

## F 320 – Termodinâmica – Lista 1

P.01. P.1.2, Sears:

Which of the following quantities are extensive and which are intensive?

- a) The magnetic moment of a gas.
- b) The electric field  $E$  in a solid.
- c) The length of a wire.
- d) The surface tension of an oil film.

P.02. P.1.6, Sears:

Two containers of gas are connected by a long, thin, thermally insulated tube. Container A is surrounded by an adiabatic boundary, but the temperature of container B can be varied by bringing it into contact with a body C at a different temperature. In Fig.1-6 from Sears, these systems are shown with a variety of boundaries. Which figure represents:

- a) an open system enclosed by an adiabatic boundary;
- b) an open system enclosed by a diathermal boundary;
- c) a closed system enclosed by a diathermal boundary;
- d) a closed system enclosed by an adiabatic boundary.

03. P.1.17, Sears:

A mixture of hydrogen and oxygen is isolated and allowed to reach a state of constant temperature and pressure. The mixture is exploded with a spark of negligible energy and again allowed to come to a state of constant temperature and pressure.

- a) Is the initial state an equilibrium state? Explain.
- b) Is the final state an equilibrium state? Explain.

04. P.1.20, Sears:

Give an example of

- a) a reversible isochoric process;
- b) a quasistatic, adiabatic, isobaric process;
- c) an irreversible isothermal process.

Be careful to specify the system in each case.

05. P.1.21, Sears:

Using the nomenclature similar to that in the Problem 04, characterize the following processes:

- a) The temperature of a gas, enclosed in a cylinder provided with a frictionless piston, is slowly increased. The pressure remains constant.
- b) A gas, enclosed in a cylinder provided with a piston, is slowly expanded. The temperature remains constant. There is a force of friction between the cylinder wall and the piston.
- c) A gas enclosed in a cylinder provided with a frictionless piston is quickly compressed.
- d) A piece of hot metal is thrown into cold water. Assume that the system is the metal which neither contracts nor expands.
- e) A pendulum with a frictionless support swings back and forth.
- f) A bullet is stopped in a target.

06. P.1.7, Zemansky:

The length of the mercury column in the old-fashioned mercury-in-glass thermometer is 15.00 cm when the thermometer is in contact with water at its triple point. Consider the length of the mercury column as the thermometric property  $X$  and let  $\theta$  be the empirical temperature determined by this thermometer.

- a) Calculate the empirical temperature when the length of the mercury column is 19.00 cm.
- b) If  $X$  can be measured with a precision of 0.01 cm, can this thermometer distinguish between the normal freezing point of water and the triple point of water?

07. P.1.7, Sears:

A water-in-glass thermoscope is to be used to determine if two separated systems are in thermal equilibrium. The density of water, shown in Fig. 1-7 from Sears, is the thermometric parameter. Suppose that when the thermoscope is inserted into each system, the water rises to the same height corresponding to a density of  $0.999945 \text{ g cm}^{-3}$ .

- a) Are the systems necessarily in thermal equilibrium?
- b) Could the height or the water in the thermoscope change if the systems are brought into thermal contact?
- c) If there is a change in part (b), would the height increase or decrease?

P.08. P.1.9, Sears:

The length of the mercury column in a certain mercury-in-glass thermometer is 5.00 cm when the thermometer is in contact with water at its triple point. Consider the length of the mercury column as the thermometric property  $X$  and let  $\theta$  be the empirical temperature determined by this thermometer.

- a) Calculate the empirical temperature, measured when the length of the mercury column is 6.00 cm.
- b) Calculate the length of the mercury column at the steam point.
- c) If  $X$  can be measured with a precision of 0.01 cm, can this thermometer be used to distinguish between the ice point and the triple point?

P.09. P.1.10, Sears:

A temperature  $t^*$  is defined by the equation  $t^* = a\theta^2 + b$ , where  $a$  and  $b$  are constants, and  $\theta$  is the empirical temperature determined by the mercury-in-glass thermometer of Problem 08.

- a) Find the numerical values of  $a$  and  $b$ , if  $t^* = 0$  at the ice point and  $t^* = 100$  at the steam point.
- b) Find the value of  $t^*$  when the length of the mercury column  $X = 7.00$  cm.
- c) Find the length of the mercury column when  $t^* = 50$ .
- d) Sketch  $t^*$  versus  $X$ .

10. P.1.13, Sears:

The pressure of an ideal gas kept at constant volume is given by the equation  $p = AT$  where  $T$  is the thermodynamic temperature and  $A$  is a constant. Let a temperature  $T^*$  be defined by  $T^* = B \ln CT$ , where  $B$  and  $C$  are constants. The pressure  $p = 0.1$  atm at the triple point of water. The temperature  $T^* = 0$  at the triple point and  $T^* = 100$  at the steam point.

