F 320 – Termodinâmica – Lista 3

01. [P.5.3, Zemansky] Mercury is poured into the open end of a J-shaped glass tube, which is closed at the short end, trapping air in that end. How much mercury can be poured in before the mercury overflows? Assume air to act like an ideal gas. The long and short arms are 1 m and 0.5 m long, respectively, and effects due to the curvature of the bottom may be neglected. Take atmospheric pressure to be 76 cm Hg.

02. [P.5.5, Zemansky] Two bulbs containing air, one of which has a volume three times the other, are connected by a tube of negligible volume and are initially at the same temperature. To what temperature must the air in the larger bulb be raised in order that the pressure be doubled? Neglect heat conduction through the air in the connecting tube.

03. [P.5.6, Zemansky] Expand the following equations in the form

$$pv = RT(1 + Bp + Cp^2 + \ldots),$$

and determine the second virial coefficient B for the a gas obeying the (a) van der Waals equation of state (see problem 2, Lista 2) and (b) the Dieterici equation of state:

$$pe^{a/RTv}(v - b) = RT.$$

04. [P.5.9, Zemansky] Prove that the work done by an ideal gas with constant heat capacities during a quasi-static adiabatic expansion is equal to (a) $W = -C_V(T_i - T_f)$ and

$$\text{(b)} \quad W = \frac{p_f V_f}{\gamma - 1}\left[1 - \left(\frac{p_i}{p_f}\right)^{(\gamma-1)/\gamma}\right].$$

05. [P.5.10, Zemansky] Show that the heat transferred during an infinitesimal quasi-static process of an ideal gas can be written

$$đQ = \frac{C_V}{nR} V\,dp + \frac{C_p}{nR} p dV.$$

Applying this equation to an adiabatic process and show that $pV^\gamma = \text{constant}$.

06. [P.5.14 Zemansky] An evacuated bottle with nonconducting walls is connected through a valve to a large supply of gas, where the pressure is p_0 and the temperature is T_0. The valve is opened slightly, and helium flows into the bottle until the pressure inside the bottle is p_0. Assuming that the helium behaves like an ideal gas with constant heat capacities, show that the final temperature of the helium in the bottle is γT_0.

P.07. [P.5.15, Zemansky] A thick-walled insulated chamber contains n_i moles of helium at high pressure p_i. It is connected through a valve with a large, almost empty container of helium at constant pressure p_0, very nearly atmospheric. The valve is opened slightly, and the helium flows slowly and adiabatically into the container until the pressures on the two sides of the valve are equal. Assuming the helium to behave like an ideal gas with constant heat capacities, show that:

(a) The final temperature of the gas in the chamber is $T_f = T_i\left(p_f/p_i\right)^{(\gamma-1)/\gamma}$;

(b) the number of moles left in the chamber is $n_f = n_i\left(p_f/p_i\right)^{1/\gamma}$;

(c) The final temperature of the gas in the container is

$$T_{f,cont} = \frac{T_i}{\gamma} \frac{1 - p_f/p_i}{1 - (p_f/p_i)^{1/\gamma}}.$$

08. [P.6.18, Sears] For an ideal gas, derive expressions

(a) for the specific entropy $s = s(T, v)$ and $s = s(p, v)$ and

(b) for the specific enthalpy $h = h(T, v)$ and $h = h(p, v)$.

09. [P.6.19, Sears] The specific entropy and enthalpy for an ideal gas can be written as

$$s = \int_{T_0}^{T_1} c_p \frac{dT}{T} - nR \ln \frac{p_1}{p_0} + s_0, \qquad h = \int_{T_0}^{T_1} c_p dT + h_0.$$

Assume that c_p for an ideal gas is given by $c_p = a + bT$, where a and b are constants.
(a) What is the expression for c_v for this gas?
(b) Obtain expressions for the specific entropy and enthalpy of an ideal gas in terms of the values in some reference state.
(c) Derive an expression for the internal energy of an ideal gas.

