

Supercondutividade.

Refs.: Caps. 17 e 18, Bruus,

Caps. 34 e 35, Ketterson,

Cap. 14, Coleman,

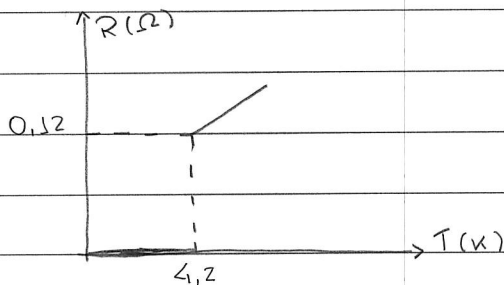
Cap. 3, Tinkem,

Sec. 48, Fetter and Walecka.

• inicial I: propriedades material supercondutor (tipo-I):

(1) resistência nula p/ $T < T_c$ ($\vec{H}_{ext} = 0$),

primeira observação: H. Onnes (1911)



(300.1)

: resistência x temperatura

p/ Hg: $T_c = 4,2\text{K}$

observação: transição ocorre na ausência de modificação estrutural

↳ transição a $T = T_c \sim$ transição eletrônica

comportamento similares: Be Al Nb Mo

T_c (K): 0,03 1,19 9,2 0,92

SC moderna: Pb NbGe₃ MgB₂

por fônons T_c (K): 7,2 23 39

Obs.: p/ detalhes, veja Tabela 14.1, Coleman.

notas: considerando a equação constitutiva:

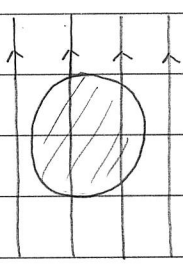
$$\vec{J} = \sigma \vec{E} \xrightarrow{p/T < T_c: \sigma \rightarrow +\infty} \vec{E} = 0$$

$$\textcircled{+} \frac{1}{c} \partial_t \vec{B} = -\vec{\nabla} \times \vec{E} \rightarrow \partial_t \vec{B} = 0 \rightarrow \vec{B} = 0 : \text{interior do material} :$$

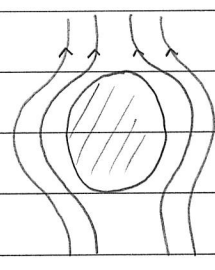
: Ok p/ SC e condutor perfeito!

(2) Efeito Meissner,

primeira observação: Meissner e Ochsenfeld (1933);
 consideram amostra material SC sob campo magnético;
 verifica-se que:



T > Tc



T < Tc

$$: H < H_c(T)$$

(300.2)

campo magnético interior SC = 0: diamagnetismo perfeito ~
 ~ \exists corrente SC na superfície de amostra

$$\text{como: } \vec{B} = \vec{H} + 4\pi \vec{M}$$

(300.3)

$$\text{e como p/ pontos E interior SC: } \vec{B} = 0 \rightarrow \vec{M} = -\frac{1}{4\pi} \vec{H} = -\frac{1}{4\pi} \vec{H}_{ext}$$

de fato: SC caracterizado pelas propriedades (300.1) e (300.2)!

comparar comportamento SC e condutor perfeito

(veja Fig. 32.1, Kittelson):

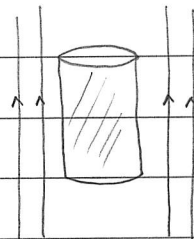
SC e condutor

perfeito :

$T < T_c$



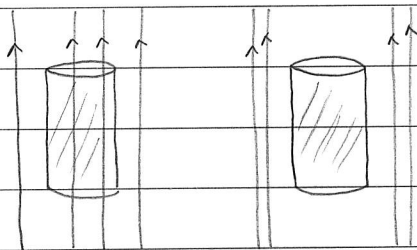
$H = 0$



$0 < H < H_c$

entrelento p/ $0 < H < H_c$:

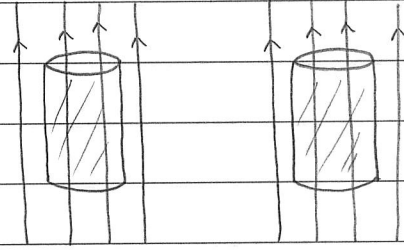
(302.1)



$T > T_c$

$T < T_c$

v. s.



$T > T_c$

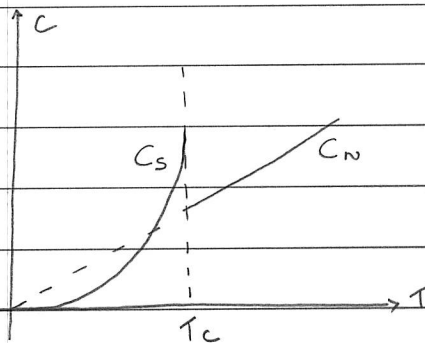
$T < T_c$

supercondutor

condutor perfeito

(3) calor específico,

Al:



(302.2)

C_N : calor específico metal normal

C_S : " " supercondutor

Definição (Mchen) metal normal: metal que não apresenta ordem magnética ou supercondutora

p/ metal normal, verifica-se que:

$$C_N = \gamma T + \beta T^3$$

↑ \sim vibrações da rede (fônons)
└ \sim elétrons

p/ supercondutor, verifica-se que:

$$C_S \sim e^{-\Delta/k_B T}, \text{ p/ } T \rightarrow 0 \quad (303.1)$$

Δ : energia de gap entre estado fundamental e estados excitados;

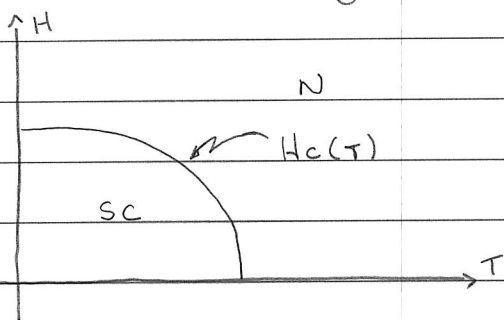
notas: descontinuidade calor específico a $T = T_c$

\hookrightarrow transição de fase metal normal - SC ($H_{ext} = 0$) =
= " " " " de 2ª ordem!

Obs.: verifica-se que $\Delta < 2k_B T_c$.

(4) \exists campo magnético crítico $H_c(T)$,

verifica-se que: diagrama H-T: diagrama de fases:



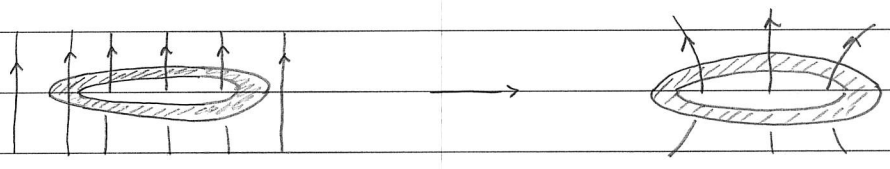
N: metal normal
SC: superconductor (303.2)

transição N-SC: transição de fase de 1ª ordem

exceto p/ $H = 0$

e p/ $T = 0$: " " " " 2ª ordem.

(5) correntes persistentes e quantização de fluxo,
 consideram amostra: anel metálico:



(304.1)

$T > T_c$
 $\vec{H} \neq 0$

2 etapas: (i) $T \rightarrow T < T_c$
 (ii) $\vec{H} = 0$

notas: como linha campo H não penetra noel p/ $T < T_c$
 \hookrightarrow após remoção H_{ext} , temos que fluxo magnético
 no interior do anel preservado \sim corrente SC
 " " " " ;

de fato, verifica-se que: $\Phi_B = n \Phi_0$, $\Phi_0 = \frac{hc}{2e} = 2.07 \cdot 10^7 \text{ Gauss cm}^2$:

: quantum de fluxo

$n \in \mathbb{N}$.

(6) efeito isotópico,

Lembrar: transição N-SC: \nearrow modificação estrutural
 " " \sim sistema eletrônico;

verifica-se que: $T_c \sim 1/\sqrt{M}$

onde M : massa do íon (sistemas puros):

: indicação relação entre SC e interação
 elétron-fônon!

inicial II: descrição termodinâmica SC:

variáveis termodinâmicas:

T : temperatura

S : entropia

\vec{B} : campo magnético (interno material)

\vec{H} : " " externo ~ condições experimentais

lembrar: $dU = Tds + \frac{1}{4\pi} \vec{H} \cdot d\vec{B}$; $U = U(S, B)$

$$dF = -SdT + \frac{1}{4\pi} \vec{H} \cdot d\vec{B} ; F = F(T, B)$$

$$dG = -SdT - \frac{1}{4\pi} \vec{B} \cdot d\vec{H} ; G = G(T, H)$$

U : energia interna

(305.1)

$F = U - TS$: energia livre de Helmholtz

$G = F - \frac{1}{4\pi} \vec{B} \cdot \vec{H}$: " " " Gibbs

considerar: - amostra = cilindro longo, eixo // eixo \hat{z}

$\vec{H} = \vec{H}_{ext}$ // eixo \hat{z} ;

variáveis termodinâmicas independentes: T e H

↳ considerar: energia livre de Gibbs;

lembrar: $S = -\left(\frac{\partial G}{\partial T}\right)_H$ e $\vec{B} = -4\pi \left(\frac{\partial G}{\partial H}\right)_T$ (305.2)

pr $T = cte$, temos que:

$$dG = -\frac{1}{4\pi} B dH \rightarrow G(T, H) - G(T, 0) = -\frac{1}{4\pi} \int_0^H B(H') dH' \quad (305.3)$$

· p/ metal normal: $M = 0$

$$\oplus \text{ Eq. (300.3): } B = H + 4\pi M = H$$

$$\hookrightarrow \text{ Eq. (305.3): } G_N(T, H) - G_N(T, 0) = -\frac{1}{4\pi} \int_0^H H' dH' = -\frac{1}{8\pi} H^2 \quad (306.1)$$

· p/ SC: $B = 0$

$$\hookrightarrow \text{ Eq. (305.3): } G_S(T, H) = G_S(T, 0) \quad (306.2)$$

· p/ transição de fase: $T = cte$ e $H = H_c(T)$: veja diagrama de fases (303.2),

temos a condição:

$$G_S(T, H_c) = G_N(T, H_c) \quad (306.3)$$

· notan.: Eqs. (306.1) - (306.3):

$$G_S(T, H_c) = G_S(T, 0) = G_N(T, 0) - \frac{1}{8\pi} H_c^2 = G_N(T, H_c)$$

$$\hookrightarrow G_S(T, 0) - G_N(T, 0) = -\frac{1}{8\pi} H_c^2 < 0 \quad (306.4)$$

↑ energia de condensação estado SC

· notan.: Eq. (306.2) \ominus Eq. (306.1):

$$G_S(T, H) - G_N(T, H) - \underbrace{(G_S(T, 0) - G_N(T, 0))}_{-\frac{1}{8\pi} H_c^2} = \frac{1}{8\pi} H^2$$

(306.5)

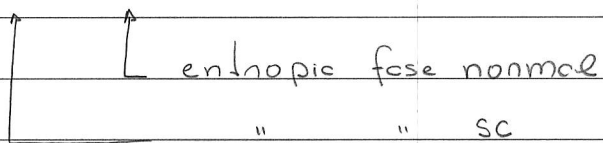
$$\hookrightarrow G_S(T, H) - G_N(T, H) = \frac{1}{8\pi} (H^2 - H_c^2) < 0 \text{ p/ } H < H_c(T),$$

i.e., como $G_S(T, H) < G_N(T, H)$ p/ $H < H_c(T)$

↳ SC: fase de equilíbrio: veja diagrama de fases (303.2)!

• notan Eqs. (305.2) e (306.5):

$$-S_S + S_N = -\frac{1}{4\pi} H_c(T) \frac{dH_c(T)}{dT}$$



$$\text{↳ } S_N - S_S = -\frac{1}{4\pi} H_c(T) \frac{dH_c(T)}{dT} \quad (307.1)$$

⊕ diagrama de fases (303.2):

se $H_c(T) \neq 0$, temos que $\frac{dH_c}{dT} < 0$

↳ Eq. (307.1) → $S_N - S_S > 0 \rightarrow S_S < S_N$: descontinuidade da entropia: OK a transição de fase de 1ª ordem;

se $H = 0 \rightarrow H_c = 0 \rightarrow \Delta S = S_N - S_S = 0$

: OK a transição de

se $T = 0 \rightarrow \frac{dH_c}{dT} = 0 \rightarrow \Delta S = 0$ fase de 2ª ordem!

• sobre o calor específico,

lembra: $C_H = T \left(\frac{\partial S}{\partial T} \right)_H$: capacidade térmica a H cte (307.2)

$$\text{Eq. (307.1): } C_{H,S} - C_{H,N} = \frac{T}{4\pi} \left(\left(\frac{dH_c}{dT} \right)^2 + H_c \frac{d^2 H_c}{dT^2} \right) \quad (307.3)$$

em particular, p/ $T = T_c$ e $H = 0 \rightarrow H_c = 0$, temos que:

$$(C_{H,S} - C_{H,N})_{T=T_c} = \frac{T_c}{4\pi} \left(\frac{dH_c}{dT} \right)_{T=T_c}^2 : \text{veja Fig. (303.2)} \quad (308.1)$$

início III: função de Green de Matsubara p/ fônons:

Lembrar hamiltoniano (137.1):

$$H = \sum_s \sum_{\vec{q} \in BZ} \hbar \omega_s(\vec{q}) (a_{\vec{q}s}^\dagger a_{\vec{q}s} + 1/2), \quad (308.2)$$

cujos estado fundamental $|0\rangle =$ vácuo bósons a : $a_{\vec{q}s}|0\rangle = 0$.

Eq (137.2): op. de campo na representação de Heisenberg:

$$\phi_s(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{q}} C_{\vec{q}}^s e^{i\vec{q} \cdot \vec{r}} (a_{\vec{q}s} e^{-i\omega_s(\vec{q})t} + a_{-\vec{q}s}^\dagger e^{i\omega_s(\vec{q})t}) \quad (308.3)$$

onde $C_{\vec{q}}^s = \sqrt{\hbar/2m\omega_s(\vec{q})}$

considerando $t \rightarrow -i\beta$, temos que

$$\phi_s(\vec{r}, \beta) = \frac{1}{\sqrt{V}} \sum_{\vec{q}} C_{\vec{q}}^s e^{i\vec{q} \cdot \vec{r}} (a_{\vec{q}s} e^{-\omega_s(\vec{q})\beta} + a_{-\vec{q}s}^\dagger e^{+\omega_s(\vec{q})\beta}) \quad (308.4)$$

p/ fônons: $K = H - \mu N = H$, pois $\mu = 0$

↳ Definição: função de Green de Matsubara p/ fônons:

$$D_s(\vec{r}, \beta; \vec{r}', \beta') = -\frac{1}{Z} T_n \left[e^{\beta H} \tau_c (\phi_s(\vec{r}, \beta) \phi_s(\vec{r}', \beta')) \right] : \quad (308.5)$$

: acompanham c/ Eqs. (137.3) e (244.1).

