F 320 – Termodinâmica – Lista 1

01. [P.1.2, Sears] Which of the following quantities are extensive and which are intensive?
(a) The magnetic moment of a gas. (b) The electric field E in a solid. (c) The length of a wire. (d) The surface tension of an oil film.

P.02. [P.1.6, Sears] Two containers of gas are connected by a long, thin, thermally insulated tube. Container A is surrounded by an adiabatic boundary, but the temperature of container B can be varied by bringing it into contact with a body C at a different temperature. In Fig.1-6 from Sears, these systems are shown with a variety of boundaries. Which figure represents:
(a) an open system enclosed by an adiabatic boundary;
(b) an open system enclosed by a diathermal boundary;
(c) a closed system enclosed by a diathermal boundary;
(d) a closed system enclosed by an adiabatic boundary.

03. [P.1.17, Sears] A mixture of hydrogen and oxygen is isolated and allowed to reach a state of constant temperature and pressure. The mixture is exploded with a spark of negligible energy and again allowed to come to a state of constant temperature and pressure.
(a) Is the initial state an equilibrium state? Explain.
(b) Is the final state an equilibrium state? Explain.

04. [P.1.20, Sears] Give an example of
(a) a reversible isochoric process;
(b) a quasistatic, adiabatic, isobaric process;
(c) an irreversible isothermal process.
Be careful to specify the system in each case.

P.05. [P.1.21, Sears] Using the nomenclature similar to that in the problem 04 and characterize the following processes:
(a) The temperature of a gas, enclosed in a cylinder provided with a frictionless piston, is slowly increased. The pressure remains constant.
(b) A gas, enclosed in a cylinder provided with a piston, is slowly expanded. The temperature remains constant. There is a force of friction between the cylinder wall and the piston.
(c) A gas enclosed in a cylinder provided with a frictionless piston is quickly compressed.
(d) A piece of hot metal is thrown into cold water. Assume that the system is the metal which neither contracts nor expands.
(e) A pendulum with a frictionless support swings back and forth.
(f) A bullet is stopped in a target.

06. [P.1.7, Zemansky] The length of the mercury column in the old-fashioned mercury-in-glass thermometer is 15.00 cm when the thermometer is in contact with water at its triple point. Consider the length of the mercury column as the thermometric property X and let θ be the empirical temperature determined by this thermometer.
(a) Calculate the empirical temperature when the length of the mercury column is 19.00 cm.
(b) If X can be measured with a precision of 0.01 cm, can this thermometer distinguish between the normal freezing point of water and the triple point of water?

07. [P.1.7, Sears] A water-in-glass thermoscope is to be used to determine if two separated systems are in thermal equilibrium. The density of water, shown in Fig. 1-7 from Sears, is the thermometric parameter. Suppose that when the thermoscope is inserted into each system, the water rises to the same height corresponding to a density of 0.999945 g cm^{-3}.
(a) Are the systems necessarily in thermal equilibrium?
(b) Could the height or the water in the thermoscope change if the systems are brought into thermal contact?
(c) If there is a change in part (b), would the height increase or decrease?

P.08. [P.1.9, Sears] The length of the mercury column in a certain mercury-in-glass thermometer is 5.00 cm when the thermometer is in contact with water at its triple point. Consider the length of the mercury column as the thermometric property X and let θ be the empirical temperature determined by this thermometer.
(a) Calculate the empirical temperature, measured when the length of the mercury column is 6.00 cm.
(b) Calculate the length of the mercury column at the steam point.
(c) If X can be measured with a precision of 0.01 cm, can this thermometer be used to distinguish between the ice point and the triple point?

P.09. [P.1.10, Sears] A temperature t^* is defined by the equation $t^* = a\theta^2 + b$, where a and b are constants, and θ is the empirical temperature determined by the mercury-in-glass thermometer of problem 08.
(a) Find the numerical values of a and b, if $t^* = 0$ at the ice point and $t^* = 100$ at the steam point.
(b) Find the value of t^* when the length of the mercury column $X = 7.00$ cm.
(c) Find the length of the mercury column when $t^* = 50$.
(d) Sketch t^* versus X.

