F 320 – Termodinâmica – Lista 2

P.01. [P.4.2, Zemansky] A combustion experiment is performed by burning a mixture of fuel and oxygen in a constant-volume container surrounded by a water bath. During the experiment, the temperature of the water rises. If the system is the mixture of fuel and oxygen: (a) Has heat been transferred? (b) Has work been done? (c) What is the sign of ΔU?

02. [P.4.3, Zemansky] A liquid is irregularly stirred in a well-insulated container and thereby experiences a rise in temperature. If the system is the liquid: (a) Has heat been transferred? (b) Has work been done? (c) What is the sign of ΔU?

03. [P.3.24, Sears] A mixture of hydrogen and oxygen is enclosed in a rigid insulating container and exploded by a spark. The temperature and pressure both increase. Neglect the small amount of energy provided by the spark itself. (a) Has there been a flow of heat into the system? (b) Has any work been done by the system? (c) Has there been any change in internal energy U of the system?

04. [P.3.2,Zemansky] Show that the work done by an ideal gas during the quasi-static isothermal expansion from an initial pressure p_i to a final pressure p_f is given by $W = nRT \ln{(p_f/p_i)}$.

P.05. [P.3.4, Zemansky] Calculate the work done upon expansion of 1 mol of a gas quasi-statically and isothermally from volume V_i to a volume V_f, when the equation of state is

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where $v = V/n$, a and b are the van der Waals constants.

P.06. [P.3.5, Zemansky and P.4.20, Sears]

(a) For an ideal gas, show that in a reversible adiabatic process:

$Tp^{(1-\gamma)/\gamma} = \text{constant}$ and $TV^{(\gamma-1)} = \text{constant}$.

(b) During a quasi-static expansion of a gas in an adiabatic container, the pressure and the volume at any moment is given by $pV^{\gamma} = K$, where γ, and K are constants. Show that the work done in expanding the gas from a state (p_i, V_i) to a state (p_f, V_f) is

$$W = -\frac{p_i V_i - p_f V_f}{\gamma - 1}.$$

If the initial pressure and volume are 10^6 Pa and 10^{-3}m^3, respectively, and the final values are 2×10^5 Pa and 3.16×10^{-3} m^3, respectively, how much work is done on a gas having $\gamma = 1.4$?

07. [P.3.3, Sears] An ideal gas originally at a temperature T_1 and pressure p_1 is compressed reversibly against a piston to a volume equal to one-half of its original volume. The temperature of the gas is varied during the compression so that at each instant the relation $p = AV$ is satisfied, where A is a constant. (a) Draw a diagram of the process in the $p - V$ plane. (b) Find the work done on the gas, in terms of n, R, and T_1.

08. [P.3.15, Sears] On a pV diagram starting from an initial state (p_0, V_0), plot an adiabatic expansion to $2V_0$, an isothermal expansion to $2V_0$, and an isobaric expansion to $2V_0$.

(a) Use this graph to determine in which process the least work is done by the system.

(b) If, instead, the substance was compressed to $V_0/2$, in which process would the least work be done?

(c) Plot the processes of parts (a) and (b) on a $p - T$ diagram starting from (p_0, T_0). Indicate expansions and compressions and be careful to show relative positions at the end points of each process.

09. [P.4.8, Zemansky] A container of volume V contains n moles of gas at high pressure. Connected to the container is a capillary tube through which the gas may leak slowly out to the atmosphere, where the pressure is p_0. Surrounding the container and capillary is a water bath, in which is immersed an electrical resistor. The gas is allowed to leak slowly through the capillary into the atmosphere while electrical energy is dissipated in the resistor at such a rate that the temperature of the gas, the container, the capillary, and the water is kept equal to that of the outside air. Show that, after as much gas as possible has leaked out during time interval t, the change in internal energy

$$\Delta U = \mathcal{E}It - p_0(nv_0 - V)$$

where v_0 is the molar volume of the gas at atmospheric pressure, $\mathcal{E}$ is the potential difference across the resistor, and I is the current in the resistor.