Obs.: similar ao caso $T=0$, vamos omitir o índice $s \rightarrow$ vamos considerar apenas 1 modo, pois o hamiltoniano (308.2) descreve sistema de fônons não-interagentes!

notar Eq. (308.5):

$$-\mathcal{D}^0(\vec{r}, \tau; \vec{r}', \tau') = \underbrace{\theta(\tau - \tau') \langle \phi(\vec{r}, \tau) \phi(\vec{r}', \tau') \rangle}_{(I)} + \underbrace{\theta(\tau' - \tau) \langle \phi(\vec{r}', \tau') \phi(\vec{r}, \tau) \rangle}_{(II)},$$

onde $\langle \hat{O} \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} \hat{O})$: Eq. (244.3)

temos que:

$$(I) = \frac{1}{V} \sum_{\vec{k}, \vec{q}} C_{\vec{k}} C_{\vec{q}} e^{i(\vec{k} \cdot \vec{r} + \vec{q} \cdot \vec{r}')} *$$

$$* \langle (a_{\vec{k}} e^{-\omega_{\vec{k}} \tau} + a_{-\vec{k}}^{\dagger} e^{\omega_{\vec{k}} \tau}) (a_{\vec{q}} e^{-\omega_{\vec{q}} \tau'} + a_{-\vec{q}}^{\dagger} e^{\omega_{\vec{q}} \tau'}) \rangle$$

$$e^{-\omega_{\vec{k}} \tau + \omega_{\vec{q}} \tau'} \langle a_{\vec{k}} a_{-\vec{q}}^{\dagger} \rangle + e^{\omega_{\vec{k}} \tau - \omega_{\vec{q}} \tau'} \langle a_{-\vec{k}}^{\dagger} a_{\vec{q}} \rangle$$

$$\delta_{\vec{k}, -\vec{q}} (1 + n_{BE}(\omega_{\vec{k}}))$$

$$\delta_{\vec{k}, -\vec{q}} n_{BE}(\omega_{\vec{k}})$$

$$\equiv n_{\vec{k}}$$

(309.1)

$$\begin{aligned} \hookrightarrow (I) = \frac{1}{V} \sum_{\vec{k}} C_{\vec{k}}^2 e^{i\vec{k} \cdot (\vec{r} - \vec{r}')} & \left(e^{-\omega_{\vec{k}} (\tau - \tau')} (1 + n_{\vec{k}}) + \right. \\ & \left. + e^{\omega_{\vec{k}} (\tau - \tau')} n_{\vec{k}} \right) \end{aligned}$$

similar p/ (II) : $\vec{n} \leftrightarrow \vec{n}'$ e $\tau \leftrightarrow \tau'$;

$$\hookrightarrow \mathcal{D}^0(\vec{n}-\vec{n}', \tau-\tau') = -\frac{1}{v} \sum_{\vec{k}} C_{\vec{k}}^2 * \left(\theta(\tau-\tau') e^{i\vec{k}\cdot(\vec{n}-\vec{n}')} \left(e^{-\omega_{\vec{k}}(\tau-\tau')} (1+n_{\vec{k}}) + e^{\omega_{\vec{k}}(\tau-\tau')} n_{\vec{k}} \right) + \theta(\tau'-\tau) e^{-i\vec{k}\cdot(\vec{n}-\vec{n}')} \left(e^{\omega_{\vec{k}}(\tau-\tau')} (1+n_{\vec{k}}) + e^{-\omega_{\vec{k}}(\tau-\tau')} n_{\vec{k}} \right) \right) \quad (310.1)$$

como (veja Eq. (248.2)):

$$\mathcal{D}^0(\vec{n}, \tau) = \frac{1}{\beta h} \sum_{n \text{ par}} \int \frac{d^3 \vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{n}} e^{-i\omega_n \tau} \mathcal{D}^0(\vec{q}, \omega_n) \quad (310.2)$$

$$\stackrel{e}{=} \mathcal{D}^0(\vec{q}, \omega_n) = \int_0^{\beta h} d\tau \int d^3 \vec{n} e^{-i\vec{q}\cdot\vec{n}} e^{i\omega_n \tau} \mathcal{D}^0(\vec{n}, \tau),$$

temos que:

$$\mathcal{D}^0(\vec{q}, \omega_n) = -\frac{1}{v} \sum_{\vec{k}} \underbrace{\int d^3 \vec{n} e^{-i(\vec{q}-\vec{k})\cdot\vec{n}}}_{\sqrt{\delta_{\vec{k}, \vec{q}}}} + C_{\vec{k}}^2 * \left(\underbrace{\int_0^{\beta h} d\tau e^{(i\omega_n - \omega_{\vec{k}})\tau} (1+n_{\vec{k}})}_{-1} + \underbrace{\int_0^{\beta h} d\tau e^{(i\omega_n + \omega_{\vec{k}})\tau} n_{\vec{k}}}_{+1} \right) \quad \text{: veja Eq. (254.1)}$$

$$\frac{-1}{i\omega_n - \omega_{\vec{q}}} \qquad \frac{+1}{i\omega_n + \omega_{\vec{q}}}$$

$$\hookrightarrow \mathcal{D}^0(\vec{q}, \omega_n) = C_{\vec{q}}^2 \left(\frac{-1}{i\omega_n + \omega_{\vec{q}}} + \frac{1}{i\omega_n - \omega_{\vec{q}}} \right) \quad (310.3)$$

$$= C_{\vec{q}}^2 \cdot \frac{2\omega_{\vec{q}}}{(i\omega_n)^2 - \omega_{\vec{q}}^2}$$

Interação elétron-fônon e diagramas de Feynman.

considerar: elétrons \oplus fônons não-interagentes;

hamiltoniano: Eqs. (252.1) e (308.2):

$$H_0 = H_0 - \mu N = \sum_{\vec{k}, \alpha} \underbrace{(E_{\vec{k}} - \mu)}_{\epsilon_{\vec{k}}} C_{\vec{k}, \alpha}^{\dagger} C_{\vec{k}, \alpha} + \sum_s \sum_{\vec{q} \in BZ} \hbar \omega_s(\vec{q}) (a_{\vec{q}, s}^{\dagger} a_{\vec{q}, s} + 1/2) \quad (311.1)$$

nesse caso, em termos de números de ocupação $n_{\vec{k}, \alpha}$ e $N_{\vec{q}, s}$, os estados do sistema são dados por:

$$|\psi\rangle = \underbrace{|n_{\vec{k}_1, \alpha_1}, n_{\vec{k}_2, \alpha_2}, \dots\rangle}_{\text{elétrons}} \underbrace{|N_{\vec{q}_1, s_1}, N_{\vec{q}_2, s_2}, \dots\rangle}_{\text{fônons}} \quad (311.2)$$

Lembrar: interação elétron-fônon (108.2):

$$H_{e-ph} = \frac{1}{V} \sum_{\vec{k}, \vec{q}} \sum_{\alpha, s} g_s(\vec{q}) C_{\vec{k}+\vec{q}, \alpha}^{\dagger} C_{\vec{k}, \alpha} \underbrace{(a_{\vec{q}, s} + a_{-\vec{q}, s}^{\dagger})}_{\equiv A_{\vec{q}, s} = A_{-\vec{q}, s}^{\dagger}} \quad (311.3)$$

onde $g_s(\vec{q}) \equiv M_s(\vec{q}) = M_s(\vec{q}, \vec{G} = 0)$

considerando (311.1), na versão de interação modificada (256.2), temos que:

$$V_S(\tau) = H_{e-ph, S}(\tau) = \frac{1}{V} \sum_{\vec{k}, \vec{q}} \sum_{\alpha, s} g_s(\vec{q}) C_{\vec{k}+\vec{q}, \alpha}^{\dagger}(\tau) C_{\vec{k}, \alpha}(\tau) A_{\vec{q}, s}(\tau) \quad (311.4)$$

pr sistema homogêneo e interação independente de spin,

Eq. (259.2) pode ser escrita como:

$$G_{\alpha}(x, \beta) = \frac{1}{Z/Z_0} \sum_{n \geq 0} \left(\frac{-1}{\hbar} \right)^n \frac{1}{n!} \int_0^{\beta \hbar} d\beta_1 \dots d\beta_n *$$

$$* \langle T_{\tau} (V_I(\beta_1) \dots V_I(\beta_n) C \tilde{\psi}_{\alpha}(\beta) C^{\dagger} \tilde{\psi}_{\alpha}(0)) \rangle_0,$$

(312.1)

onde

$$\frac{Z}{Z_0} = \sum_{n \geq 0} \left(\frac{-1}{\hbar} \right)^n \frac{1}{n!} \int_0^{\beta \hbar} d\beta_1 \dots d\beta_n \langle T_{\tau} (V_I(\beta_1) \dots V_I(\beta_n)) \rangle_0$$

consideram $V_I(\beta) = H_e - p_{H,I}(\beta)$;

como, p/ sistema não-inteligente, possíveis estados do sistema assumem a forma (311.2), temos que:

$$\langle T_{\tau} (A \tilde{q}_1 s_1(\beta_1) A \tilde{q}_2 s_2(\beta_2) \dots A \tilde{q}_n s_n(\beta_n)) *$$

$$* C \tilde{\psi}_1^{\dagger} + \tilde{q}_1^{\dagger} \alpha_1(\beta_1) C \tilde{\psi}_1(\beta_1) \dots C \tilde{\psi}_n^{\dagger} + \tilde{q}_n^{\dagger} \alpha_n(\beta_n) C \tilde{\psi}_n(\beta_n) *$$

$$* C \tilde{\psi}_{\alpha}(\beta) C^{\dagger} \tilde{\psi}_{\alpha}(0) \rangle_0 = (I)$$

(312.2)

$$= \langle \text{fônons ops. } A \rangle_0 \langle \text{férmions ops. } C \rangle_0$$

notas: como $A \tilde{q}_{\alpha}$: linear em termos ops. $A \tilde{q}_{\alpha}$, temos que apenas termos $n \rightarrow 2n$ da série (312.1) são, em princípio, não nulos!

p/ $n=2$, temos que:

$$(I) = \langle T_{\tau} (A \tilde{q}_1 s_1(\beta_1) A \tilde{q}_2 s_2(\beta_2)) \rangle_0 *$$

(II)

$$* \langle T_{\tau} (C \tilde{\psi}_1^{\dagger} + \tilde{q}_1^{\dagger} \alpha_1(\beta_1) C \tilde{\psi}_1(\beta_1) C \tilde{\psi}_2^{\dagger} + \tilde{q}_2^{\dagger} \alpha_2(\beta_2) C \tilde{\psi}_2(\beta_2) C \tilde{\psi}_{\alpha}(\beta) C^{\dagger} \tilde{\psi}_{\alpha}(0)) \rangle_0$$

• Detalhes Eq. (352.1):

$$\text{Lembrando Eq. (252.2): } \Psi_{\alpha}(\vec{n}, \tau) = \frac{1}{\sqrt{V/2}} \sum_{\vec{u}} e^{i\vec{u} \cdot \vec{n}} C_{\vec{u}\alpha}(\tau)$$

$$\Psi_{\alpha}^{\dagger}(\vec{n}, \tau) = \frac{1}{\sqrt{V/2}} \sum_{\vec{u}} e^{-i\vec{u} \cdot \vec{n}} C_{\vec{u}\alpha}^{\dagger}(\tau)$$

$$\hookrightarrow \Psi_{\alpha}(\vec{n}, \tau) \Psi_{\beta}^{\dagger}(\vec{n}', \tau') = \frac{1}{V} \sum_{\vec{u}, \vec{q}} e^{i\vec{u} \cdot \vec{n} - i\vec{q} \cdot \vec{n}'} C_{\vec{u}\alpha}(\tau) C_{\vec{q}\beta}^{\dagger}(\tau')$$

p/ sistema homogêneo e $\tau' = 0$, Eq. (259.2) assume a forma:

$$G_{\alpha\beta}(\vec{n} - \vec{n}', \tau) = \frac{1}{V} \sum_{\vec{u}} e^{i\vec{u} \cdot \vec{n}} *$$

$$* \frac{(-1)^n}{Z} T_n \left[e^{-\beta K_0} \sum_{n \geq 0} \left(\frac{-1}{\hbar} \right)^n \frac{1}{n!} \int_0^{\beta \hbar} d\tau_1 \dots d\tau_n * \right.$$

$$\left. * T_{\tau} \left(V_I(\tau_1) \dots V_I(\tau_n) C_{\vec{u}\alpha}(\tau) C_{\vec{u}\beta}^{\dagger}(0) \right) \right]$$

$$G_{\alpha\beta}(\vec{u}, \tau)$$

$$\hookrightarrow G_{\alpha\beta}(\vec{u}, \tau) = - \frac{1}{Z/Z_0} \sum_{n \geq 0} \left(\frac{-1}{\hbar} \right)^n \frac{1}{n!} \int_0^{\beta \hbar} d\tau_1 \dots d\tau_n *$$

$$* \frac{1}{Z_0} T_n \left[e^{-\beta K_0} T_{\tau} \left(V_I(\tau_1) \dots V_I(\tau_n) C_{\vec{u}\alpha}(\tau) C_{\vec{u}\beta}^{\dagger}(0) \right) \right]$$

$$= \langle T_{\tau} \left(V_I(\tau_1) \dots V_I(\tau_n) C_{\vec{u}\alpha}(\tau) C_{\vec{u}\beta}^{\dagger}(0) \right) \rangle_0$$

e

$$(II) = \delta \vec{q}_1 - \vec{q}_2 \delta s_1, s_2 \langle T_\tau (A \vec{q}_1 s_1(\tau_1) A - \vec{q}_1 s_1(\tau_2)) \rangle_0$$

$$= -D_{s_1}^0(\vec{q}_1, \tau_1 - \tau_2) : \text{veja Eqs. (308.3) p/ } C_{\vec{q}}^s = 1, \text{ Eqs. (309.1) e (310.2) !}$$

$$\hookrightarrow -g_\alpha^{(2)}(\vec{u}, \tau) = \left(\frac{-1}{\hbar}\right)^2 \cdot \frac{1}{2!} \cdot \frac{1}{v^2} \sum_{\vec{u}_1, \vec{u}_2, \vec{q}} \sum_{\mu, \nu} \sum_s (-1) g_s(\vec{q}) g_s(-\vec{q}) *$$

$$* \int_0^{\beta \hbar} d\tau_1 d\tau_2 D_s^0(\vec{q}, \tau_1 - \tau_2) *$$

$$* (+1) \langle T_\tau (C^{\dagger} \vec{u}_1 + \vec{q}, \mu(\tau_1) C^{\dagger} \vec{u}_2 - \vec{q}, \nu(\tau_2) C^{\dagger} \vec{u}_2 \nu(\tau_2) C \vec{u}_2 \mu(\tau_1) C \vec{u}_2(\tau) C^{\dagger} \vec{u}_2(0)) \rangle_0$$

$$\hookrightarrow -g_\alpha^{(2)}(\vec{u}, \tau) \rightarrow g_\alpha^{(1)}(\vec{u}, \tau) = -\frac{1}{\hbar} \int_0^{\beta \hbar} d\tau_1 \langle T_\tau (W_I(\tau_1) C \vec{u}_2(\tau) C^{\dagger} \vec{u}_2(0)) \rangle_0$$

onde

(313.1)

$$W_I(\tau_1) = \frac{1}{2v} \sum_{\vec{u}_1, \vec{u}_2, \vec{q}} \sum_{\mu, \nu} \sum_s \int_0^{\beta \hbar} d\tau'_1 |g_s(\vec{q})|^2 D_s^0(\vec{q}, \tau_1 - \tau'_1) * \frac{1}{\hbar v}$$

$$* C^{\dagger} \vec{u}_1 + \vec{q}, \mu(\tau_1) C^{\dagger} \vec{u}_2 - \vec{q}, \nu(\tau_2) C^{\dagger} \vec{u}_2 \nu(\tau_2) C \vec{u}_2 \mu(\tau_1) C \vec{u}_2(\tau) C^{\dagger} \vec{u}_2(0) \quad (313.2)$$

$$\stackrel{e}{=} g_s(-\vec{q}) = g_s^*(\vec{q}) : \text{veja Eq. (107.2)}$$

notas:

W_I : interação efetiva elétron-elétron mediada pelos fônons;

$$\text{como acoplamento} = \frac{1}{\hbar v} |g_s(\vec{q})|^2 D_s^0(\vec{q}, \tau - \tau')$$

(313.3)

\hookrightarrow interação efetiva é não-local no tempo!

· verifica-se que a Eq. (332.1) assume a forma
(veja Sec. 17.2, Bruns):

$$G_2(\vec{x}, \vec{z}) = - \frac{Z_0}{Z} \sum_{n \geq 0} \left(\frac{-1}{\hbar}\right)^n \int_0^{\beta \hbar} d\tau_1 \dots d\tau_n +$$


$$* \langle T_\tau (W_I(\tau_1) \dots W_I(\tau_n) C \vec{x}_2(\tau) C^\dagger \vec{x}_2(0)) \rangle_0$$

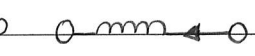
onde (334.1)

$$\frac{Z_0}{Z} = \sum_{n \geq 0} \left(\frac{-1}{\hbar}\right)^n \int_0^{\beta \hbar} d\tau_1 \dots d\tau_n \langle T_\tau (W_I(\tau_1) \dots W_I(\tau_n)) \rangle_0$$

· como W_I assume a forma (43.3) do termo de interação entre pares de partículas

↳ regras p/ determinação diagramas de Feynman de ordem n no espaço de momentos - $T \neq 0$ = regras (268.1), exceto pelas modificações:

(1) ... n linhas de interação 

(2) cada linha de interação  corresponde

(334.2)

ao potencial (333.2): $\frac{1}{\hbar v} |g_s(\vec{q})|^2 D_s^0(\vec{q}, \nu_n)$

· próxima etapa: considerar interação coulombiana:

- elétron - elétron,
- ion - ion,
- ion - elétron;

↳ Q: Qual procedimento deve ser utilizado p/ o tratamento simultâneo dessas 3 interações?