10. [P.1.13, Sears] The pressure of an ideal gas kept at constant volume is given by the equation $p = AT$ where T is the thermodynamic temperature and A is a constant. Let a temperature T^* be defined by $T^* = B \ln CT$, where B and C are constants. The pressure $p = 0.1$ atm at the triple point of water. The temperature $T^* = 0$ at the triple point and $T^* = 100$ at the steam point.
(a) Find the values of A, B, and C.
(b) Find the value of T^* when $p = 0.15$ atm.
(c) Find the value of p when $T^* = 50$.
(d) What is the value of T^* at absolute zero?
(e) Sketch a graph of T^* versus the Celsius temperature t for $-200°C < t < +200°C$.

11. [P.2.1, Zemansky] The equation of state of an ideal gas is $PV = nRT$, where n and R are constants. Show that the volume expansivity β is equal to $1/T$ and that the isothermal compressibility κ is equal to $1/P$.

P.12. [P.2.2, Zemansky and P.2.22, Sears] The equation of state of a van der Waals gas is given as

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT,$$

where a, b, and R are constants.
(a) Calculate the following quantities: (i) $(\partial P/\partial v)_T$ and (ii) $(\partial P/\partial T)_v$.
From parts (i) and (ii), calculate $(\partial v/\partial T)_P$.
(b) Show that the compressibility of a van der Waals gas is

$$\kappa = \frac{v^2(v-b)^2}{RTv^3 - 2a(v-b)^2}.$$

(c) What is the expression for κ if $a = b = 0$.

P.13. [P.2.25, Sears] A substance has an isothermal compressibility $\kappa = aT^3/p^2$ and an expansivity $\beta = bT^2/p$, where a and b are constants. Find the equation of state of the substance and the ratio a/b.

14. [P.2.13, Sears] In all so-called diatomic gases, some of the molecules are dissociated into separated atoms, the fraction dissociated increasing with increasing temperature. The gas as a whole thus consists of a diatomic and a monatomic portion. Even though each component may act as an ideal gas, the mixture does not, because the number of moles varies with the temperature. The degree of dissociation δ of a diatomic gas is defined as the ratio of the mass m_1 of the monatomic portion to the total mass m of the system, i.e., $\delta = m_1/m$. Show that the equation of state of the gas is

$$pV = (\delta + 1)\frac{m}{M_2}RT,$$

where M_2 is the molecular weight of the diatomic component.

15. [P.2.6, Zemansky] Consider a wire that undergoes an infinitesimal change from an initial equilibrium state to a final equilibrium state.
(a) Show that the change of tension is equal to

$$df = -\alpha AY\, dT + \frac{AY}{L} dL.$$

(b) A nickel wire of cross-sectional area 0.0085 cm^2 under a tension of 20 N and a temperature of 20° C is stretched between two rigid supports 1 m apart. If the temperature is reduced to 8°C, what is the final tension?
Assume that α and Y remain constant at the values of 1.33×10^{-5} K^{-1} and 2.1×10^9 Pa, respectively.

P.16. [P.2.7, Zemansky] The equation of state of an ideal elastic substance is

$$\mathcal{F} = KT \left(\frac{L}{L_0} - \frac{L_0^2}{L^2} \right),$$

where K is a constant and L_0 (the value of L at zero tension) is a function of temperature only.
(a) Show that the isothermal Young's modulus is given by

$$Y = \frac{\mathcal{F}}{A} + \frac{3KTL_0^2}{AL^2}.$$

(b) Show that the isothermal Young's modulus at zero tension is given by $Y_0 = 3KT/A$.
(c) Show that the linear expansivity is given by

$$\alpha = \alpha_0 - \frac{\mathcal{F}}{AYT} = \alpha_0 - \frac{1}{T} \frac{L^3/L_0^3 - 1}{L^3/L_0^3 - 2},$$

where $\alpha_0 = (1/L_0)(dL_0/dT)$ is the value of the linear expansivity at zero tension.
(d) Assume the following values for a sample of rubber: $T = 300K$, $K = 1.333 \times 10^{-2} N/K$, $A = 1 \times 10^{-6} m^2$, $\alpha_0 = 5 \times 10^{-4} K^{-1}$. When this sample is stretched to length $L = 2L_0$, calculate $\mathcal{F}$, Y, and α.

17. [P.2.17, Sears] Show that $\beta = 3\alpha$ for an isotropic solid.

18. [P.2.11, Zemansky] Calculate $\left(\frac{\partial E}{\partial T}\right)_p$ and $\left(\frac{\partial p}{\partial T}\right)_E$ for a dielectric material obeying the equation

$$p = VE\left(a + \frac{b}{T}\right).$$

P.19. [P.2.4, Zemansky]

(a) A block of copper at a pressure of 1 atm (approximately 100 kPa) and a tem- perature of $5°$C is kept at constant volume. If the temperature is raised to $10°$C, what will be the final pressure?