P.10. [P.6.3, Zemansky] Figure P6-1 represents a simplified pV diagram of the Joule ideal-gas cycle. All processes are quasi-static and C_p is constant. Prove that the thermal efficiency of an engine performing this cycle is

$$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}.$$

11. [P.6.4, Zemansky] Figure P6-2 represents a simplified pV diagram of the Sargent ideal-gas cycle. All processes are quasi-static and the heat capacities are constant. Prove that the thermal efficiency of an engine performing this cycle is

$$\eta = 1 - \gamma \frac{T_4 - T_1}{T_3 - T_2}.$$

P.12. [P.6.7, Zemansky] A vessel contains 10^{-3} m^3 of helium gas at 3 K and 10^3 Pa. Take the zero of internal energy of helium to be at this state.
(a) The temperature is raised at constant volume to 300 K. Assuming helium to behave like an ideal monatomic gas, how much heat is absorbed, and what is the internal energy of the helium? Can this energy be regarded as the result of heating or working?
(b) The helium is now expanded adiabatically to 3 K. How much work is done, and what is the new internal energy? Has heat been converted to work without compensation, thus violating the second law?
(c) The helium is now compressed isothermally to its original volume. What are the quantities of heat and work in this process? What is the thermal efficiency of the cycle? Plot the cycle on a $p-V$ diagram.

P.13. [P.6.10 and 6.12, Zemansky and P.5.23, Sears]
(a) A storage battery is connected to a motor, which is used to lift a weight. The battery remains at constant temperature by receiving heat from the outside air. Is this a violation of the second law? Why?
(b) There are many paramagnetic solids that have internal energies which depend only on temperature, like an ideal gas. In an isothermal decrease of the magnetic field, heat is absorbed from one reservoir and converted completely into work. Is this a violation of the second law? Explain.
(c) When there is a heat flow out of a system during a reversible isothermal process, the entropy of the system decreases. Why does this not violate the second law?

P.14. [P.5.12, Sears, P.8.2 and 8.3, Zemansky] (a) Show that the partial derivatives

$$\left(\frac{\partial T}{\partial S}\right)_p = \frac{T}{C_p}, \qquad \left(\frac{\partial T}{\partial S}\right)_V = \frac{T}{C_V}$$

are independent of the equation of state for the hydrostatic system. As a consequence, in a T-S diagram of, the slope of an isochoric curve is T/C_v while the slope of an isobaric curve is T/C_p.
(b) On a single T-S diagram, sketch curves for the following reversible processes for an ideal gas starting from the same initial stale: (i) an isothermal expansion, (ii) an adiabatic expansion, (iii) an isobaric expansion, and (iv) an isochoric process in which heat is added.

P.15. [P.4.28 and 4.31, Sears]

(a) Sketch a Carnot cycle for an ideal gas on a (i) u-v diagram, (ii) u-T diagram, (iii) u-h diagram, (iv) p-T diagram.

(b) Show that for Carnot engines operating between the same high temperature reservoirs and different low temperature reservoirs, the engine operating over the largest temperature difference has the greatest efficiency.

(c) Is the more effective way to increase the efficiency of a Carnot engine to increase the temperature of the hotter reservoir, keeping the temperature of the colder reservoir constant, or vice versa?

(d) Repeat parts (b) and (c) to find the optimum coefficient of performance for a Carnot refrigerator.

16. [P.7.1, Zemansky] Take an ideal monatomic gas ($\gamma = 5/3$) around the Carnot cycle, where $T_H = 600$ K and $T_L = 300$ K. Point 1 at the beginning of the adiabatic compression has pressure $p_1 = p_0$ (atmospheric pressure) and volume $V_1 = 50$ liters. Point 3 has a volume $V_3 = 75$ liters. The resulting Carnot cycle is shown in Fig. P7-1. Calculate the values of volume and pressure at all four points.

17. [P.7.4, Zemansky] A Carnot engine absorbs 100 J of heat from a reservoir at the temperature of the normal boiling point of water and rejects heat to a reservoir at the temperature of the triple point of water. Find the heat rejected, the work done by the engine, and the thermal efficiency.