A: resumo proposta:

(1) interação ion-ion: jellium model

↳ determinação de interação elétron-fônon

↳ determinação interação efetiva elétron-elétron;

(2) interação efetiva elétron-elétron: RPA

↳ determinação de interação efetiva elétron-elétron na RPA

= " " propagador renormalizado dos fônons.

Em detalhes:

(1) interação ion-ion: jellium model,

considerar: sistema de N ions interagentes @ volume V

⊕ distribuição uniforme e estática de elétrons

~ neutralidade de carga do sistema, i.e.:

$$Ze \rho_{ion} = e \rho_{el} ; e > 0 \tag{315.1}$$

↑ densidade elétrons = cte :
↑ " ions = cte

: jellium model p/ ions: companas modelo (49.1) p/ elétrons;

nesse caso, verifica-se que (veja pg. 315.1) as excitações

do sistema = oscilações (variações) na densidade dos ions

$\delta \rho_I(\vec{r}, t)$ c/ frequência:

$$\Omega = \sqrt{\frac{4\pi (Ze)^2 \rho_{ion}}{M_{ion}}} : \text{frequência de plasma} \tag{315.2}$$

: veja Eq. (23.3)

↳ nesse modelo, os fônons são descritos pelo hamiltoniano:

• Sobre as oscilações de plasma: tratamento semiclassico
(pr detalhes, veja Sec. 15, Fetter):

consideramos: gás uniforme de ions,
 $\rho_{ion} = cte$: densidade sistema em equilibrio
 ρ_{ee} = distribuição uniforme e estatica de electrons,
 $\rho_{ee} = cte$: " " " " ;
Eq. (315.1) \rightarrow neutricidade de carga do sistema;

pr o sistema de ions, temos que:

$$\rho_{ion}(\vec{n}, t) = \rho_{ion}^0 + \delta\rho_{ion}(\vec{n}, t) \rightarrow \rho(\vec{n}, t) = \rho^0 + \delta\rho(\vec{n}, t)$$

↑
variação densidade ions w.r.t. ρ_{ion}^0 !

$\delta\rho(\vec{n}, t) \neq 0 \rightarrow \exists$ campo electrico no sistema;

- Eq. de Maxwell:

$$\vec{\nabla} \cdot \vec{E}(\vec{n}, t) = 4\pi Ze (\rho(\vec{n}, t) - \rho^0) = 4\pi Ze \delta\rho(\vec{n}, t); \quad (315.3)$$

: interação Coulomb!

- 2ª Lei de Newton:

$$M \frac{d}{dt}(\rho \vec{u}) = M \left(\frac{\partial}{\partial t}(\rho \vec{u}) + \vec{u} \cdot \vec{\nabla}(\rho \vec{u}) \right) = Ze \rho(\vec{n}, t) \vec{E}(\vec{n}, t)$$

$\approx M \rho^0 \frac{\partial}{\partial t} \vec{u} \qquad \approx \rho^0$

$$\hookrightarrow \frac{\partial}{\partial t} \vec{u} \approx Ze \vec{E} / M \quad (315.4)$$

- eq. de continuidade:

$$\frac{\partial}{\partial t} \rho(\vec{n}, t) + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \rightarrow \frac{\partial}{\partial t} \delta\rho(\vec{n}, t) \approx -\rho^0 \vec{\nabla} \cdot \vec{u}$$

$$\hookrightarrow \partial_t^2 \delta p(\vec{n}, t) = -\rho^0 \vec{v} \cdot (\partial_t \vec{v})$$

(315.5)

$$\frac{Z_e}{M} \vec{v} \cdot \vec{E}$$

$$\langle \pi Z_e \delta p(\vec{n}, t) \rangle$$

$$\hookrightarrow \partial_t^2 \delta p(\vec{n}, t) = - \frac{\langle \pi (Z_e)^2 \rho^0 \rangle}{M} \delta p(\vec{n}, t)$$

$$s.c. \delta p(\vec{n}, t) = \delta p(\vec{n}) e^{-i\Omega t}$$

$$\hookrightarrow \Omega^2 = \frac{\langle \pi (Z_e)^2 \rho^0 \rangle}{M}$$

$$H_{ph} = \sum_{\vec{q}} \hbar \Omega (a_{\vec{q}}^\dagger a_{\vec{q}} + 1/2) \quad (316.1)$$

notas: hamiltoniano (316.1) NOT OR o/ hamiltoniano (308.2), pois, p/ o caso 1 ion/célula unitária, temos que:

$$\lim_{q \rightarrow 0} \omega_s(\vec{q}) = c q \neq \Omega = c v_e !$$

Obs.: como mostrado na pg. 315.1:

oscilações de plasma ~ interação Coulomb
 ions longo alcance!

• sobre a interação elétron-fônon:

considerar apenas fônons longitudinais:

nesse caso $\hat{E}^s(\vec{q}) = \hat{q}$ e índice s pode ser omitido,

$$Eq. (107.2): g(\vec{q}) = i e \sqrt{\frac{\hbar N}{2 M_{ion} \Omega}} \underbrace{\vec{q} \cdot \hat{E}(\vec{q})}_{q} \sqrt{q}^{s-e}; \quad \sqrt{q}^{s-e} = \frac{4\pi Z e^2}{q^2}$$

$$\hookrightarrow \frac{1}{\hbar v} |g(\vec{q})|^2 = \frac{e^2 \cdot \hbar N}{\hbar v 2 M_{ion} \Omega^2} \frac{1}{\Omega^2} \frac{q^2 (4\pi Z e^2)^2 \cdot \Omega}{q^4} = \frac{1}{2} \cdot \frac{4\pi e^2}{q^2} \cdot \Omega \underbrace{\sqrt{q}}_{\frac{p_{ion}^0 \cdot M}{4\pi (Z e^2)^2 p_{ion}^0}}$$

↳ Eq. (310.3) c/ $C_q = 1$ e $\omega_{\vec{q}} = \Omega \oplus Eq. (313.3):$

$$\frac{1}{\hbar v} |g(\vec{q})|^2 D^0(\vec{q}, \nu n) = \frac{4\pi e^2}{q^2} \cdot \frac{\Omega^2}{(\nu n)^2 - \Omega^2} \quad (316.1)$$

: acoplamento da interação (base) elétron-elétron mediada pelos fônons considerando os ions descritos pelo jellium model!

hamiltoniano do sistema interagente ions ⊕ e elétrons:

$$H = H_{ee} + V_{el-eel} + H_{ph} + V_{el-ph} \tag{317.1}$$

onde

$$H_{ee} = \sum_{\vec{k}\alpha} \xi_{\vec{k}} C_{\vec{k}\alpha}^\dagger C_{\vec{k}\alpha}$$

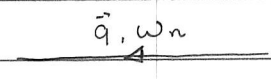
$$V_{el-eel} = \frac{1}{2V} \sum_{\alpha\beta} \sum_{\vec{k}\vec{p}\vec{q}} V_{\vec{q}} C_{\vec{k}+\vec{q}\alpha}^\dagger C_{\vec{p}-\vec{q}\beta}^\dagger C_{\vec{p}\beta} C_{\vec{k}\alpha} \tag{Eq. (51.2)}$$

$$H_{ph} : \text{Eq. (316.1)}$$

$$V_{el-ph} : \text{Eq. (311.3)}$$

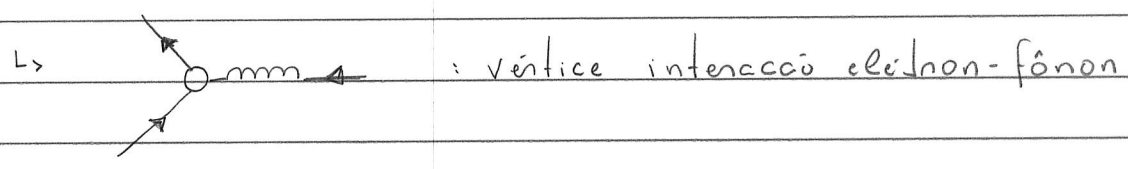
sobre a interação elétron-fônon:

considerar a representação (314.2):

$G^\circ(\vec{q}, \omega_n)$:  : propagação elétron

$D^\circ(\vec{q}, \omega_n)$:  : " fônon

$g(\vec{q})$: acoplamento elétron-fônon

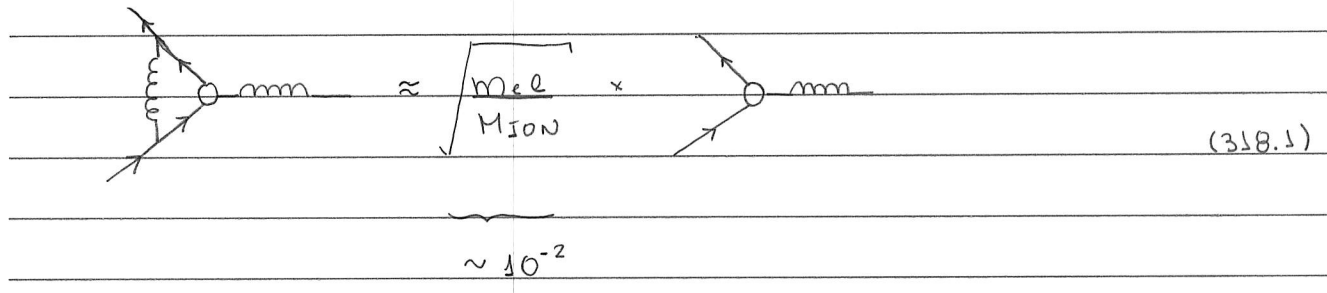


Teorema de Migdal

(p/ detalhes, veja Sec. 17.3.1, Bruus e Sec. 47, Fetter):

verifica-se que a renormalização do vértice da interação elétron-fônon devido H_{ph} é $\ll 1$;

por exemplo, em 1ª ordem, verifica-se que a renormalização do vértice é dada por:



de fato, teorema Migdal \sim teorema Born-Oppenheimer;

\hookrightarrow próxima etapa: determinar a interação efetiva elétron-elétron devido V_{el-el} e V_{el-ph} dado que o vértice elétron-fônon não é renormalizado;

• Eqs. (313.2) e (313.2):

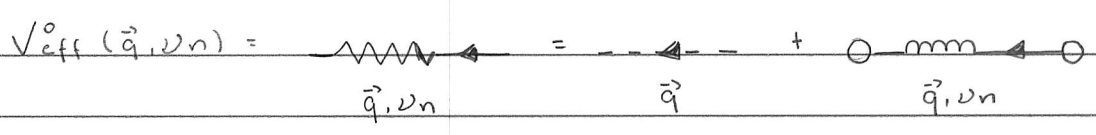
$$V_{eff}^0(\vec{q}, \nu_n) = V_{\vec{q}} + W(\vec{q}, \nu_n)$$

$$= \frac{4\pi e^2}{q^2} + \frac{1}{\hbar v} |g(\vec{q})|^2 D^0(\vec{q}, \nu_n)$$

Eq. (316.1) $\approx \frac{4\pi e^2}{q^2} \frac{\Omega^2}{(i\nu_n)^2 - \Omega^2}$

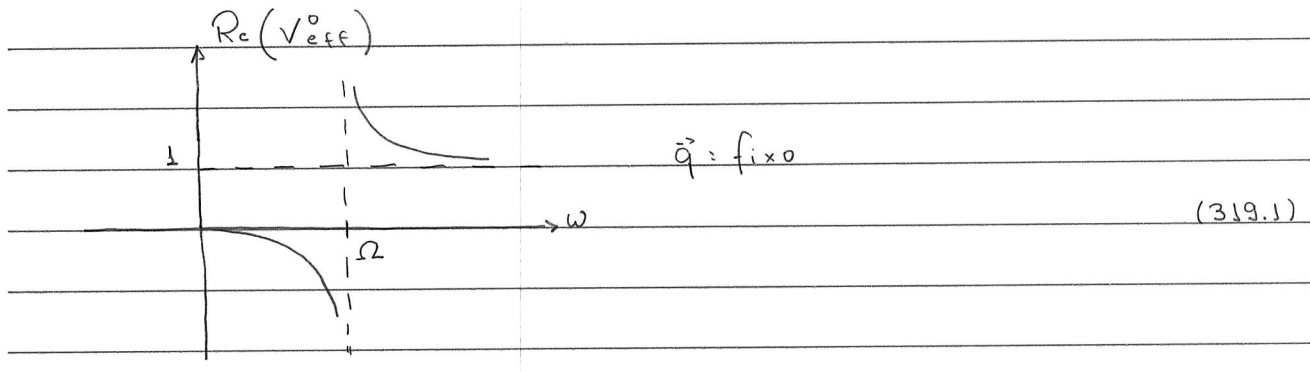
$$\hookrightarrow V_{eff}^0(\vec{q}, \nu_n) = V_{\vec{q}} \frac{(i\nu_n)^2}{(i\nu_n)^2 - \Omega^2} \quad (318.2)$$

ou



• em termos de frequências reais, $i\nu_n \rightarrow \omega + i\eta$, temos que:

$$V_{eff}^0(\vec{q}, \omega) = V_{\vec{q}} \frac{\omega^2}{\omega^2 - \Omega^2 + i\eta} \quad (318.3)$$



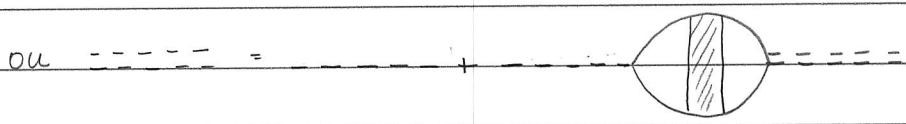
notas: $Re(V_{eff}^0) < 0$ p/ $\omega < \Omega$

$$Re(V_{eff}^0) = \sqrt{q} \text{ p/ } \omega \rightarrow +\infty.$$

(2) interação efetiva elétron-elétron na RPA,

Lembran Eq. (191.1):

$$U^R(q) = U(q) + U(q)\pi(q)U^R(q)$$



na RPA, a polarização total

$$\pi(q) \approx \pi^0(q) = : veja Eq. (282.3)$$

L> p/ potenciael efetivo (318.3), demos que:

$$V_{eff}^{RPA}(\vec{q}, \nu_n) = V_{eff}^0(\vec{q}, \nu_n) + V_{eff}^0(\vec{q}, \nu_n)\pi^0(\vec{q}, \nu_n)V_{eff}^{RPA}(\vec{q}, \nu_n)$$

$$\text{ou } V_{eff}^{RPA}(\vec{q}, \nu_n) = \frac{V_{eff}^0(\vec{q}, \nu_n)}{1 - V_{eff}^0(\vec{q}, \nu_n)\pi^0(\vec{q}, \nu_n)} : (319.2)$$

: interação efetiva elétron-elétron na RPA!

