8: 02, 04, 07, 09, 11, 14, 17, 20, 21, 22, 24, 30 (28)

P 9.15, Zangwill. A current I flows up the z-axis and is intercepted by an origin-centered sphere with radius R and conductivity σ. The current enters and exits the sphere through small conducting electrodes which occupy the portion of the sphere's surface defined by $\theta \leq \alpha$ and $\pi - \alpha \leq \theta \leq \pi$. Derive an expression for the resistance of the sphere to the flowing current. Assume that $\alpha \ll 1$ and comment on the limit $\alpha \to 0$. Hint:

$$(2l+1)\int_{x_1}^{x_2} dx P_l(x) = \left[P_{l+1}(x) - P_{l-1}(x)\right]_{x_1}^{x_2}.$$

P 10.6, Zangwill. Two Approaches to the Field of a Current Sheet.
a) Use the Biot-Savart law to find $\mathbf{B}(\mathbf{r})$ everywhere for a current sheet at $x = 0$ with $\mathbf{k} = k\hat{z}$.
b) Check your answer to part (a) by superposing the magnetic field from an infinite number of straight current-carrying wires.

P 10.11, Zangwill. A current I starts at $z = -\infty$ and flows up the z-axis as a linear filament until its hits an origin-centered sphere of radius R. The current spreads out uniformly over the surface of the sphere and flows up lines of longitude from the south pole to the north pole. The recombined current flows thereafter as a linear filament up the z-axis to $z = +\infty$.
a) Find the current density on the sphere.
b) Use explicitly stated symmetry arguments and Ampère's law in integral form to find the magnetic field at every point in space.
c) Check that your solution satisfies the magnetic field matching conditions at the surface of the sphere.

P 11.1, Zangwill. A current distribution produces the vector potential

$$\mathbf{A}(r,\theta,\phi) = \hat{\phi}\frac{\mu_0}{4\pi}\frac{A_0 \sin\theta}{r}\exp -\lambda r.$$

What is the magnetic moment associated with this current distribution?

P 11.15, Zangwill. A superconductor has the property that its interior has $B = 0$ under all conditions. Let a sphere (radius R) of this kind sit in a uniform magnetic field $\mathbf{B}_0$.
a) Place a fictitious point magnetic dipole $\mathbf{m}$ at the center of the sphere. Find $\mathbf{m}$ from the matching condition on the normal component of $\mathbf{B}$.
b) In reality, the dipole field in part (a) is created by a current density $\mathbf{K}$ which appears on the surface of the sphere. Find $\mathbf{K}$ from the matching condition on the tangential component of $\mathbf{B}$.
c) Confirm your answer in part (a) by computing the magnetic dipole moment associated with $\mathbf{K}$ from part (b).

Cap. 6: Campos magnéticos na matéria

Reitz and Milford, 4. ed ou (c1982, português):
Cap. 9: 01, 02, 03, 06, 07, 13, 14, 15

P 13.21, Zangwill. In 1935, the brothers Fritz and Heinz London described superconductivity using a phenomenological constitutive equation where a length $\delta > 0$ relates the current density to the Coulomb vector potential:

$$\mathbf{J} = -\frac{1}{\mu_0 \delta^2}\mathbf{A}.$$

a) Use the London constitutive equation to derive a differential equation for $\mathbf{B}(\mathbf{r})$.
b) The London theory predicts that $\mathbf{B}$ is not strictly zero at every point inside a superconductor. To see this, consider a slab of superconductor which is infinite in the x- and y-directions and lies between $z = -d$ and $z = d$. Compute $\mathbf{B}(z)$ inside the superconductor when the slab is placed in a static and uniform magnetic field $\mathbf{B}_0 = B_0\hat{x}$.
c) Find the current density inside the superconductor.

Cap. 7: Eletrodinâmica

Reitz and Milford, 4. ed ou (c1982, português):
Cap. 9: 01, 06, 09, 11, 13, 16, 18, 24 (22), 26 (24)
Cap. 16: 01

P 14.3, Zangwill. A surface current density $\mathbf{k} = -k\hat{x}$ flows in the half-plane ($x > 0$, $z = 0$). The current accumulates on the line $x = 0$ which bounds the half-plane.
a) Find $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ in the quasi-electrostatic approximation.
Hint: Use symmetry and the Ampère-Maxwell law in integral form to find the magnetic field.
b) Confirm that your solution satisfies the full set of Maxwell equations without approximation.