- a) Find the values of  $A$ ,  $B$ , and  $C$ .
- b) Find the value of  $T^*$  when  $p = 0.15$  atm.
- c) Find the value of  $p$  when  $T^* = 50$ .
- d) What is the value of  $T^*$  at absolute zero?
- e) Sketch a graph of  $T^*$  versus the Celsius temperature  $t$  for  $-200^\circ\text{C} < t < +200^\circ\text{C}$ .

P.11. P.2.1, Zemansky:

The equation of state of an ideal gas is  $PV = nRT$ , where  $n$  and  $R$  are constants. Show that the volume expansivity  $\beta$  is equal to  $1/T$  and that the isothermal compressibility  $\kappa$  is equal to  $1/P$ .

P.12. P.2.2, Zemansky and P.2.22, Sears:

The equation of state of a van der Waals gas is given as

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where  $a$ ,  $b$ , and  $R$  are constants.

a) Calculate the following quantities:

$$\left(\frac{\partial p}{\partial v}\right)_T, \quad \left(\frac{\partial p}{\partial T}\right)_v, \quad \text{and} \quad \left(\frac{\partial v}{\partial T}\right)_p$$

b) Show that the compressibility of a van der Waals gas is

$$\kappa = \frac{v^2(v - b)^2}{RTv^3 - 2a(v - b)^2}.$$

c) What is the expression for  $\kappa$  if  $a = b = 0$ .

P.13. P.2.25, Sears:

A substance has an isothermal compressibility  $\kappa = aT^3/p^2$  and an expansivity  $\beta = bT^2/p$ , where  $a$  and  $b$  are constants. Find the equation of state of the substance and the ratio  $a/b$ .

14. P.2.13, Sears:

In all so-called diatomic gases, some of the molecules are dissociated into separated atoms, the fraction dissociated increasing with increasing temperature. The gas as a whole thus consists of a diatomic and a monatomic portion. Even though each component may act as an ideal gas, the mixture does not, because the number of moles varies with the temperature. The degree of dissociation  $\delta$  of a diatomic gas is defined as the ratio of the mass  $m_1$  of the monatomic portion to the total mass  $m$  of the system, i.e.,  $\delta = m_1/m$ . Show that the equation of state of the gas is

$$pV = (\delta + 1) \frac{m}{M_2} RT,$$

where  $M_2$  is the molecular weight of the diatomic component.

P.15. P.2.6, Zemansky:

Consider a wire that undergoes an infinitesimal change from an initial equilibrium state to a final equilibrium state.

- a) Show that the change of tension is equal to

$$df = -\alpha AY dT + \frac{AY}{L} dL.$$

- b) A nickel wire of cross-sectional area  $0.0085 \text{ cm}^2$  under a tension of 20 N and a temperature of  $20^\circ \text{C}$  is stretched between two rigid supports 1 m apart. If the temperature is reduced to  $8^\circ \text{C}$ , what is the final tension?

Assume that  $\alpha$  and  $Y$  remain constant at the values of  $1.33 \times 10^{-5} \text{ K}^{-1}$  and  $2.1 \times 10^9 \text{ Pa}$ , respectively.

P.16. P.2.7, Zemansky:

The equation of state of an ideal elastic substance is

$$\mathcal{F} = KT \left( \frac{L}{L_0} - \frac{L_0^2}{L^2} \right),$$

where  $K$  is a constant and  $L_0$  (the value of  $L$  at zero tension) is a function of temperature only.

- a) Show that the isothermal Young's modulus is given by

$$Y = \frac{\mathcal{F}}{A} + \frac{3KTL_0^2}{AL^2}.$$

- b) Show that the isothermal Young's modulus at zero tension is given by  $Y_0 = 3KT/A$ .  
c) Show that the linear expansivity is given by

$$\alpha = \alpha_0 - \frac{\mathcal{F}}{AYT} = \alpha_0 - \frac{1}{T} \frac{L^3/L_0^3 - 1}{L^3/L_0^3 - 2},$$

where  $\alpha_0 = (1/L_0)(dL_0/dT)$  is the value of the linear expansivity at zero tension.

- d) Assume the following values for a sample of rubber:  
 $T = 300\text{K}$ ,  $K = 1.333 \times 10^{-2} \text{ N/K}$ ,  $A = 1 \times 10^{-6} \text{ m}^2$ ,  $\alpha_0 = 5 \times 10^{-4} \text{ K}^{-1}$ . When this sample is stretched to length  $L = 2L_0$ , calculate  $\mathcal{F}$ ,  $Y$ , and  $\alpha$ .