Eq. (319.2) pode ser escrita como:

$$V_{\text{eff}}^{\text{RPA}}(\vec{q}, \nu_n) : \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

(320.1)

entretanto, é interessante reescrever (320.1) de seguinte forma (veja pg. 320.1):

$$\text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---}$$

(320.2)

ou

$$V_{\text{eff}}^{\text{RPA}}(\vec{q}, \nu_n) = V^{\text{RPA}}(\vec{q}, \nu_n) + \frac{1}{\hbar\nu} |g^{\text{RPA}}(\vec{q})|^2 D^{\text{RPA}}(q, \nu_n)$$

onde

$V^{\text{RPA}}(\vec{q}, \nu_n)$: interação elétron-elétron (blindada) na RPA:

$$V^{\text{RPA}}(\vec{q}, \nu_n) = \frac{V_{\vec{q}}}{1 - \sqrt{q} \tilde{\Pi}^0(q, \nu_n)} ; \quad V_{\vec{q}} = \frac{4\pi e^2}{q^2} \quad (320.3)$$

$g^{\text{RPA}}(\vec{q}, \nu_n)$: acoplamento elétron-fônon não normalizado na RPA:

$$g^{\text{RPA}} = \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \dots$$

$$= \text{---} \text{---} \text{---} * \left(\text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots \right) + \text{---} \text{---} \text{---}$$

$\tilde{\Pi}^{\text{RPA}}$: veja Eq. (191.3)

$$\hookrightarrow g^{\text{RPA}}(\vec{q}, \nu_n) = g(\vec{q}) \left(1 + \sqrt{q} \tilde{\Pi}^{\text{RPA}}(\vec{q}, \nu_n) \right) \quad (320.3)$$

Sobre a Eq. (320.2):

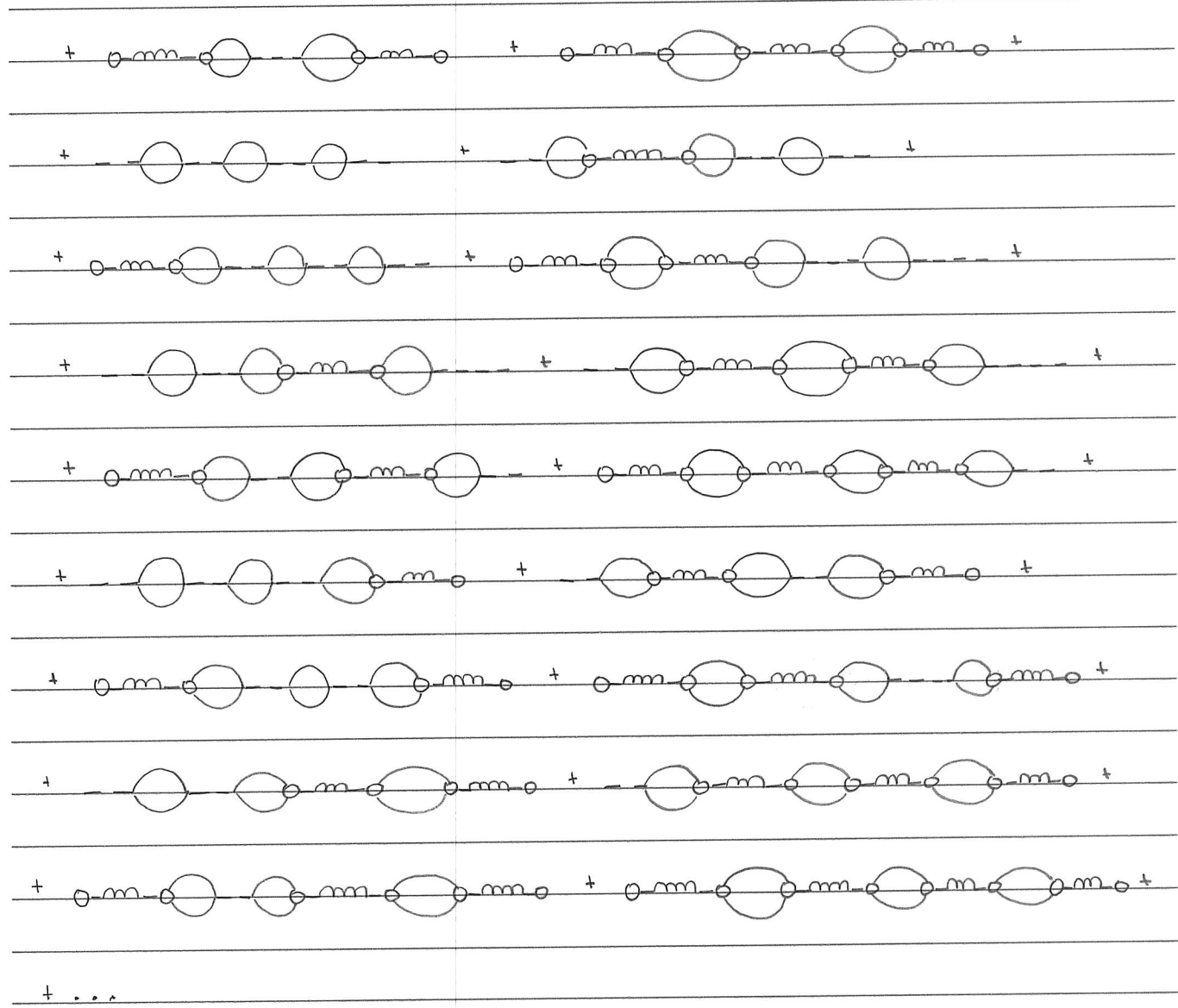
utilizando a Eq. (320.1), temos que:

$$\begin{aligned}
 \text{Diagram 1} &= \text{Diagram 2} + \text{Diagram 3} + \\
 &+ \text{Diagram 4} + \text{Diagram 5} + \dots = (I),
 \end{aligned}$$

considerando a Eq. (318.2), temos que:

$$\begin{aligned}
 (I) &= \text{Diagram 1} + \text{Diagram 2} + \\
 &+ (\text{Diagram 1} + \text{Diagram 2}) \text{Diagram 3} + \\
 &+ (\text{Diagram 1} + \text{Diagram 2}) \text{Diagram 3} \text{Diagram 4} + \dots
 \end{aligned}$$

$$\begin{aligned}
 (I) &= \text{Diagram 1} + \text{Diagram 2} + \\
 &+ \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \\
 &+ \text{Diagram 7} + \text{Diagram 8} + \text{Diagram 9} + \text{Diagram 10} + \dots
 \end{aligned}$$

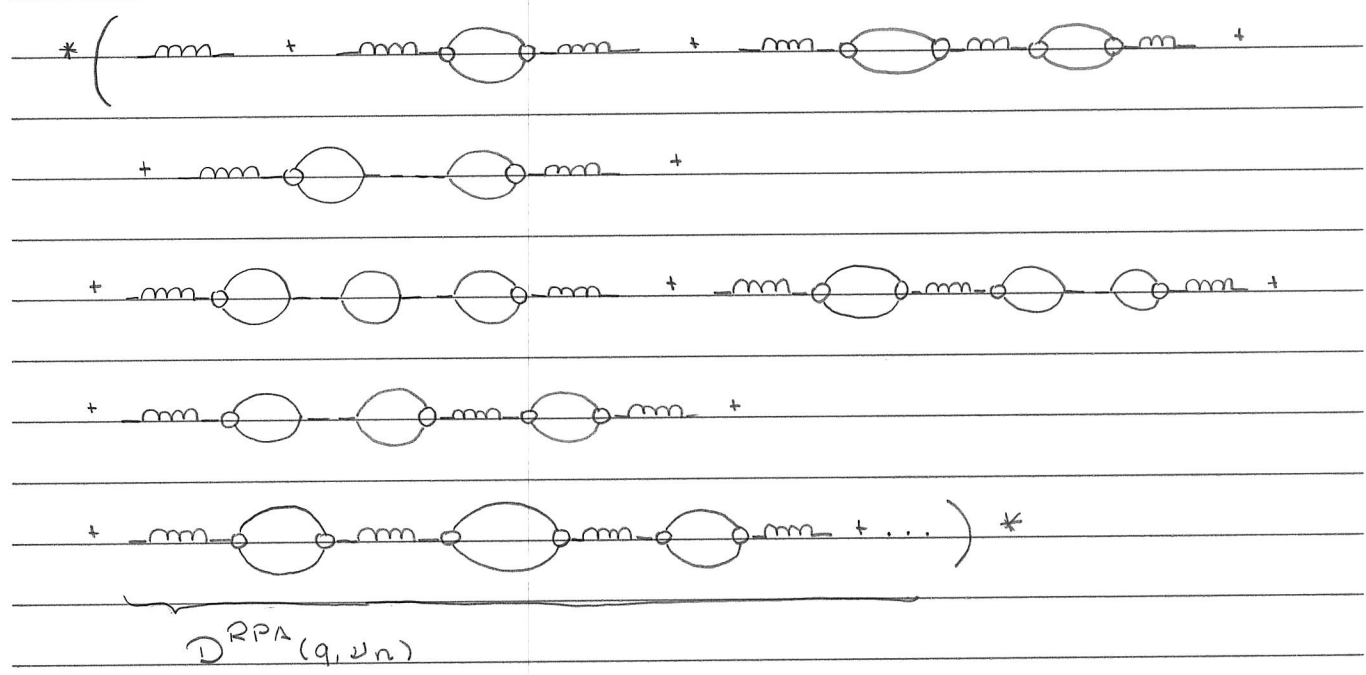


notan:

$$(I) = \text{---} + \text{---} + \text{---} + \text{---}$$

$$\underbrace{\text{---} + \text{---} + \text{---}}_{V_{\vec{q}}^{RPA}} + \dots +$$

$$+ \left(\text{---} + \text{---} + \text{---} + \text{---} + \dots \right) * \underbrace{\hspace{10em}}_{g^{RPA}(\vec{q})}$$



$$L \rightarrow (I) = V^{RPA}(\vec{q}, \nu_n) + \frac{1}{\hbar \nu} |g^{RPA}(\vec{q})|^2 D^{RPA}(\vec{q}, \nu_n)$$

↳ Eq. (320.3): $g^{RPA} = g (1 + \sqrt{q} \pi^{RPA})$

$$1 + \frac{\sqrt{q} \pi^0}{\epsilon^{RPA}} = \frac{1}{\epsilon^{RPA}}$$

↳ $g^{RPA}(\vec{q}, \omega_n) = \frac{g(\vec{q})}{\epsilon^{RPA}(\vec{q}, \omega_n)}$ (322.1)

• Eqs. (310.3) e (316.1)

↳ Eq. (321.1): $\mathcal{D}^{RPA}(\vec{q}, \omega_n) = \frac{2\Omega}{(i\omega_n)^2 - \Omega^2 (1 + \sqrt{q} \pi^{RPA}(\vec{q}, \omega_n))}$

$$\equiv \omega_{\vec{q}}^2 = \Omega^2 / \epsilon^{RPA}(\vec{q}, \omega_n):$$

: frequência dos fônons renormalizada devido interação elétron-elétron blindada no RPA. (322.2)

↳ acoplamento efetivo elétron-elétron (316.1) assume a forma:

$$\frac{1}{\hbar v} |g^{RPA}|^2 \mathcal{D}^{RPA} = \frac{1}{\hbar v} |g(\vec{q})|^2 \cdot \frac{1}{\epsilon^{RPA}} \cdot \frac{2\Omega / \epsilon^{RPA}}{(i\omega_n)^2 - \omega_{\vec{q}}^2}$$

Eq. (316.1) $\rightarrow \frac{1}{2} \sqrt{q} \Omega$

$$= \frac{\sqrt{q}}{\epsilon^{RPA}(\vec{q}, \omega_n)} \cdot \frac{\omega_{\vec{q}}^2}{(i\omega_n)^2 - \omega_{\vec{q}}^2} :$$

$$\underbrace{\frac{\sqrt{q}}{\epsilon^{RPA}(\vec{q}, \omega_n)}}_{\sqrt{RPA}(\vec{q}, \omega_n)} \quad (322.3)$$

: Similar Eq. (316.1) c/ as renormalizações:

$$v \rightarrow v^{RPA} \quad e \quad \Omega \rightarrow \Omega / \sqrt{\epsilon^{RPA}}$$

sobre a frequência dos fônons normalizada $\omega_{\vec{q}}$,

no limite estático, temos que:

$$\epsilon^{RPA}(q, \omega=0) = 1 + \frac{4\kappa_F}{\pi a_0} \frac{1}{q^2} + g(q/\kappa_F) \quad = \text{Eq. (214.3)}$$

$$\text{onde } g(x) = \frac{1}{2} - \frac{1}{2x} \left(1 - \frac{1}{4} x^2 \right) \ln \left| \frac{1-x/2}{1+x/2} \right|$$

como $g(0) = 1$, no limite $q \rightarrow 0$, temos que:

$$\epsilon^{RPA}(q, 0) \approx \frac{4\kappa_F}{\pi a_0} \frac{1}{q^2} = \left(\frac{q_{TF}}{q} \right)^2 : \text{veja Eq. (216.2)}$$

Eqs. (315.2) e (322.2)

$$\hookrightarrow \omega_{\vec{q}}^2 = \frac{\Omega^2}{\epsilon^{RPA}(\vec{q}, 0)} = \frac{4\pi (Ze)^2 \rho_{ion}^0}{M_{ion}} \cdot \frac{\pi}{4\kappa_F} \frac{\hbar^2}{m^2} q^2$$

$$= \frac{4\pi Ze^2}{M} \cdot \underbrace{Z \rho_{ion}^0}_{\rho_{el}} \cdot \frac{\pi}{4} \cdot \frac{1}{\kappa_F^3} \cdot m \cdot \frac{\hbar^2 \kappa_F^2}{m^2} \cdot \frac{1}{c^2} q^2 = \frac{Zm}{3M} \underbrace{V_F^2}_{\frac{1}{3\pi^2 \rho_{el}}} q^2$$

$$\hookrightarrow \omega_{\vec{q}} = \underbrace{\sqrt{\frac{Zm}{3M}}}_{V_{som}} V_F q : \text{frequência fônons acústicos} \quad (323.1)$$

Obs.: veja comentário final Sec. 17.4, Bruus.

· Instabilidade de Cooper,

nesse caso, há duas formulações possíveis;

(1) análise via função de onda de um par de elétrons:

considerar: função de onda Ψ elétrons:

$$\Psi(\vec{n}_1, \vec{n}_2) = \phi_{\vec{q}}(\vec{n}) e^{i\vec{q} \cdot \vec{R}} \chi_S(\sigma_1, \sigma_2)$$

↑ estado de singlete

onde $\vec{R} = \frac{1}{2}(\vec{n}_1 + \vec{n}_2)$

(324.1)

e $\vec{n} = \vec{n}_1 - \vec{n}_2$;

considerar: momento par $\vec{q} = 0$ e

$$\phi_{\vec{q}=0}(\vec{n}) = \phi(\vec{n}) = \sum_{|\vec{k}| > k_F} g(\vec{k}) e^{i\vec{k} \cdot \vec{n}} = \sum_{\vec{k}} g(\vec{k}) e^{i\vec{k} \cdot \vec{n}_1} e^{-i\vec{k} \cdot \vec{n}_2}$$

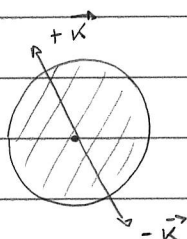
↑ superposição ondas planas
momentos \vec{k} e $-\vec{k}$;

como $\chi_S(\sigma_1, \sigma_2)$: singlete

↳ $\Psi(\vec{n}_1, \vec{n}_2)$: par elétrons ~ estados $|\vec{k} \uparrow\rangle$ e $|\vec{k} \downarrow\rangle$;

de fato, 2 elétrons acima de superfície de Fermi:

notar restrição da soma sob momentos!



(324.2)

· eq. de Schrödinger p/ sistema de Ψ elétrons acima FS:

$$\left(\frac{-\hbar^2}{2m} (\nabla_1^2 + \nabla_2^2) + V(\vec{n}_1, \vec{n}_2) \right) \Psi(\vec{n}_1, \vec{n}_2) = \underbrace{\left(\epsilon + \frac{2\hbar^2 k_F^2}{2m} \right)}_E \Psi(\vec{n}_1, \vec{n}_2) \quad (324.3)$$

considerando $V(\vec{n}_1, \vec{n}_2) = V(\vec{n}_1 - \vec{n}_2) = V(\vec{n})$, temos que

$$\sum_{\vec{p}} \left(\frac{\hbar^2 p^2}{m} + V(\vec{n}) \right) g(\vec{p}) e^{i\vec{p} \cdot \vec{n}} = (\epsilon + 2\epsilon_F) \sum_{\vec{p}} g(\vec{p}) e^{i\vec{p} \cdot \vec{n}}$$

$$\int d^3 n e^{-i\vec{k} \cdot \vec{n}} * :$$

$$\hookrightarrow \sum_{\vec{p}} \frac{\hbar^2 p^2}{m} g(\vec{p}) \underbrace{\int d^3 n e^{i(\vec{p} - \vec{k}) \cdot \vec{n}}}_{L^3 \delta_{\vec{k}, \vec{p}}} + \sum_{\vec{p}} g(\vec{p}) \underbrace{\int d^3 n V(\vec{n}) e^{i(\vec{p} - \vec{k}) \cdot \vec{n}}}_{= L^3 V(\vec{k}, \vec{p})} =$$

$$= (\epsilon + 2\epsilon_F) \sum_{\vec{p}} g(\vec{p}) \underbrace{\int d^3 n e^{i(\vec{p} - \vec{k}) \cdot \vec{n}}}_{L^3 \delta_{\vec{k}, \vec{p}}}$$

$$\hookrightarrow \frac{\hbar^2 k^2}{m} g(\vec{k}) + \sum_{\vec{p}} V(\vec{k}, \vec{p}) g(\vec{p}) = (\epsilon + 2\epsilon_F) g(\vec{k}) :$$

(325.1)

: eq. de Bethe - Goldstone p/ problema 2 electrons

a fim de resolver (325.1), vamos considerar:

$$V(\vec{k}, \vec{p}) = \begin{cases} -V, & |\epsilon_{\vec{k}} - \epsilon_F| < \hbar\omega_D \text{ e } |\epsilon_{\vec{p}} - \epsilon_F| < \hbar\omega_D \\ 0, & \text{c.c.} \end{cases}$$

(325.2)

onde ω_D : frequência de Debye, veja pg.

$$\hookrightarrow \text{Eq. (325.1)}: \left(\frac{\hbar^2 k^2}{m} - \epsilon - \epsilon_F \right) g(\vec{k}) = V \sum'_{\vec{p}} g(\vec{p})$$

← nota restricao!

$$\hookrightarrow \sum'_{\vec{k}} g(\vec{k}) = -V \sum'_{\vec{k}} \frac{1}{\epsilon + \epsilon_F - \hbar^2 k^2 / m} + \left(\sum'_{\vec{p}} g(\vec{p}) \right)$$

(1)

se $(I) \neq 0$, temos que:

$$I = V \sum_{\vec{k}} \frac{1}{2 \left(\frac{\hbar^2 k^2}{2m} - E_F \right) - E} \quad (326.1)$$

se $N(\epsilon)$: densidade de estados, temos que

$$I = V \int_0^{\hbar\omega_D} d^3 \epsilon N(\epsilon) \frac{1}{2\epsilon - E} \approx N(0)V \int_0^{\hbar\omega_D} d^3 \epsilon \frac{1}{2\epsilon - E} \quad (326.2)$$

densidade de estados no FS! $\frac{1}{2} \ln \left(\frac{\epsilon - 2\hbar\omega_D}{\epsilon} \right)$

considerando $|E| \ll \hbar\omega_D$ OK!

$$\hookrightarrow I = \frac{1}{2} N(0)V \ln \left(-\frac{2\hbar\omega_D}{E} \right) \rightarrow E \approx -2\hbar\omega_D e^{-2/N(0)V} \quad (326.3)$$

notar Eq. (326.3):

- $E < 0 \rightarrow$ formação estado ligado \approx elétrons acima FS:
: par de Cooper;
- \exists estado ligado \sim indicação \exists gap de energia no espectro de excitação;
- \exists estado ligado \neq intensidade da interação efetiva V ;
- não é possível determinar E a partir série de potências em V : indicação resultado não-perturbativo.