P.18. [P.4.33, Sears] One mol of an ideal gas for which $c_v = 3R/2$ is the working substance of a Carnot engine. During the isothermal expansion the volume doubles. The ratio of the final volume to the initial volume in the adiabatic expansion is 5.7. The work output of the engine is 9×10^2 J in each cycle. Compute the temperature of the reservoirs between which the engine operates.

P.19. [P.5.20, Sears] Two identical finite systems of constant heat capacity C_p are initially at temperatures T_1 and T_2 where $T_2 > T_1$.

(a) These systems are used as the reservoirs of a Carnot engine which does an infinitesimal amount of work $đW$ in each cycle. Show that the final equilibrium temperature of the reservoirs is $(T_1 T_2)^{1/2}$.

(b) Show that the final temperature of the systems if they are brought in contact in a rigid adiabatic enclosure is $(T_1 + T_2)/2$.

(c) Which final temperature is greater?

(d) Show that the total amount of work done by the Carnot engine in part (a) is $C_p(T_2^{1/2} - T_1^{1/2})^2$.

(e) Show that the total work available in the process of part (b) is zero.

20. [P.6.2, Lemons] A reversible refrigerator engine extracts heat from the inside of a refrigerator compartment kept at $8°$C and rejects unwanted heat Q_H to its $20°$C exterior. Find the work required to extract one calorie from the interior of the refrigerator compartment.

21. [P.6.5, Lemons] A building is maintained at temperature T_H with a reversible heat pump operating between the building and a colder environment at temperature $T_C < T_H$. The heat pump consumes electrical power at a constant rate $\dot{W}$. The building also loses heat according to Newton law of cooling, that is, at a rate $\alpha(T_H - T_C)$ where α is constant. Show that the building temperature is maintained at

$$T_H = T_C + \frac{\dot{W}}{2\alpha}\left[1 + \sqrt{1 + \frac{4\alpha T_c}{\dot{W}}}\right].$$

22. [P.6.6, Lemons] A Carnot engine generates work at a rate $\dot{W}$ by operating between reservoirs at temperatures T_C and $T_H > T_C$. The lower-temperature reservoir is a finite body with surface area A that maintains its temperature by radiating electromagnetic energy into space at a rate $\sigma_B A T_C^4$, where σ_B is a universal constant.
(a) Express $\dot{W}$ in terms of T_C, T_H, σ_B, and A.
(b) Suppose one wants to minimize the area A of the colder reservoir while generating work at a given rate $\dot{W}$ by extracting heat from a reservoir at given temperature T_H. What is this minimum area A and what is the temperature T_C of the colder reservoir that minimizes the area?

23. [P.8.1, Zemansky]
(a) Derive the expression for the efficiency of a Carnot engine directly from a T-S diagram.
(b) Compare the efficiencies of cycles A and B of Fig. P8-1.

P.24. [P.8.7, Zemansky]
(a) One kilogram of water at 273 K is brought into contact with a heat reservoir at 373 K. When the water has reached 373 K, what is the entropy change of the water, of the heat reservoir, and of the universe?
(b) If the water had been heated from 273 to 373 K by first bringing it into contact with a reservoir at 323 K and then with a reservoir at 373 K, what would have been the entropy change of the universe?
(c) Explain how the water might be heated from 273 to 373 K with almost no change of entropy of the universe.

25. [P.8.8, Zemansky] A body of constant heat capacity C_p and at a temperature T_i is put in contact with a reservoir at a higher temperature T_f. The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to

$$\Delta S = C_p \left[x - \ln(1+x) \right],$$

where $x = -(T_f - T_i)/T_f$. Prove that the entropy change is positive.

P.26. [P.5.5, Sears] A 50-ohm resistor carrying a constant current of 1 A is kept at a constant temperature of $27°$C by a stream of cooling water. In a time interval or 1s,
(a) what is the change in entropy of the resistor?
(b) what is the change in entropy of the universe?