P.17. P.2.17, Sears:

Show that  $\beta = 3\alpha$  for an isotropic solid.

P.18. P.2.11, Zemansky:

Calculate

$$\left(\frac{\partial E}{\partial T}\right)_p \quad \text{and} \quad \left(\frac{\partial p}{\partial T}\right)_E$$

for a dielectric material obeying the equation

$$P = \frac{1}{V}p = \left(a + \frac{b}{T}\right)E = \chi_e(T)E.$$

P.19. P.2.4, Zemansky:

- a) A block of copper at a pressure of 1 atm (approximately 100 kPa) and a temperature of 5°C is kept at constant volume. If the temperature is raised to 10°C, what will be the final pressure?
- b) If the vessel holding the block of copper has a negligibly small thermal expansivity and can withstand a maximum pressure of 1000 atm, what is the highest temperature to which the system may be raised?

The volume expansivity  $\beta$  and isothermal compressibility  $\kappa$  are not always listed in handbooks of data. However,  $\beta$  is three times the linear expansion coefficient  $\alpha$  and  $\kappa$  is the reciprocal of the bulk modulus  $B$ . For this problem, assume that the volume expansivity and isothermal compressibility remain practically constant within the temperature range of 0 to 20°C at the values of  $4.95 \times 10^{-5} \text{ K}^{-1}$  and  $6.17 \times 10^{-12} \text{ Pa}^{-1}$ , respectively.

20. P.2.3, Sears:

A cylinder provided with a movable piston contains an ideal gas at a pressure  $p_1$ , specific volume  $v_1$ , and temperature  $T_1$ . The pressure and volume are simultaneously increased so that at every instant  $p$  and  $v$  are related by the equation  $p = Av$ , where  $A$  is a constant.

- a) Express the constant  $A$  in terms of the pressure  $p_1$ , the temperature  $T_1$ , and the gas constant  $R$ .
- b) Construct the graph representing the process above in the  $p - v$  plane.
- c) Find the temperature when the specific volume has doubled, if  $T_1 = 200 \text{ K}$ .

21. P.2.8, Sears:

Figure 2-20 from Sears shows five processes,  $a - b$ ,  $b - c$ ,  $c - d$ ,  $d - a$  and  $a - c$ , plotted in the  $p - V$  plane for an ideal gas in a closed system. Show the same processes (a) in the  $p - T$  plane and (b) in the  $T - V$  plane.

P.22. P.2.11, Sears:

A quantity of air is contained in a cylinder provided with a movable piston. Initially the pressure of the air is  $p_0$ , the volume is  $V_0$ , and the temperature is  $T_0$ . Assume air is an ideal gas.

- What is the final volume of the air if it is allowed to expand isothermally until the pressure is  $p_1 = p_0/2$ , the piston moving outward to provide for the increased volume of the air?
- What is the final temperature of the air if the piston is held fixed at its initial position and the system is cooled until the pressure is  $p_1 = p_0/2$ ?
- What are the final temperature and volume of the air if it is allowed to expand isothermally from the initial conditions until the pressure is  $p_1 = 3p_0/4$  and then it is cooled at constant volume until the pressure is  $p_2 = p_0/2$ ?
- What are the final temperature and volume of the air if an isochoric cooling to  $3p_0/4$  is followed by an isothermal expansion to  $p_0/2$ ?
- Plot each of these processes on  $p - V$ ,  $p - T$ , and  $T - V$  diagrams.

P.23. P.2.12, Sears:

A volume  $V$  at temperature  $T$  contains  $n_A$  moles of ideal gas A and  $n_B$  moles of ideal gas B. The gases do not react chemically.

- Show that the total pressure  $p$  of the system is given by  $p = p_A + p_B$ , where  $p_A$  and  $p_B$  are the pressures that each gas would exert if it were in the volume alone. The quantity  $p_A$  is called the partial pressure of gas A and the above equation is known as Dalton law of partial pressures.
- Show that  $p_A = x_A p$ , where  $x_A$  is the fraction of moles of A in the system.

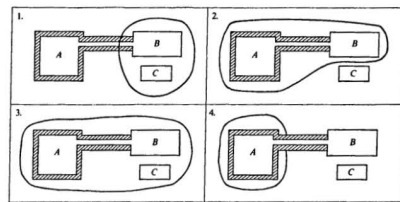


Figure 1-6

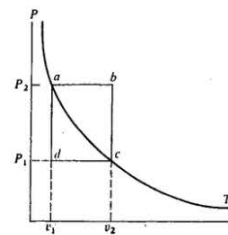


Figure 2-20