• é interessante estimar a distância relativa entre os elétrons do par; verifica-se que (veja Sec. 34.3, Ketterson):

$$\langle n^2 \rangle = \frac{\int d^3n n^2 |\varphi(\vec{n})|^2}{\int d^3n |\varphi(\vec{n})|^2} = \frac{4}{3} \frac{\hbar^2 v_F^2}{e^2} \quad (327.1)$$

considerando $|E| \sim k_B T_c$, $T_c \sim 10\text{K}$ e $v_F \sim 10^8\text{cm/s}$, temos que:

$$\langle n^2 \rangle^{1/2} \sim 10^4 \text{ \AA} \sim \text{comprimento de coerência } \xi_0 \quad (327.2)$$

(introduzido por Pippard)

Lembrar: volume ocupado por $1e^- \sim (2\text{ \AA})^3 = L^3$

$$L \gg L \ll \xi_0$$

Obs.: se momento do par $\vec{q} \neq 0$, verifica-se que a energia de ligação do par é dada por:

$$E(q) = E(q=0) + \frac{1}{2} \hbar v_F q; \quad |E(0)| \sim k_B T_c \quad (327.3)$$

Como $E(q) < 0$, temos que q_{MAX} p/ o estado ligado é dado por:

$$0 = E(0) + \frac{1}{2} \hbar v_F q_{\text{MAX}} \rightarrow q_{\text{MAX}} = \frac{-2E(0)}{\hbar v_F} \sim \frac{1}{\xi_0};$$

: \exists estado ligado $\sim q < 1/\xi_0$;

$$E_{\text{MAX}} \sim q=0.$$

(2) análise diagnóstica:

é possível obter uma indicação de formação do par de Cooper acima de FS via os resultados obtidos através da análise diagnóstica do problema elétrons \oplus íons interagentes;

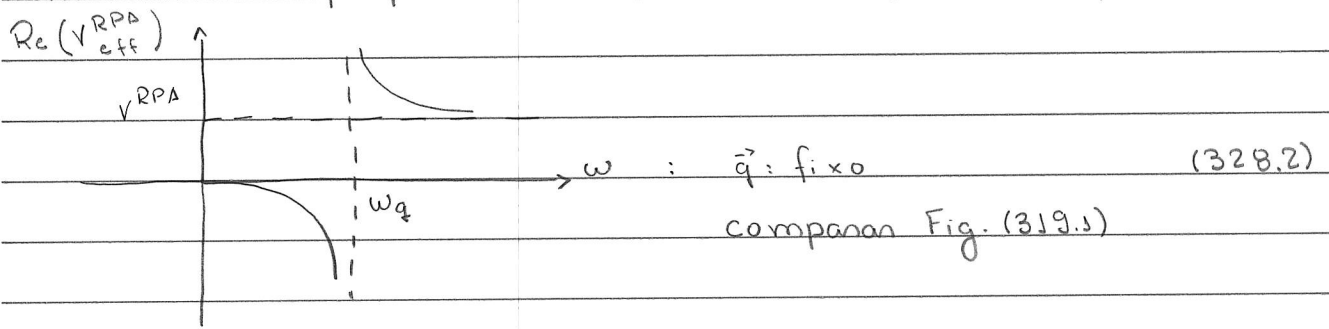
lembrando interaçãõ efetiva elétron-elétron Eqs. (320.2) e (322.3):

$$V_{eff}^{RPA}(\vec{q}, \omega_n) = V^{RPA}(\vec{q}, \omega_n) + \frac{1}{\hbar v} |g^{RPA}(\vec{q})|^2 D^{RPA}(\vec{q}, \omega_n)$$

$$V^{RPA}(\vec{q}, \omega_n) \cdot \frac{\omega_q^2}{(i\omega_n)^2 - \omega_q^2}$$

$$= V^{RPA}(\vec{q}, \omega_n) \cdot \frac{(i\omega_n)^2}{(i\omega_n)^2 - \omega_q^2}; \quad (328.1)$$

em termos de frequências reais, $i\omega_n \rightarrow \omega + i\eta$, temos que:



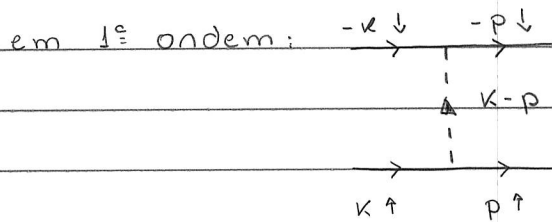
notas: $Re(V_{eff}^{RPA}) < 0$, p/ $\omega < \omega_q$

(328.3)

$$Re(V_{eff}^{RPA}) \rightarrow V^{RPA}, \text{ p/ } \omega \rightarrow +\infty$$

considerar: espalhamento entre elétrons ~ estados de partícula

$$\text{única } |\vec{k}\uparrow\rangle \text{ e } |\vec{k}\downarrow\rangle;$$



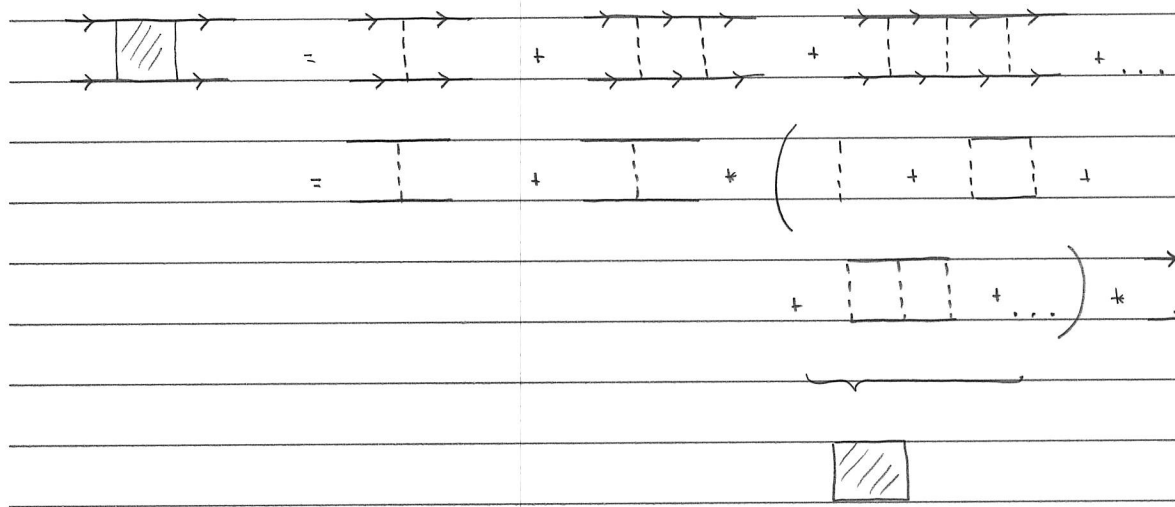
$$: K = (\vec{k}, \omega_m)$$

$$p = (\vec{p}, \omega_e)$$

$$q = (\vec{q}, \omega_n)$$

onde (notar notação): $\text{---} = \text{---} = V_{eff}^{RPA}$

considerando processos de ordem superior, temos que:



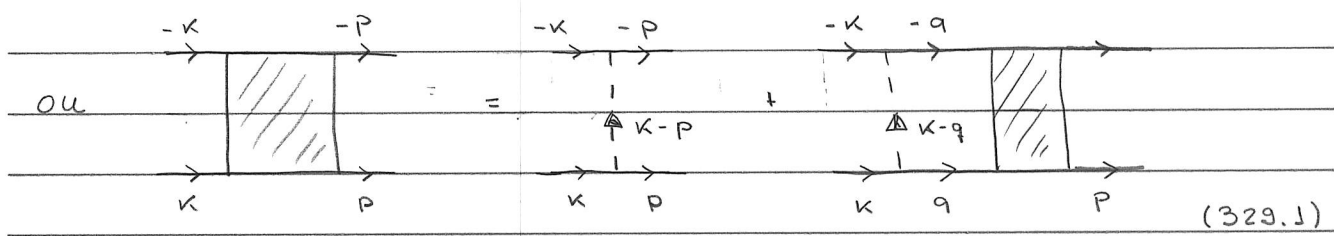
\hookrightarrow : equação de Bethe-Salpeter,
 veja Fig. 11.8 e Eq. (11.30),
 Fetter.

considerando:

$\Gamma(k, p) =$: vértice de espalhamento =
 : generalização de amplitude
 de espalhamento em um meio

⊕ negros (268.1):

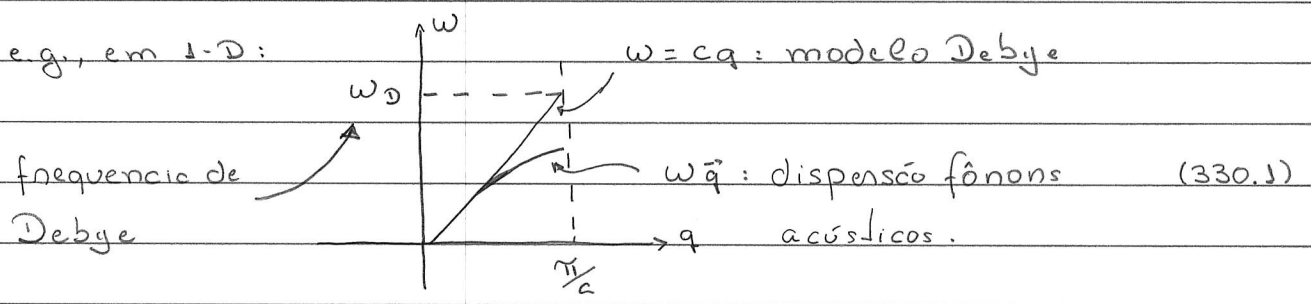
$$\Gamma(k, p) = U(k-p) + \frac{(-1)}{\beta h} \sum_n \int \frac{d^3 q}{(2\pi)^3} U(k-q) \Gamma(q, p) g^{\circ}(q) g^{\circ}(-q)$$



onde $U(k) = \overset{\text{RPA}}{V_{\text{eff}}(k)}$!

lembrar: modelo de Debye p/ fônons acústicos:
 (veja Sec. 4.3, KeHenson):

e.g., em 1-D:



como $\text{Re}(V_{\text{eff}}^{\text{RPA}}(\vec{q}, \nu_n)) < 0$ apenas p/ $|\nu_n| < \omega_q$, a fim

de resolver Eq. (329.1), vemos consideram:

$$V_{\text{eff}}^{\text{RPA}}(\vec{q}, \nu_n) = \begin{cases} -V, & |\nu_n| < \omega_D \\ 0, & |\nu_n| > \omega_D \end{cases} ; U > 0 \quad (330.2)$$

$$\epsilon = \Gamma(\kappa, \rho) = \Gamma(\vec{\kappa}, \omega_m; \vec{\rho}, \omega_e) \approx \Gamma(\vec{\kappa}, \vec{\rho})$$

↳ Eq. (329.1):

$$\Gamma(\vec{\kappa}, \vec{\rho}) = -V + \frac{V}{\beta \hbar} \sum_{|\nu_n| < \omega_D} \int \frac{d^3 q}{(2\pi)^3} g^0(q) g^0(-q) \Gamma(\vec{q}, \vec{\rho})$$

Como R.H.S. da eq. acima $\nrightarrow \vec{\kappa} \rightarrow \Gamma(\vec{\kappa}, \vec{\rho}) = \Gamma(\vec{\rho})$

$$\epsilon = \Gamma(\vec{q}, \vec{\rho}) = \Gamma(\vec{\rho})$$

$$\rightarrow \Gamma(\vec{\rho}) = -V + \Gamma(\vec{\rho}) \frac{V}{\beta \hbar} \sum_n \int \frac{d^3 q}{(2\pi)^3} g^0(q) g^0(-q)$$

↳ podemos consideram $\Gamma(\vec{\rho}) = \Gamma = \text{cte. i.e.,}$

$$\Gamma = \frac{-V}{1 - \frac{V}{\beta \hbar} \sum_n \int \frac{d^3 q}{(2\pi)^3} g^0(q) g^0(-q)} \quad (330.3)$$

notas:

- p/ altas temperaturas: $k_B T \gg \hbar \omega_D$ ou $\frac{1}{\beta} \gg \hbar \omega_D$,

temos que $\Gamma \rightarrow -V$;

- $\exists T_c$ tal que $\Gamma \rightarrow -\infty$, pois, eventualmente:

$$\chi = \frac{V}{\beta \hbar} \sum_n \int \frac{d^3 q}{(2\pi)^3} g^0(q) g^0(-q); \tag{331.1}$$

Lembran M.D.: divergência na amplitude de espalhamento
~ ressonância ou formação de estados ligados;
p/ gás de elétrons: formação de estados ligados
de pares de elétrons (par de Cooper) p/
elétrons c/ energia $|E_{\vec{n}} - E_F| < \hbar \omega_D$:

= instabilidade de Cooper: interação elétron-fônon $\propto V_{eff}^{RPA}$
↳ instabilidade estado fundamental = estado de Fermi
↳ formação novo " " " !

Eq. (331.1): permite determinar T_c abaixo da qual a instabilidade de Cooper é observada; temos que:

$$\chi = \frac{V}{\beta \hbar} \sum_n \int \frac{d^3 q}{(2\pi)^3} \frac{1}{i\omega_n - \epsilon_q} \cdot \frac{1}{-i\omega_n - \epsilon_q}$$

$$\frac{1}{2} \int d\epsilon N(\epsilon) \frac{1}{\omega_n^2 + \epsilon^2}; N(\epsilon): \text{densidade de estados / spin}$$

$$\approx \frac{1}{2} N(0) \int_{-\infty}^{+\infty} \frac{1}{\omega_n^2 + \epsilon^2} d\epsilon$$

densidade de estados na FS \nearrow $\frac{\pi}{|\omega_n|}$

como $\omega_n = (2n+1)\pi / \beta \hbar$, temos que:

$$J = \frac{\sqrt{N(0)}}{2} \sum_{n=0}^{n_{MAX}} 2 \cdot \frac{1}{2n+1} \quad ; \quad n_{MAX} = \frac{1}{2\pi} \beta \hbar \omega_D$$

$$\approx \int_0^{\Lambda} dt \frac{1}{t+1/2} \quad \text{p/ } k_B T_c \ll \hbar \omega_D \text{ ou } \beta \hbar \omega_D \gg 1$$

$$= \ln \left(\frac{\Lambda+1/2}{1/2} \right) \approx \ln(2\Lambda) = \ln \left(\frac{1}{\pi} \beta \hbar \omega_D \right)$$

$$\hookrightarrow \frac{1}{\pi} \hbar \omega_D = e^{2/\sqrt{N(0)}} \frac{k_B T_c}{\pi}$$

$$\hookrightarrow k_B T_c \approx \frac{1}{\pi} \hbar \omega_D e^{-2/\sqrt{N(0)}} \quad : \text{ comparan Eq. (326.3)}$$

(332.1)

notas: em principio, energia caracteristica $k_B T_c \sim \hbar \omega_D$;
entretanto, temos que $k_B T_c \ll \hbar \omega_D$!