(b) If the vessel holding the block of copper has a negligibly small thermal expansivity and can withstand a maximum pressure of 1000 atm, what is the highest temperature to which the system may be raised?

The volume expansivity β and isothermal compressibility κ are not always listed in handbooks of data. However, β is three times the linear expansion coefficient α and κ is the reciprocal of the bulk modulus B. For this problem, assume that the volume expansivity and isothermal compressibility remain practically constant within the temperature range of 0 to $20°$C at the values of 4.95×10^{-5} K^{-1} and 6.17×10^{-12} Pa^{-1}, respectively.

20. [P.2.3, Sears] A cylinder provided with a movable piston contains an ideal gas at a pressure p_1, specific volume v_1, and temperature T_1. The pressure and volume are simultaneously increased so that at every instant p and v are related by the equation $p = Av$, where A is a constant.

(a) Express the constant A in terms of the pressure p_1, the temperature T_1, and the gas constant R.

(b) Construct the graph representing the process above in the $p - v$ plane.

(c) Find the temperature when the specific volume has doubled, if $T_1 = 200$ K.

P.21. [P.2.8, Sears] Figure 2-20 from Sears shows five processes, $a - b$, $b - c$, $c - d$, $d - a$ and ac, plotted in the $p - v$ plane for an ideal gas in a closed system. Show the same processes (a) in the $p - T$ plane and (b) in the $T - v$ plane.

P.22. [P.2.11, Sears] A quantity of air is contained in a cylinder provided with a movable piston. Initially the pressure of the air is 2×10^7 N m^2, the volume is 0.5 m^3 and the temperature is 300 K. Assume air is an ideal gas.

(a) What is the final volume of the air if it is allowed to expand isothermally until the pressure is 1×10^7 Nm^2, the piston moving outward to provide for the increased volume of the air?

(b) What is the final temperature of the air if the piston is held fixed at its initial position and the system is cooled until the pressure is 1×10^7 Nm^2?

(c) What are the final temperature and volume of the air if it is allowed to expand isothermally from the initial conditions until the pressure is 1.5×10^7 Nm^2 and then it is cooled at constant volume until the pressure is 1×10^7 Nm^2? (d) What are the final temperature and volume of the air if an isochoric cooling to 1.5×10^7 Nm^2 is followed by an isothermal expansion to 1×10^7 Nm^2?

(e) Plot each of these processes on a $T - V$ diagram.

P.23. [P.2.12, Sears] A volume V at temperature T contains n_A moles of ideal gas A and n_B moles of ideal gas B. The gases do not react chemically.

(a) Show that the total pressure p of the system is given by $p = p_A + p_B$, where p_A and p_B are the pressures that each gas would exert if it were in the volume alone. The quantity p_A is called the partial pressure of gas A and the above equation is known as Dalton law of partial pressures.

(b) Show that $p_A = x_A p$, where x_A is the fraction of moles of A in the system.

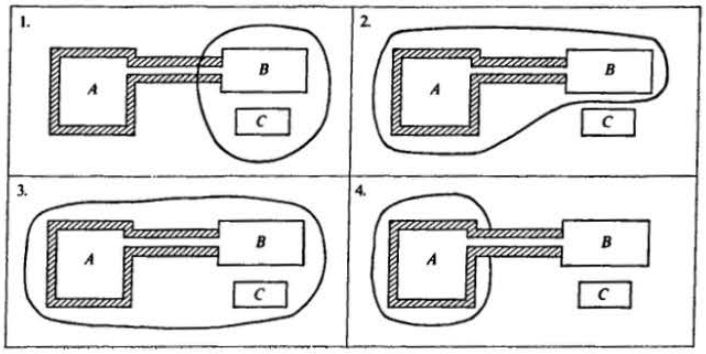

Figure 1–6

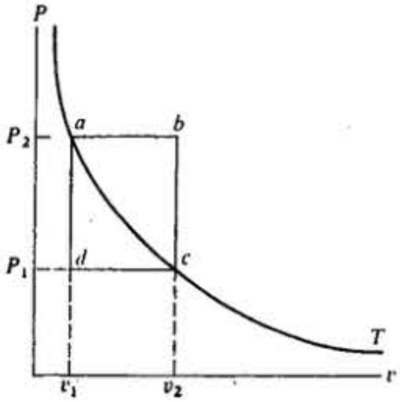

Figure 2–20