10. [P.4.10, Zemansky] Regarding the internal energy of a hydrostatic system to be a function of T and p, derive the following equations:

a) $$đQ = \left[\left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p\right] dT + \left[\left(\frac{\partial U}{\partial p}\right)_T + p\left(\frac{\partial V}{\partial p}\right)_T\right] dp.$$

b) $$\left(\frac{\partial U}{\partial T}\right)_p = C_p - pV\beta.$$

c) $$\left(\frac{\partial U}{\partial p}\right)_T = pV\kappa - (C_p - C_V)\frac{\kappa}{\beta}.$$

11. [P.4.5, Sears] Show that

$$\left(\frac{\partial h}{\partial p}\right)_T = -c_p\left(\frac{\partial T}{\partial p}\right)_h, \qquad \left(\frac{\partial u}{\partial p}\right)_v = c_v\left(\frac{\partial T}{\partial p}\right)_v, \qquad \left(\frac{\partial h}{\partial v}\right)_p = c_p\left(\frac{\partial T}{\partial v}\right)_p.$$

12. [P.4.8, Sears] The equation of state of a certain gas is $(p+b)v = RT$. (a) Show that the specific enthalpy of the gas of can be written as $h = (a+R)T + \text{constant}$. (b) Find c_p.

13. [P.4.2, Sears] The equation of state of a certain gas is $(p+b)v = RT$ and its specific internal energy is given by $u = aT + bv + u_0$. (a) Find c_v. (b)Show that $c_p - c_v = R$.

P.14. [P.4.14, Zemansky] One mole of a gas obeys the van der Waals equation of state (see problem 5) and its molar internal energy is given by $u = cT - a/v$, where a, b, c, and R are constants. Calculate the molar heat capacities c_v and c_p.

P.15. [P.4.11, Sears] An ideal gas for which $c_v = 5R/2$ is taken from point a to point b along the three paths a-c-b, a-d-b, and a-b, see Fig. 4.8, Sears. Let $p_2 = 2p_1$ and $v_2 = 2v_1$.
(a) Compute the heat supplied to the gas, per mole, in each of the three processes. Express the answer in terms of R and T_1.
(b) Compute the molar specific heat capacity of the gas in terms of R for the the process a-b.

P.16. [P.4.21, Sears] Figure 4.9 from Sears represents a cylinder with thermally insulated walls containing a movable frictionless thermally insulated piston. On each side of the piston are n moles of an ideal gas. The initial pressure p_0, volume V_0, and temperature T_0 are the same on both sides of the piston. The value of γ for the gas is 1.50, and c_v is independent of temperature. By means of a heating coil in the gas on the left side of the piston, heat is supplied slowly to the gas on this side. It expands and compresses the gas on the right side until its pressure has increased to $27p_0/8$. In terms of n, c_v, and T_0: (a) How much work is done on the gas on the right side?
(b) What is the final temperature of the gas on the right?
(c) What is the final temperature of the gas on the left?
(d) How much heat flows into the gas on the left?

P.17. [P.4.23, Sears]
(a) Show that the work done on an ideal gas to compress it isothermally is greater than that necessary to compress the gas adiabatically if the pressure change is the same in the two processes and
(b) that the isothermal work is less than the adiabatic work if the volume change is the same in the two processes.
(c) Plot these processes on a pV diagram and explain physically why the isothermal work should be greater than the adiabatic work in part (a) and why it should be less in part (b).

P.18. [P.1.3, Oliveira] Consider a gas whose internal energy $U = 3pV/2$. Determine the work done by the gas when it is adiabatically expanded from a state (p_1, V_1) to a state (p_2, V_2). Determine the heat that the gas is supplied to the gas in a isochoric process from state 2 to a state 3, whose energy is equal to the energy of state 1. Finally, consider a quasistatic process from state 1 to state 3 such that the energy is constant during the process and determine work and the heat involved.

19. [P.3.8, Sears] Show that the work done in an arbitrary process on a gas can be expressed as

$$\text{đ}W = pV\beta dT - pV\kappa dp.$$

In particular, find the work of an ideal gas in the arbitrary process.

P.20. [P.3.6, Sears] An ideal gas and a block of copper have equal volumes V_0 at temperature T_0 and pressure p_0. The pressure on both system is increased reversibly and isothermally to $5p_0$. (a) Explain with the aid of a $p - V$ diagram why the work is not the same in the two processes. (b) In which process is the work done greater?

21. [P.4.15, Zemansky] The equation of state for a monatomic solid is $pv + f(v) = \Gamma u$, where v is the molar volume, Γ is the Grüneisen constant, and u is the molar internal energy due to lattice vibrations. Prove that

$$\Gamma = \frac{\beta v}{c_v \kappa},$$

where κ is the isothermal compressibility. This equation, known as the Grüneisen relation, plays an important role in solid-state theory.