Teoria BCS,

Bandeira-Cooper-Schrieffer (1957);

Lembrar: hamiltoniano (51.2) gás de elétrons interagente.

$$H = \sum_{\vec{k}, \alpha} \epsilon_{\vec{k}}^0 C_{\vec{k}\alpha}^{\dagger} C_{\vec{k}\alpha} + \frac{1}{2V} \sum_{\vec{k}, \vec{p}, \vec{q}} \sum_{\alpha, \beta} V_{\vec{q}} C_{\vec{k}+\vec{q}\alpha}^{\dagger} C_{\vec{p}-\vec{q}\beta}^{\dagger} C_{\vec{p}\beta} C_{\vec{k}\alpha}$$

onde $\epsilon_{\vec{k}}^0 = \hbar^2 k^2 / 2m$

Teoria Líquido de Fermi: interação elétron-elétron
 ↳ renormalização $\epsilon_{\vec{k}}^0 \rightarrow \epsilon_{\vec{k}}$: energia quasipartícula;

interação elétron-fônon → interação efetiva elétron-elétron
 (328.3) aditiva p/ baixas frequências: $\omega < \omega_D$;

↳ hamiltoniano efetivo, interação aditiva entre quasipartículas.

$$H_{BCS} = \sum_{\vec{k}, \alpha} \epsilon_{\vec{k}} C_{\vec{k}\alpha}^{\dagger} C_{\vec{k}\alpha} + \sum_{\vec{k}, \vec{p}} V_{\vec{k}, \vec{p}} C_{\vec{k}\uparrow}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger} C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \tag{333.1}$$

onde
$$V_{\vec{k}, \vec{p}} = \begin{cases} -V, & |\vec{k}|, |\vec{p}| < \hbar\omega_D \\ 0, & \text{c.c.} \end{cases} \quad ; \text{ Eq. (330.2)}$$

$$\epsilon_{\vec{k}} = \hbar^2 k^2 = \epsilon_{\vec{k}} - \mu$$

próximo etapa: análise $H_{BCS} = H_{BCS} - \mu N$ em uma aproximação de campo médio (HF):!

hipótese: $\langle C_{\vec{k}\uparrow}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger} \rangle \neq 0$ (333.2)

notar: $C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger$: op. creac one par de Cooper!

notar:

$$C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger = \langle C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger \rangle + \left(C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger - \langle C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger \rangle \right) \quad (334.1)$$

$$C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} = \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle + \left(C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} - \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle \right)$$

$$\hookrightarrow C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} =$$

$$= - \langle C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger \rangle \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle + \langle C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger \rangle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} +$$

$$+ \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger + \left(\quad \right) + \left(\quad \right)$$

fluctua ione w.r.t. $\langle \quad \rangle \ll 1$.

Definici one:

$$\Delta_{\vec{k}} = - \sum_{\vec{p}} V_{\vec{k},\vec{p}} \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle \quad (334.2)$$

\hookrightarrow hamiltoniano (333.1) assume a forma:

$$K_{BCS}^{MF} = \sum_{\vec{k},\alpha} \epsilon_{\vec{k}} C_{\vec{k}\alpha}^\dagger C_{\vec{k}\alpha} - \sum_{\vec{k}} \left(\Delta_{\vec{k}} C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger + \Delta_{\vec{k}}^* C_{-\vec{k}\downarrow} C_{\vec{k}\uparrow} \right)$$

$$+ \sum_{\vec{k}} \Delta_{\vec{k}} \langle C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger \rangle \quad (334.3)$$

$$\equiv E_0$$

Como:

$$\sum_{\vec{k}, \alpha} \gamma_{\vec{k}} C_{\vec{k}\alpha}^{\dagger} C_{\vec{k}\alpha} = \sum_{\vec{k}} \gamma_{\vec{k}} C_{\vec{k}\uparrow}^{\dagger} C_{\vec{k}\uparrow} + \underbrace{\gamma_{\vec{k}} C_{-\vec{k}\downarrow}^{\dagger} C_{-\vec{k}\downarrow}}_{1 - C_{-\vec{k}\downarrow}^{\dagger} C_{-\vec{k}\downarrow}}$$

é possível expressar (334.3) em forma matricial:

$$K = E_0 + \sum_{\vec{k}} \gamma_{\vec{k}} + \sum_{\vec{k}} (C_{\vec{k}\uparrow}^{\dagger} \ C_{-\vec{k}\downarrow}) \underbrace{\begin{pmatrix} \gamma_{\vec{k}} & -\Delta_{\vec{k}} \\ -\Delta_{\vec{k}}^{\dagger} & -\gamma_{\vec{k}} \end{pmatrix}}_{\hat{H}_{\vec{k}}} \begin{pmatrix} C_{\vec{k}\uparrow} \\ C_{-\vec{k}\downarrow}^{\dagger} \end{pmatrix} \quad (335.1)$$

Definição: spinor de Nambu:

$$\hat{\psi}_{\vec{k}} = \begin{pmatrix} C_{\vec{k}\uparrow} \\ C_{-\vec{k}\downarrow}^{\dagger} \end{pmatrix} \quad (335.2)$$

o hamiltoniano (335.1) pode ser diagonalizado via uma transformação (canônica) de Bogoliubov:

$$\begin{aligned} C_{\vec{k}\uparrow} &= U_{\vec{k}}^{\dagger} \gamma_{\vec{k}\uparrow} + V_{\vec{k}} \gamma_{-\vec{k}\downarrow}^{\dagger} & C_{\vec{k}\uparrow}^{\dagger} &= U_{\vec{k}} \gamma_{\vec{k}\uparrow}^{\dagger} + V_{\vec{k}}^{\dagger} \gamma_{-\vec{k}\downarrow} \\ C_{-\vec{k}\downarrow}^{\dagger} &= -V_{\vec{k}}^{\dagger} \gamma_{\vec{k}\uparrow} + U_{\vec{k}} \gamma_{-\vec{k}\downarrow}^{\dagger} & C_{-\vec{k}\downarrow} &= -V_{\vec{k}} \gamma_{\vec{k}\uparrow}^{\dagger} + U_{\vec{k}}^{\dagger} \gamma_{-\vec{k}\downarrow} \end{aligned} \quad \rightarrow \quad (335.3)$$

onde $|U_{\vec{k}}|^2 + |V_{\vec{k}}|^2 = 1$.

verifica-se que (veja pg. 335.1):

$$K = E_0 + \sum_{\vec{k}} \gamma_{\vec{k}} + \sum_{\vec{k}} (\gamma_{\vec{k}\uparrow}^{\dagger} \ \gamma_{-\vec{k}\downarrow}) \begin{pmatrix} E_{\vec{k}} & 0 \\ 0 & -E_{\vec{k}} \end{pmatrix} \begin{pmatrix} \gamma_{\vec{k}\uparrow} \\ \gamma_{-\vec{k}\downarrow}^{\dagger} \end{pmatrix}$$

• sobre a diagonalização hamiltoniana (335.1):

Eq. (335.3) em forma matricial:

$$\underbrace{\begin{pmatrix} C_{\vec{k}\uparrow} \\ C_{-\vec{k}\downarrow} \end{pmatrix}}_{\hat{\Psi}_{\vec{k}}} = \underbrace{\begin{pmatrix} u_{\vec{k}} & v_{\vec{k}} \\ -v_{\vec{k}}^* & u_{\vec{k}} \end{pmatrix}}_{\hat{U}_{\vec{k}}^{\dagger}} \underbrace{\begin{pmatrix} \psi_{\vec{k}\uparrow} \\ \psi_{-\vec{k}\downarrow} \end{pmatrix}}_{\hat{\Phi}_{\vec{k}}}$$

$$\hookrightarrow \hat{\Psi}_{\vec{k}}^{\dagger} \hat{H}_{\vec{k}} \hat{\Psi}_{\vec{k}} = \hat{\Phi}_{\vec{k}}^{\dagger} \hat{U}_{\vec{k}} \hat{H}_{\vec{k}} \hat{U}_{\vec{k}}^{\dagger} \hat{\Phi}_{\vec{k}} = \hat{\Phi}_{\vec{k}}^{\dagger} \hat{H}'_{\vec{k}} \hat{\Phi}_{\vec{k}}$$

onde

$$H'_{\vec{k},11} = \zeta_{\vec{k}} (|u_{\vec{k}}|^2 - |v_{\vec{k}}|^2) + \Delta_{\vec{k}} u_{\vec{k}} v_{\vec{k}}^* + \Delta_{\vec{k}}^* v_{\vec{k}} u_{\vec{k}}^* = (I)$$

$$H'_{\vec{k},12} = 2\zeta_{\vec{k}} u_{\vec{k}} v_{\vec{k}} - \Delta_{\vec{k}} (u_{\vec{k}})^2 + \Delta_{\vec{k}}^* (v_{\vec{k}})^2 = (II)$$

$$H'_{\vec{k},21} = (H'_{\vec{k},12})^*$$

$$H'_{\vec{k},22} = -H'_{\vec{k},11}$$

como $|u_{\vec{k}}|^2 + |v_{\vec{k}}|^2 = 1$, podemos considerar

$$u_{\vec{k}} = e^{i\phi_{\vec{k}}/2} \cos \theta_{\vec{k}} \quad \text{e} \quad v_{\vec{k}} = e^{-i\phi_{\vec{k}}/2} \sin \theta_{\vec{k}} \quad (335.4)$$

$$\hookrightarrow (I) = \zeta_{\vec{k}} (\cos^2 \theta_{\vec{k}} - \sin^2 \theta_{\vec{k}}) + \Delta_{\vec{k}} e^{i\phi_{\vec{k}}} \sin \theta_{\vec{k}} \cos \theta_{\vec{k}} +$$

$$+ \Delta_{\vec{k}}^* e^{-i\phi_{\vec{k}}} \sin \theta_{\vec{k}} \cos \theta_{\vec{k}}$$

$$= \zeta_{\vec{k}} \cos 2\theta_{\vec{k}} + \frac{1}{2} (\Delta_{\vec{k}} e^{i\phi_{\vec{k}}} + \Delta_{\vec{k}}^* e^{-i\phi_{\vec{k}}}) \sin 2\theta_{\vec{k}}$$

$$(II) = 2\zeta_{\vec{k}} \sin\theta_{\vec{k}} \cos\theta_{\vec{k}} - \Delta_{\vec{k}} e^{i\phi_{\vec{k}}} \cos^2\theta_{\vec{k}} + \Delta_{\vec{k}}^* e^{-i\phi_{\vec{k}}} \sin^2\theta_{\vec{k}}$$

$$\text{use } \cos 2\theta_{\vec{k}} = \frac{\zeta_{\vec{k}}}{E_{\vec{k}}} \quad \text{e} \quad e^{i\phi_{\vec{k}}} \sin 2\theta_{\vec{k}} = \frac{\Delta_{\vec{k}}}{E_{\vec{k}}} \quad ; \quad E_{\vec{k}} = \sqrt{\zeta_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2}$$

temos que:

$$(I) = \frac{\zeta_{\vec{k}}^2}{E_{\vec{k}}} + \frac{1}{2} \left(\frac{\Delta_{\vec{k}} \cdot \Delta_{\vec{k}}^*}{E_{\vec{k}}} + \frac{\Delta_{\vec{k}}^* \cdot \Delta_{\vec{k}}}{E_{\vec{k}}} \right) = E_{\vec{k}}$$

$$(II) = \zeta_{\vec{k}} \sin 2\theta_{\vec{k}} - \underbrace{\left(E_{\vec{k}} e^{-i\phi_{\vec{k}}} \sin 2\theta_{\vec{k}} \right)}_{\Delta_{\vec{k}}} e^{i\phi_{\vec{k}}} \cos^2\theta_{\vec{k}} +$$

$$\underbrace{\left(E_{\vec{k}} e^{i\phi_{\vec{k}}} \sin 2\theta_{\vec{k}} \right)}_{\Delta_{\vec{k}}^*} e^{-i\phi_{\vec{k}}} \sin^2\theta_{\vec{k}}$$

$$= \sin 2\theta_{\vec{k}} \left(\zeta_{\vec{k}} - \underbrace{E_{\vec{k}} \cos 2\theta_{\vec{k}}}_{\zeta_{\vec{k}}/E_{\vec{k}}} \right) = 0$$

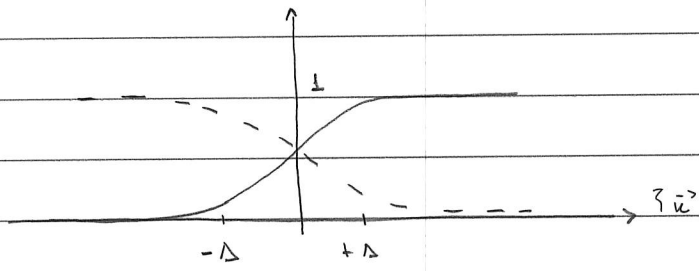
notas:

$$|u_{\vec{k}}|^2 = \cos^2\theta_{\vec{k}} = \frac{1}{2} (1 + \cos 2\theta_{\vec{k}}) = \frac{1}{2} \left(1 + \frac{\zeta_{\vec{k}}}{E_{\vec{k}}} \right)$$

$$|v_{\vec{k}}|^2 = \sin^2\theta_{\vec{k}} = \frac{1}{2} (1 - \cos 2\theta_{\vec{k}}) = \frac{1}{2} \left(1 - \frac{\zeta_{\vec{k}}}{E_{\vec{k}}} \right) \quad (335.5)$$

$$u_{\vec{k}} v_{\vec{k}}^{\dagger} = e^{i\phi_{\vec{k}}} \sin\theta_{\vec{k}} \cos\theta_{\vec{k}} = \frac{\Delta_{\vec{k}}}{2E_{\vec{k}}}$$

notan:



$|u_{\vec{\nu}}|^2$: —

$|v_{\vec{\nu}}|^2$: - - - -

(335.6)

$$K = E_0 + \sum_{\vec{k}} \zeta_{\vec{k}} - E_{\vec{k}} + \sum_{\vec{k}} E_{\vec{k}} (r_{\vec{k}\uparrow}^\dagger r_{\vec{k}\uparrow} + r_{\vec{k}\downarrow}^\dagger r_{\vec{k}\downarrow})$$

$$K = \sum_{\vec{k}} (\zeta_{\vec{k}} - E_{\vec{k}} + \Delta_{\vec{k}} \langle C_{\vec{k}\uparrow}^\dagger C_{\vec{k}\downarrow}^\dagger \rangle) + \sum_{\vec{k}, \sigma} E_{\vec{k}} r_{\vec{k}\sigma}^\dagger r_{\vec{k}\sigma}$$

(336.1)

$$\text{onde } E_{\vec{k}} = \sqrt{\zeta_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2}$$

e

$$|u_{\vec{k}}|^2 = \frac{1}{2} \left(1 + \frac{\zeta_{\vec{k}}}{E_{\vec{k}}} \right); \quad |v_{\vec{k}}|^2 = \frac{1}{2} \left(1 - \frac{\zeta_{\vec{k}}}{E_{\vec{k}}} \right) \quad \text{e} \quad u_{\vec{k}} v_{\vec{k}}^\dagger = \frac{\Delta_{\vec{k}}^\dagger}{2 E_{\vec{k}}}$$

• sobre a determinação de $\Delta_{\vec{k}}$:

$$\text{como: } \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle = \langle (-v_{\vec{p}} r_{\vec{p}\uparrow}^\dagger + u_{\vec{p}}^\dagger r_{-\vec{p}\downarrow}) (u_{\vec{p}} r_{\vec{p}\uparrow} + v_{\vec{p}} r_{-\vec{p}\downarrow}^\dagger) \rangle$$

$$= v_{\vec{p}} u_{\vec{p}}^\dagger \left(\langle r_{-\vec{p}\downarrow} r_{-\vec{p}\downarrow}^\dagger \rangle - \langle r_{\vec{p}\uparrow}^\dagger r_{\vec{p}\uparrow} \rangle \right)$$

$$= 1 - r_{-\vec{p}\downarrow}^\dagger r_{-\vec{p}\downarrow}$$

$$= v_{\vec{p}} u_{\vec{p}}^\dagger \left(1 - \langle r_{\vec{p}\uparrow}^\dagger r_{\vec{p}\uparrow} \rangle - \langle r_{-\vec{p}\downarrow}^\dagger r_{-\vec{p}\downarrow} \rangle \right)$$

$$\hookrightarrow \langle C_{-\vec{p}\downarrow} C_{\vec{p}\uparrow} \rangle = v_{\vec{p}} u_{\vec{p}}^\dagger \text{ à } T=0$$