27. A thermally insulated 50-ohm resistor carries a current or 1 A for 1 s. The initial temperature of the resistor is $10°$C, its mass is 5g, and its specific heat capacity is 850 J/Kg K.
(a) What is the change in entropy of the resistor?
(b) What is the change in entropy or the universe?

28. [P.8.10, Zemansky] According to Debye's law, the molar heat capacity at constant volume of a diamond varies with the temperature as follows:

$$c_v = 3R\frac{4\pi^4}{5}\left(\frac{T}{\Theta}\right)^3.$$

What is the entropy change in units of R of a diamond of 1.2 g mass when it is heated at constant volume from 10 to 350 K? The molar mass of diamond is 12 g and $\Theta = 2230$ K.

P.29. [P.7.6, Lemons] Consider two identical, finite blocks of metal each with constant heat capacity C. Initially, one is at temperature T_H and the other is colder, with temperature $T_C < T_H$.
(a) The two blocks are placed in thermal contact and allowed to equilibrate. According to Problem 19 their final temperature is $(T_H + T_C)/2$. What is the total change in the entropy of the two blocks?
(b) Imagine the two blocks, again at temperatures T_H and $T_C < T_H$, are allowed to equilibrate by means of a reversible heat engine operating between the two blocks. According to Problem 19 their final temperature is $\sqrt{T_H T_C}$. What is the total change in the entropy of the two blocks?

30. [P.8.11, Zemansky] A thermally insulated cylinder, closed at both ends, is fitted with a frictionless heat-conducting piston that divides the cylinder into two parts. Initially, the piston is clamped in the center with 1 liter of air at 300 K and 2 atm pressure on one side and 1 liter of air at 300 K at 1 atm pressure on the other side. The piston is released and reaches equilibrium in pressure and temperature at a new position. Compute the final pressure and temperature and increase of entropy if air is assumed to be the ideal gas. What irreversible process has taken place?

P.31. [P.8.12, Zemansky] An adiabatic cylinder, closed at both ends, is fitted with a frictionless adiabatic piston that divides the cylinder into two parts. Initially the pressure, volume, and temperature are the same on both sides of the piston (P_0, V_0, and T_0). The gas is ideal with C_V independent of temperature and $\gamma = 1.5$. By means of a heating coil in the gas on the left side, heat is slowly supplied to the gas on the left until the pressure reaches $27P_0/8$. In terms of nR, V_0, and T_0:

(a) What is the final volume on the right side?

(b) What is the final temperature on the right side?

(c) What is the final temperature on the left side?

(d) How much heat must be supplied to the gas on the left side? (Ignore the coil!)

(e) How much work is done on the gas on the right side?

(f) What is the entropy change of the gas on the right side?

(g) What is the entropy change of the gas on the left side?

(h) What is the entropy change of the universe?

32. [P.5.11, Sears] A body of finite mass is originally at a temperature T_2, which is higher than that of a heat reservoir at a temperature T_1. An engine operates in infinitesimal cycles between the body and the reservoir until it lowers the temperature of the body from T_2 to T_1. In this process there is a heat flow Q out of the body. Prove that the maximum work obtainable from the engine is $Q + T_1(S_1 - S_2)$, where $S_1 - S_2$ is the decrease in entropy of the body.

33. [P.5.21, Sears] A mass m of a liquid at a temperature T_1 is mixed with an equal mass of the same liquid at a temperature T_2 The system is thermally insulated. Show that the entropy change of the universe is

$$2mc_p \ln \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}$$

and prove that this is necessarily positive.

34. [P.6.22, Sears] The pressure on a block of copper at a temperature of $0°$C is increased isothermally and reversibly from 1 atm to 1000 atm. Assume that β, κ, and ρ are constant and equal respectively to 5×10^{-5} K^{-1}, 8×10^{-12} m^2/N, and 8.9×10^3 Kg/m^3.

(a) Calculate the work done on the copper per kilogram and the heat evolved.

(b) How do you account for the fact that the heat evolved is greater than the work done?

(c) What would be the rise in temperature of the copper, if the compression were adiabatic rather than isothermal?

Explain approximations made.