22. [P.4.17, Zemansky] The molar heat capacity at constant volume of a metal at low temperatures varies with the temperature according to the equation

$$\frac{C_V}{n} = \left(\frac{124.8}{\Theta}\right)^3 T^3 + \gamma T,$$

where Θ is the Debye temperature, γ is a constant, and C_V/n is measured in units of mJ/mol K. The first term on the left is the contribution attributable to lattice vibrations and the second term is due to the contribution of free electrons. For copper, $\Theta = 343$ K and $\gamma = 0.688$ mJ/mol K^2. How much heat per mole is transferred during a process in which the temperature changes from 2 to 3 K?

P.23. [P.3.8, Zemansky] The tension in a wire is increased quasi-statically and isothermally from f_i to f_j. If the length, cross-sectional area, and isothermal Young's modulus of the wire remain practically constant, show that the work done is

$$W = \frac{L}{2AY}\left(f_f^2 - f_i^2\right).$$

The tension in a copper wire 1 m long and 0.001 cm^2 in area is increased quasi- statically and isothermally at $20°$C from 10 to 100 N. How much work is done if the isothermal Young's modulus at 20°C is 1.23×10^{11} N/m^2?

24. [P.3.9, Zemansky] The equation of state of an ideal elastic substance is

$$f = KT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right),$$

where K is a constant and L_0 (the value of L at zero tension) is a function of temperature only. Calculate the work necessary to compress the substance from $L = L_0$ to $L = L_0/2$ quasi-statically and isothermally.

25. [P.3.10, Zemansky] Show that the work required to blow a spherical soap bubble of radius r in an isothermal, quasi-static process in the atmosphere is equal to $8\pi\gamma r^2$.

P.26. [P.3.12, Zemansky] A dielectric has an equation of state $p = \chi E V$, where p is the total electric dipole moment and χ is a function of temperature only. Show that the work done in an isothermal, quasi-static change of state is given by

$$W = \frac{1}{2V\chi}\left(p_f^2 - p_i^2\right) = \frac{V\chi}{2}\left(E_f^2 - E_i^2\right).$$

27. [P.3.13, Zemansky] Prove that the work done during a quasi-static isothermal change of state of a paramagnetic substance obeying Curie's law is given by

$$W = \frac{\mu_0 T}{2C}\left(m_f^2 - m_i^2\right) = \frac{\mu_0 C}{2T}\left(H_f^2 - H_i^2\right),$$

where m is the total magnetic dipole moment and C is the Curie constant.

28. [P.3.19, Zemansky] Calculate the work necessary to isothermally and reversibly remove a paramagnetic slender rod from a close fitting coaxial solenoid of zero resistance while the magnetic intensity H remains constant. Assume that the rod obeys Curie's law.

P.29. [P.4.12, Zemansky] For a stretched wire, show that

$$C_L = \left(\frac{\partial U}{\partial T}\right)_L \qquad C_f = \left(\frac{\partial U}{\partial T}\right)_f - fL\alpha.$$

For a paramagnetic solid obeying Curie law, show that

$$C_m = \left(\frac{\partial U}{\partial T}\right)_m \qquad C_H = \left(\frac{\partial U}{\partial T}\right)_H + \frac{m^2}{C},$$

where C is the Curie constant.

30. [P.4.27, Sears] The equation of state for radiant energy in equilibrium with the temperature of the walls of a cavity of volume V is $p = aT^4/3$. The energy equation is $U = aT^4V$. Show that the heat supplied in an isothermal doubling of the volume of the cavity is $4aT^4V/3$.

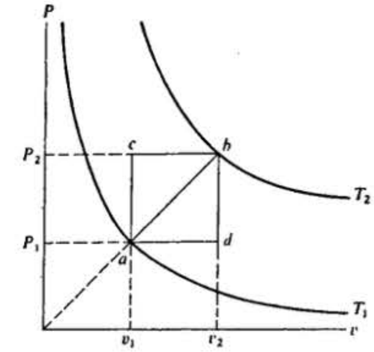

Figure 4–8

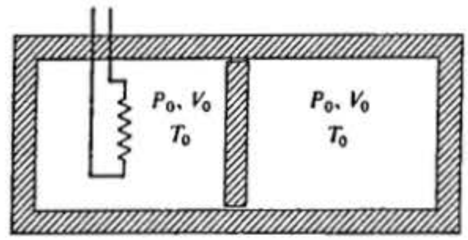

Figure 4–9