\hookrightarrow Eqs. (334.2) e (336.1):

$$\Delta_{\vec{k}} = - \sum_{\vec{p}} v_{\vec{k}, \vec{p}} \langle r_{\vec{p}}^\dagger u_{\vec{p}}^\dagger \rangle = - \sum_{\vec{p}} v_{\vec{k}, \vec{p}} \frac{\Delta_{\vec{p}}}{2 \sqrt{\zeta_{\vec{p}}^2 + |\Delta_{\vec{p}}|^2}} :$$

(336.2)

: equação do gap!

em particular, p/ o potencial (330.2), temos que:

$$\Delta_{\vec{p}} = \frac{V}{2} \sum_{\vec{p}'} \frac{\Delta_{\vec{p}'}}{\sqrt{\xi_{\vec{p}'}^2 + |\Delta_{\vec{p}'}|^2}}$$

$$\hookrightarrow \Delta_{\vec{p}} = \begin{cases} \Delta, & |\xi_{\vec{p}}| < \hbar\omega_D \\ 0, & \text{c.c.} \end{cases} \quad ; \quad \Delta \in \mathbb{R} \quad (337.1)$$

substituindo $\sum_{\vec{p}'}$ $\rightarrow \int d^3$, temos que:

$$\Delta = \frac{V}{2} \int_{-\hbar\omega_D}^{+\hbar\omega_D} d^3 N(\xi) \frac{\Delta}{\sqrt{\xi^2 + \Delta^2}} \quad ; \quad N(\xi): \text{densidade de estados}$$

p/ solução não trivial $\Delta \neq 0$, temos que:

$$1 \approx V N(0) \int_0^{\hbar\omega_D} d\xi \frac{1}{\sqrt{\xi^2 + \Delta^2}} = V N(0) \sinh^{-1} \left(\frac{\hbar\omega_D}{\Delta} \right)$$

densidade de

estados no

nível de Fermi

$$\hookrightarrow \frac{\hbar\omega_D}{\Delta} = \frac{1}{2} \left(e^{1/VN(0)} - e^{-1/VN(0)} \right);$$

no limite de acoplamento fraco $VN(0) \ll 1$, temos que:

$$\hookrightarrow \Delta(T=0) \approx 2\hbar\omega_D e^{-1/VN(0)} \quad ; \quad \text{energia de gap a } T=0: \quad (337.2)$$

: companha energia do estado ligado do par de Cooper, Eq. (326.3)!

• sobre a energia do estado fundamental: E_s^0 :

como $\langle \vec{u} | \psi_{BCS} \rangle = 0$, onde

$|\psi_{BCS}\rangle$: estado fundamental BCS

$$\hookrightarrow \text{Eq. (336.1)}: E_s^0 = \sum_{\vec{u}} (\zeta_{\vec{u}} - E_{\vec{u}} + \Delta_{\vec{u}} \langle C_{\vec{u}\uparrow}^\dagger C_{-\vec{u}\downarrow}^\dagger \rangle);$$

é interessante escrever E_s^0 como:

$$E_s^0 = \sum_{\vec{u}} (\zeta_{\vec{u}} - E_{\vec{u}} + 2\Delta_{\vec{u}} \langle C_{\vec{u}\uparrow}^\dagger C_{-\vec{u}\downarrow}^\dagger \rangle) - \sum_{\vec{u}} \Delta_{\vec{u}} \langle C_{\vec{u}\uparrow}^\dagger C_{-\vec{u}\downarrow}^\dagger \rangle$$

$$u_{\vec{u}} v_{\vec{u}}^\dagger = \frac{\Delta_{\vec{u}}}{2E_{\vec{u}}}$$

$$= \sum_{\vec{u}} (\zeta_{\vec{u}} - E_{\vec{u}} + \frac{\Delta_{\vec{u}}^2}{E_{\vec{u}}}) - \Delta \sum_{\vec{u}} u_{\vec{u}} v_{\vec{u}}^\dagger$$

$$\zeta_{\vec{u}} - E_{\vec{u}} + \frac{E_{\vec{u}}^2 - \zeta_{\vec{u}}^2}{E_{\vec{u}}} \quad \Delta/V: \text{Eq. (336.2)}$$

$$\hookrightarrow E_s^0 = \sum_{\vec{u}} \left(\zeta_{\vec{u}} - \frac{\zeta_{\vec{u}}^2}{E_{\vec{u}}} \right) - \frac{\Delta^2}{V} \quad (338.1)$$

à $T=0$, o estado normal é caracterizado por:

$$\Delta=0 \rightarrow E_{\vec{u}} = |\zeta_{\vec{u}}| = \begin{cases} +\zeta_{\vec{u}}, & |\vec{u}| > k_F \\ -\zeta_{\vec{u}}, & |\vec{u}| < k_F \end{cases}$$

$$\hookrightarrow \text{Eq. (338.1)}: E_N^0 = \sum_{|\vec{u}| < k_F} 2\zeta_{\vec{u}} \quad (338.2)$$

Temos que:

$$E_S^0 - E_N^0 = \sum_{|\vec{k}| < k_F} \left(-\zeta_{\vec{k}} - \frac{\zeta_{\vec{k}}^2}{E_{\vec{k}}} \right) + \sum_{|\vec{k}| > k_F} \left(\zeta_{\vec{k}} - \frac{\zeta_{\vec{k}}^2}{E_{\vec{k}}} \right)$$

$$- \Delta^2 / V$$

$$= \sum_{|\vec{k}| > k_F} 2 \left(\zeta_{\vec{k}} - \frac{\zeta_{\vec{k}}^2}{E_{\vec{k}}} \right) - \frac{\Delta^2}{V}$$

Como $|\vec{k}| > k_F \rightarrow \zeta_{\vec{k}} > 0$ e $E_{\vec{k}} = \zeta_{\vec{k}}$ p/ $\Delta = 0$

$$\hookrightarrow E_S^0 - E_N^0 = 2 \int_0^{\hbar\omega_D} d\zeta N(\zeta) \left(\zeta - \frac{\zeta^2}{\sqrt{\zeta^2 + \Delta^2}} \right) - \frac{\Delta^2}{V}$$

$$\zeta = x \Delta$$

$$\approx 2N(0) \int_0^{\hbar\omega_D} d\zeta \zeta - 2N(0) \Delta^2 \int_0^{\hbar\omega_D/\Delta} dx \frac{x^2}{\sqrt{x^2 + 1}} - \frac{\Delta^2}{V}$$

$$2N(0) \frac{1}{2} (\hbar\omega_D)^2 \quad (3)$$

$$(I) = -2N(0) \Delta^2 \cdot \frac{1}{2} \left(\frac{\hbar\omega_D}{\Delta} \right) \left(1 + \left(\frac{\hbar\omega_D}{\Delta} \right)^2 \right)^{1/2} + 2N(0) \Delta^2 \frac{1}{2} \sinh^{-1} \left(\frac{\hbar\omega_D}{\Delta} \right)$$

$$\frac{\left(\frac{\hbar\omega_D}{\Delta} \right)^2 \left(1 + \left(\frac{\Delta}{\hbar\omega_D} \right)^2 \right)^{1/2}}{VN(0)} = 1$$

$$\approx 1 + \frac{1}{2} \left(\frac{\Delta}{\hbar\omega_D} \right)^2$$

$$= -N(0) \Delta^2 (\hbar\omega_D)^2 - \frac{1}{2} N(0) \Delta^2 + \frac{\Delta^2}{V}$$

$$\hookrightarrow E_S^0 - E_N^0 = \left(\frac{\Delta^2}{V} - \frac{1}{2} N(0) \Delta^2 \right) - \frac{\Delta^2}{V} = -\frac{1}{2} N(0) \Delta^2 :$$

(339.1)

: energia de condensação

Lembrar Eq. (306.4):

$$G_S(T, 0) - G_N(T, 0) = -\frac{1}{8\pi} H_C^2(T) : \text{energia de condensação estado supercondutor;}$$

Como $G = G(T, H)$, $F = F(T, B)$ e $G = F - \frac{1}{4\pi} \vec{B} \cdot \vec{H}$

$\hookrightarrow G(T, 0) = F(T, 0);$

de fato, $\Delta = \Delta(T)$,

à $T=0$: $F = 0 = \langle H \rangle$

veja abaixo

$$\hookrightarrow E_S^0 - E_N^0 = G_S(T, 0) - G_N(T, 0) = -\frac{1}{2} N(0) \Delta^2(T=0) = -\frac{1}{8\pi} H_C^2(T=0) \quad (340.1)$$

sobre o estado fundamental $|\psi_{SC}\rangle$,

Lembrar: instabilidade de Cooper:

\hookrightarrow estado fundamental SC \sim conjunto de pares de Cooper!

em princípio, pr sistema de N -elétrons, temos que:

$$|\psi_N\rangle = \sum_{\vec{k}_1 \dots \vec{k}_{N/2}} g(\vec{k}_1, \dots, \vec{k}_{N/2}) \psi^\dagger$$

$$\psi^\dagger = C_{\vec{k}_1 \uparrow}^\dagger C_{-\vec{k}_1 \downarrow}^\dagger \dots C_{\vec{k}_{N/2} \uparrow}^\dagger C_{-\vec{k}_{N/2} \downarrow}^\dagger |0\rangle \quad (340.2)$$

op. de criação de

vácuo

pares, veja pg. 334.

entretanto, devido dificuldades pr realização de cálculos c/

Eq. (340.2)

\hookrightarrow proposta BCS pr estado variacional:

$$|\psi_{BCS}\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c_{\vec{k}}^{\dagger} + c_{-\vec{k}}^{\dagger}) |0\rangle, \quad (341.1)$$

onde: $u_{\vec{k}}$ e $v_{\vec{k}} \in \mathbb{C}$: parâmetros variacionais

$$\langle \psi_{BCS} | \psi_{BCS} \rangle = 1 \quad \underline{\text{VERIFICAR}} \rightarrow |u_{\vec{k}}|^2 + |v_{\vec{k}}|^2 = 1$$

notas:

$$(1) \text{ se } u_{\vec{k}} = \begin{cases} 0, & k < k_F \\ 1, & k > k_F \end{cases} \quad \text{e } v_{\vec{k}} = \begin{cases} 1, & k < k_F \\ 0, & k > k_F \end{cases}$$

$\hookrightarrow |\psi_{BCS}\rangle = |FS\rangle$: Fermi sea

(2) $|v_{\vec{k}}|^2$: probabilidade de estado $|\vec{k}\uparrow\rangle$ e $|\vec{k}\downarrow\rangle$,
ocupados por par de Cooper

e $|u_{\vec{k}}|^2 = 1 - |v_{\vec{k}}|^2$: " " " " "
não ocupados " " " " "

Obs.: via procedimento variacional, verifica-se que

Eqs. (333.1) e (341.1)

$$\hookrightarrow \langle \psi_{BCS} | H_{BCS} - \mu N | \psi_{BCS} \rangle =$$

$$= 2 \sum_{\vec{k}} \xi_{\vec{k}} |v_{\vec{k}}|^2 + \sum_{\vec{k}, \vec{p}} v_{\vec{k}, \vec{p}} u_{\vec{k}} v_{\vec{k}}^{\dagger} u_{\vec{p}}^{\dagger} v_{\vec{p}} \quad (341.2)$$

minimização do funcional (341.2)

$\hookrightarrow u_{\vec{k}}$ e $v_{\vec{k}}$: Eq. (335.5)

(3) número de pares de Cooper em $|\psi_{BCS}\rangle$: indefinido:

Lembrar: H_{BCS}^{MF} : Eq. (335.1): número de partículas

não é conservado;

em principio, temos que

$$|\psi_{BCS}\rangle = \sum_N \lambda_N |\psi_N\rangle \quad \text{Eq. (340.2)} \quad (342.1)$$

verifica-se que:

$$\langle \hat{N} \rangle = \langle \psi_{BCS} | \sum_{\vec{k}, \alpha} C_{\vec{k}\alpha}^\dagger C_{\vec{k}\alpha} | \psi_{BCS} \rangle = \sum_{\vec{k}} 2 |V_{\vec{k}}|^2 ;$$

$$\langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2 = 4 \sum_{\vec{k}} |U_{\vec{k}}|^2 |V_{\vec{k}}|^2 = (\Delta N)^2 \quad (342.2)$$

$$\hookrightarrow \frac{(\Delta N)}{\langle \hat{N} \rangle} \sim \left(\frac{T_c}{T_F} \right)^{1/2} \sim 10^{-3} \rightarrow |\lambda_N|^2 : \text{ pico em } \langle \hat{N} \rangle !$$

Obs. 1: desprezando o fator de normalização, temos que:

$$|\psi_{BCS}\rangle = \prod_{\vec{k}} (1 + \phi_{\vec{k}} C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger) |0\rangle$$

$$= \exp \left(\sum_{\vec{k}} \phi_{\vec{k}} C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger \right) |0\rangle ; \quad (342.3)$$

Lembrar que

$$(C_{\vec{k}\alpha}^\dagger)^2 = 0 \quad \equiv \Lambda^\dagger$$

onde $\phi_{\vec{k}} = V_{\vec{k}} / U_{\vec{k}}$,

i.e., $|\psi_{BCS}\rangle$:: forma de um estado coerente!

expandindo a exponencial, temos que:

$$|\psi_{BCS}\rangle = \sum_{n \geq 0} \frac{1}{n!} (\Lambda^\dagger)^n |0\rangle = \sum_{n \geq 0} \frac{1}{\sqrt{n!}} \underbrace{\frac{1}{\sqrt{n!}} (\Lambda^\dagger)^n |0\rangle}_{\text{estado c/ n pares de Cooper}} ;$$

estado c/ n pares de Cooper : $|n\rangle$

i.e. de Eq. (342.1)

Obs. 2: consideramos a transformação de gauge:

$$C_{\vec{u}\alpha} \rightarrow e^{-i\varphi} C_{\vec{u}\alpha} \quad e \quad C_{\vec{u}\alpha}^{\dagger} \rightarrow e^{i\varphi} C_{\vec{u}\alpha}^{\dagger}; \quad \varphi \in \mathbb{R} \quad (343.1)$$

$$\hookrightarrow C_{\vec{u}\alpha}^{\dagger} C_{\vec{u}\alpha} \rightarrow C_{\vec{u}\alpha}^{\dagger} C_{\vec{u}\alpha}$$

$$\text{Eq. (334.2): } \Delta_{\vec{u}} = - \sum_{\vec{p}} \sqrt{u, \vec{p}} \langle C_{-\vec{p}\uparrow} C_{\vec{p}\uparrow} \rangle \rightarrow e^{-2i\varphi} \Delta_{\vec{u}}$$

$$C_{\vec{u}\uparrow}^{\dagger} C_{-\vec{u}\uparrow} \rightarrow e^{2i\varphi} C_{\vec{u}\uparrow}^{\dagger} C_{-\vec{u}\uparrow}$$

\hookrightarrow Eq. (334.3): $H_{\text{BCS}}^{\text{MF}}$: invariante sob transf. (343.1)!

$$|\psi_{\text{BCS}}\rangle \rightarrow |\psi_{\varphi}\rangle = \prod_{\vec{u}} \left(1 + e^{2i\varphi} \phi_{\vec{u}} C_{\vec{u}\uparrow}^{\dagger} C_{-\vec{u}\uparrow} \right) |0\rangle$$

$$= \exp \left(e^{2i\varphi} \sum_{\vec{u}} \phi_{\vec{u}} C_{\vec{u}\uparrow}^{\dagger} C_{-\vec{u}\uparrow} \right) |0\rangle$$

$$= \exp \left(e^{2i\varphi} \Lambda^{\dagger} \right) |0\rangle = \sum_{n \geq 0} \frac{1}{\sqrt{n!}} e^{2in\varphi} |n\rangle ;$$

: $|\psi_{\text{BCS}}\rangle$ não preserva simetria de gauge

\hookrightarrow formação estado SC \sim quebra espontânea simetria $U(1)$

$\hookrightarrow |\psi_{\text{BCS}}\rangle \sim$ condensado de pares de Cooper!

notas: op. número \hat{N} :

$$\hat{N} |\psi_{\varphi}\rangle = \sum_{n \geq 0} \frac{1}{\sqrt{n!}} (2n) e^{2in\varphi} |n\rangle = i \frac{d}{d\varphi} |\psi_{\varphi}\rangle$$

(343.2)

$$\hookrightarrow \hat{N} = i \frac{d}{d\varphi}$$

↳ devido analogia entre Eq. (343.2) e propriedades operadoras \vec{a} e \vec{p} , podemos considerar:

- $[\varphi, \hat{N}] = i$: (op.) fase e op. número; canonicamente conjugados;

- op. \hat{N} : gerador das transf. de gauge (343.1)

- $\Delta \neq \Delta N \approx 1$: como $|\psi_{BCS}\rangle$: fase definida
↳ número partículas indefinido!