35. [P.8.7, Adkins] The principal specific heat capacities of a certain perfect gas are $c_p = 1.0$ and $c_v = 0.7$ kJ/ K kg. In a reversible heat engine the gas is (a) heated at constant volume until the pressure is $6/5$ of its initial value, (b) heated at constant pressure until its volume is $5/4$ of its initial value, (c) cooled at constant volume until the pressure returns to its initial value, (d) cooled at constant pressure until the volume returns to its initial value. Find the greatest possible efficiency of this engine and the ratio of the maximum and minimum temperatures of the gas. How does the efficiency of the engine compare with that of a Carnot engine working between the same extremes of temperature?

36. [P.8.8, Adkins] A perfect gas, for which $\gamma = 1.5$, is used as the working substance in a Carnot engine operating between reservoirs at $300°$C and $50°$C. The isothermal process at the hotter reservoir consists of an expansion from a pressure of 10 atm and a volume of 1×10^{-3} m^3 to a pressure of 4 atm and a volume of 2.5×10^{-3} m^3.

(a) Between what limits of pressure and volume does it operate at the cold reservoir?

(b) Calculate the heat taken from the source and the heat rejected to the sink in each cycle and show that the efficiency is indeed $(1 - T_2/T_1)$.

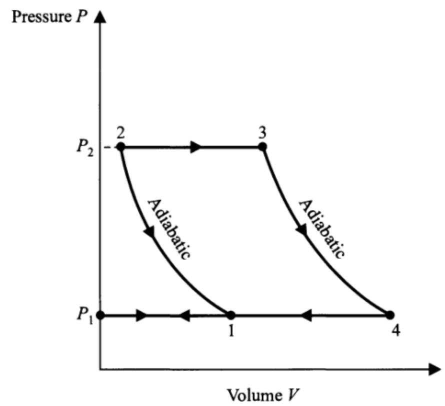

FIGURE P6-1
Joule ideal-gas cycle.

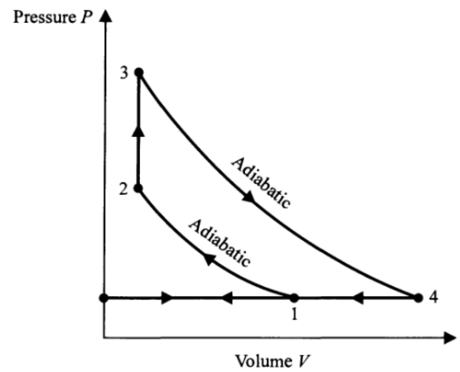

FIGURE P6-2
Sargent ideal-gas cycle.

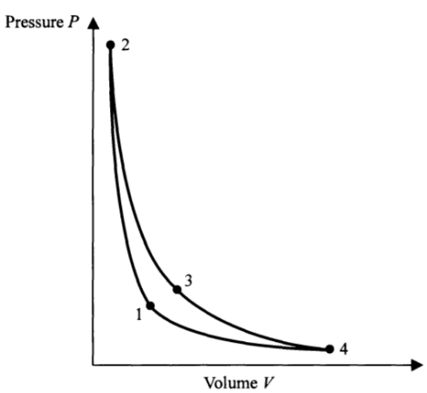

FIGURE P7-1
An accurately drawn Carnot cycle for an ideal gas with the ratio $V_3/V_1 = \frac{3}{2}$.

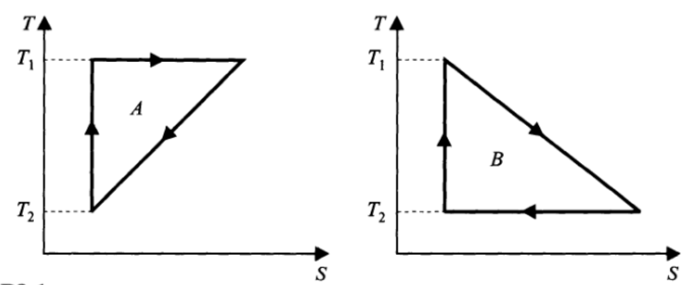

FIGURE P8-1