- sobre os estados excitados,

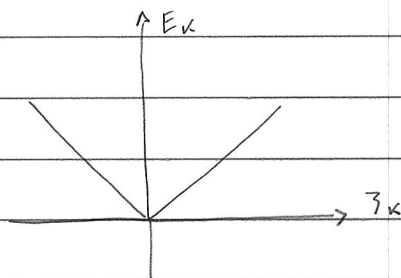
Eq. (336.1) : $E_{\vec{k}} = \sqrt{\epsilon_{\vec{k}}^2 + |\Delta_{\vec{k}}|^2}$:

: energia quasipartículas \vec{k} e momento $\hbar\vec{k}$!

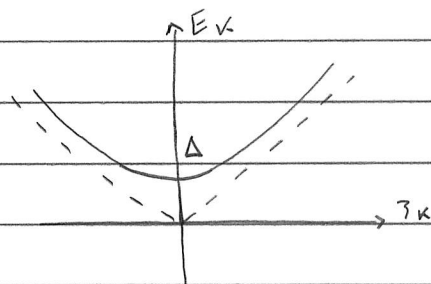
notas: p/ excitação na superfície de Fermi :

$\epsilon_{\vec{k}} = 0 \rightarrow E_{\vec{k}} = |\Delta_{\vec{k}}|$: energia de gap!

comparamos: energia das excitações em torno FS p/ metal normal e sc BCS :



metal normal :
 $\Delta_{\vec{k}} = 0 \rightarrow E_{\vec{k}} = |\Delta_{\vec{k}}|$



BCS : $\Delta_{\vec{k}} \neq 0$ (334.1)
 $E_{\vec{k}} = \sqrt{\epsilon_{\vec{k}}^2 + \Delta^2}$

considerar inverse de transf. (335.3):

$$\gamma_{\vec{k}\uparrow}^{\dagger} = U_{\vec{k}}^{\dagger} C_{\vec{k}\uparrow}^{\dagger} - V_{\vec{k}}^{\dagger} C_{-\vec{k}\downarrow}$$

(345.1)

$$\gamma_{-\vec{k}\downarrow}^{\dagger} = U_{\vec{k}}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger} + V_{\vec{k}}^{\dagger} C_{\vec{k}\uparrow}$$

de fato, temos que:

$$U_{\vec{k}}^{\dagger} + \text{Eq. (335.3)}: U_{\vec{k}}^{\dagger} C_{\vec{k}\uparrow}^{\dagger} = |U_{\vec{k}}|^2 \gamma_{\vec{k}\uparrow}^{\dagger} + U_{\vec{k}}^{\dagger} V_{\vec{k}}^{\dagger} \gamma_{-\vec{k}\downarrow}$$

⊕

$$-V_{\vec{k}}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger} = |V_{\vec{k}}|^2 \gamma_{\vec{k}\uparrow}^{\dagger} - U_{\vec{k}}^{\dagger} V_{\vec{k}}^{\dagger} \gamma_{-\vec{k}\downarrow}$$

$$\hookrightarrow (|U_{\vec{k}}|^2 + |V_{\vec{k}}|^2) \gamma_{\vec{k}\uparrow}^{\dagger} = U_{\vec{k}}^{\dagger} C_{\vec{k}\uparrow}^{\dagger} - V_{\vec{k}}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger}$$

lembrar: $\langle \psi_{BCS} | \psi_{\vec{k}\uparrow}^{\dagger} | \psi_{BCS} \rangle = 0$.

notar Eqs. (345.1) e (345.2):

$$\gamma_{\vec{k}\uparrow}^{\dagger} | \psi_{BCS} \rangle = \sum_{\vec{p}} (U_{\vec{k}} C_{\vec{k}\uparrow}^{\dagger} - V_{\vec{k}} C_{-\vec{k}\downarrow}^{\dagger}) \prod_{\vec{p}} (U_{\vec{p}} + V_{\vec{p}} C_{\vec{p}\uparrow}^{\dagger} C_{-\vec{p}\downarrow}^{\dagger}) | 0 \rangle$$

$$= \underbrace{(U_{\vec{k}} C_{\vec{k}\uparrow}^{\dagger} - V_{\vec{k}} C_{-\vec{k}\downarrow}^{\dagger})}_{(I)} (U_{\vec{k}} + V_{\vec{k}} C_{\vec{k}\uparrow}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger}) +$$

$$+ \sum_{\vec{p} \neq \vec{k}} \prod_{\vec{p} \neq \vec{k}} (U_{\vec{p}} + V_{\vec{p}} C_{\vec{p}\uparrow}^{\dagger} C_{-\vec{p}\downarrow}^{\dagger}) | 0 \rangle$$

como:

$$(I) | 0 \rangle = (U_{\vec{k}}^2 C_{\vec{k}\uparrow}^{\dagger} + U_{\vec{k}} V_{\vec{k}} (C_{\vec{k}\uparrow}^{\dagger} C_{\vec{k}\uparrow}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger} - C_{-\vec{k}\downarrow}^{\dagger}) -$$

$$C_{-\vec{k}\downarrow}^{\dagger} + C_{\vec{k}\uparrow}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger} C_{\vec{k}\uparrow})$$

$$- V_{\vec{k}}^2 C_{-\vec{k}\downarrow}^{\dagger} C_{\vec{k}\uparrow}^{\dagger} C_{-\vec{k}\downarrow}^{\dagger}) | 0 \rangle = 0$$

$$- C_{\vec{k}\uparrow}^{\dagger} (C_{-\vec{k}\downarrow}^{\dagger})^2$$

similar, verifica-se que (exercício): $\gamma_{-\vec{u}\downarrow} |\psi_{\text{BCS}}\rangle = 0$.

por os estados excitados, temos que:

$$\gamma_{\vec{u}\uparrow} |\psi_{\text{BCS}}\rangle = \underbrace{(u_{\vec{u}}^\dagger C_{\vec{u}\uparrow}^\dagger - v_{\vec{u}}^\dagger C_{-\vec{u}\downarrow})}_{(I)} (u_{\vec{u}} + v_{\vec{u}} C_{\vec{u}\uparrow}^\dagger C_{-\vec{u}\downarrow}^\dagger) |\psi_0\rangle$$

$$+ \sum_{\vec{p} \neq \vec{u}} \pi_{\vec{p}} (u_{\vec{p}} + v_{\vec{p}} C_{\vec{p}\uparrow}^\dagger C_{-\vec{p}\downarrow}^\dagger) |\psi_0\rangle$$

como:

$$(I) |\psi_0\rangle = \left(|u_{\vec{u}}|^2 C_{\vec{u}\uparrow}^\dagger + u_{\vec{u}}^\dagger v_{\vec{u}} \underbrace{C_{\vec{u}\uparrow}^\dagger C_{-\vec{u}\downarrow}^\dagger}_{=0} C_{-\vec{u}\downarrow} - u_{\vec{u}} v_{\vec{u}}^\dagger C_{-\vec{u}\downarrow} - \right.$$

$$\left. - |v_{\vec{u}}|^2 C_{-\vec{u}\downarrow} C_{\vec{u}\uparrow}^\dagger C_{-\vec{u}\downarrow}^\dagger \right) |\psi_0\rangle$$

$$- C_{\vec{u}\uparrow}^\dagger (1 - C_{-\vec{u}\downarrow}^\dagger C_{-\vec{u}\downarrow})$$

$$= (|u_{\vec{u}}|^2 + |v_{\vec{u}}|^2) C_{\vec{u}\uparrow}^\dagger |\psi_0\rangle$$

$$\hookrightarrow \gamma_{\vec{u}\uparrow} |\psi_{\text{BCS}}\rangle = C_{\vec{u}\uparrow}^\dagger \sum_{\vec{p} \neq \vec{u}} \pi_{\vec{p}} (u_{\vec{p}} + v_{\vec{p}} C_{\vec{p}\uparrow}^\dagger C_{-\vec{p}\downarrow}^\dagger) |\psi_0\rangle :$$

(346.1)

: 1 elétron adicionado ao estado $|\vec{u}\uparrow\rangle$

é par $(\vec{u}\uparrow; -\vec{u}\downarrow)$ removido do condensado de pares!

similar, verifica-se que (exercício):

$$\gamma_{-\vec{u}\downarrow} |\psi_{\text{BCS}}\rangle = C_{-\vec{u}\downarrow}^\dagger \sum_{\vec{p} \neq \vec{u}} \pi_{\vec{p}} (u_{\vec{p}} + v_{\vec{p}} C_{\vec{p}\uparrow}^\dagger C_{-\vec{p}\downarrow}^\dagger) |\psi_0\rangle \quad (346.2)$$

· Eqs. (346.1) e (346.2): indica relação one-to-one entre quasipartículas $\gamma_{\vec{u}\uparrow}$ e férmions $C_{\vec{u}\uparrow}^\dagger \sim$ metal normal;

↳ podemos considerar:

$$N_S(E) dE = N_N(\zeta) d\zeta \quad (347.1)$$

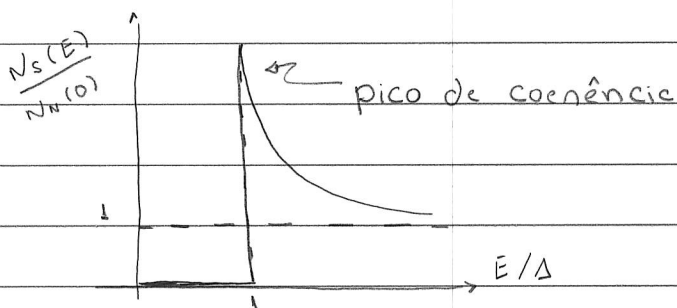
↑ densidade de estados metal normal
 " " " SC

pt excitações em torno FS, podemos considerar $N_N(\zeta) = N_N(0)$;

$$\text{Como } E^2 = \zeta^2 + \Delta^2 \rightarrow 2E dE = 2\zeta d\zeta$$

$$\text{↳ } N_S(E) = N_N(0) \frac{d\zeta}{dE} = \frac{E}{\sqrt{E^2 - \Delta^2}} \theta(E - |\Delta|) : \quad (347.2)$$

: densidade de estado SC BCS!



(347.3)

Obs.: $N_S(E)$ pode ser medida via a determinação da corrente de tunelamento em junções NORMAL-SC, pois

$$\frac{dI}{dV} \propto N_S(E);$$

veja, e.g., Fig. 14.10, Coleman.

propriedades a temperatura finita.

$$\text{Eq. (334.2):} \quad \Delta \vec{x} = - \sum_{\vec{p}} v_{\vec{x}, \vec{p}} \langle C_{-\vec{p}} + C_{\vec{p}} \rangle$$

como (veja pg. 336):

$$\langle C_{-\vec{p}} + C_{\vec{p}} \rangle = v_{\vec{p}} u_{\vec{p}}^+ \left(1 - \underbrace{\langle r_{\vec{p}}^{\dagger} r_{\vec{p}} \rangle}_{N_{FD}(E_{\vec{p}})} - \underbrace{\langle r_{-\vec{p}} + r_{-\vec{p}} \rangle}_{N_{FD}(E_{\vec{p}})} \right)$$

$$\begin{aligned} \epsilon \quad 1 - 2N_{FD}(x) &= 1 - \frac{2}{e^{\beta x} + 1} = \frac{e^{\beta x} - 1}{e^{\beta x} + 1} = \frac{e^{\beta x/2} - e^{-\beta x/2}}{e^{\beta x/2} + e^{-\beta x/2}} \\ &= \text{tgh}\left(\frac{1}{2}\beta x\right) \end{aligned}$$

$$\hookrightarrow \Delta \vec{x} = -\frac{1}{2} \sum_{\vec{p}} v_{\vec{x}, \vec{p}} \frac{\Delta \vec{p}}{E_{\vec{p}}} \text{tgh}\left(\frac{1}{2}\beta E_{\vec{p}}\right) \quad (348.1)$$

em particular, p/ potencial (330.2), temos que:

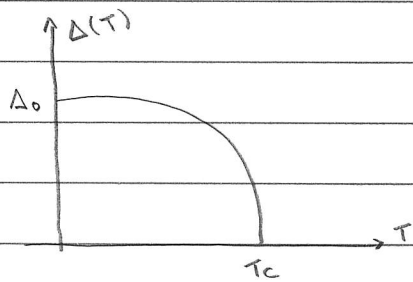
$$\Delta = v \sum_{\vec{p}} \frac{\Delta}{E_{\vec{p}}} \text{tgh}\left(\frac{1}{2}\beta E_{\vec{p}}\right)$$

ϵ , similar pg. 337, temos que:

$$1 = v \int_{-k_{\text{WD}}}^{+k_{\text{WD}}} d\tau N(\tau) \frac{1}{2\sqrt{\tau^2 + \Delta^2(\tau)}} \text{tgh}\left(\frac{1}{2k_{\text{BT}}} \sqrt{\tau^2 + \Delta^2(\tau)}\right) \quad (348.2)$$

Eq. (348.2) \rightarrow determinação $\Delta(\tau)$;

verifica-se que (solução numérica, veja Fig. 18.4(a), Bruus p/ detalhes):



$$\Delta_0 = \Delta(T=0)$$

(349.1)

Eq. (348.2) \rightarrow determinação da temperatura crítica T_c ;
 como $\Delta(T_c) = 0$, temos que:

$$1 \approx \sqrt{N(0)} \int_{-h\omega_D}^{+h\omega_D} d\zeta \frac{1}{2\zeta} \operatorname{tgh} \left(\frac{|\zeta|}{2k_B T_c} \right)$$

$$= \sqrt{N(0)} \int_0^{h\omega_D} d\zeta \frac{1}{\zeta} \operatorname{tgh} \left(\frac{\zeta}{2k_B T_c} \right)$$

$$x = \frac{\zeta}{2k_B T_c} \rightarrow dx = \frac{1}{2k_B T_c} d\zeta$$

$$= \sqrt{N(0)} \int_0^{\Lambda} dx \frac{1}{x} \operatorname{tgh} x ; \quad \Lambda = \frac{h\omega_D}{2k_B T_c}$$

(I)

no limite de acoplamento fraco (*), verificou-se que
 (veja Sec. 18.4.2, Bruus):

$$(I) \approx \ln \left(\frac{h\omega_D}{2k_B T_c} \right) + \ln \frac{4e^r}{\pi} ; \quad r \approx 0.577 : \text{cte de Euler}$$

Obs (*): Lembra pg. 337: acoplamento fraco: $\sqrt{N(0)} \ll 1$;

nesse caso, Eq. (337.2) $\rightarrow h\omega_D \gg 1$;
 $\Delta(0)$

como $\Delta(0) \sim k_B T_c \rightarrow \frac{h\omega_D}{k_B T_c} \gg 1$.

temos que:

$$J = \sqrt{N(0)} \ln \left(\frac{4e^f}{2\pi} \cdot \frac{h\nu_D}{k_B T_c} \right) \\ \approx 1,13$$

$$\hookrightarrow k_B T_c = 1,13 h\nu_D e^{-1/\sqrt{N(0)}} \quad (350.1)$$

comparando Eqs. (337.2) e (350.1), temos que:

$$\Delta(T=0) = 1,764 k_B T_c \quad (350.2)$$

Obs.: experimento:

	Sn	Pb	In	BCS
$2\Delta(T=0)/k_B T_c$	3,46	4,29	3,63	3,53

Obs.: comparan Eq. (350.2) c/ Eq. (332.1): T_c determinada via análise da instabilidade de Cooper!

notas Eqs. (337.2) e (350.1):

$$N(0) \approx k_B T_c \alpha \exp(-1/\sqrt{N(0)})$$

\hookrightarrow não é possível determinação perturbativa considerando FS como o estado fundamental, é necessária uma nova hipótese sobre o estado fundamental!

Eq. (348.2): p/ $T \approx T_c$, verifica-se que:

$$\frac{\Delta(T)}{\Delta(0)} \approx 1,74 \left(1 - \frac{T}{T_c} \right)^{1/2} \quad (350.3)$$

: componente do parâmetro de ordem em uma teoria de campo médio!

: sobre o calor específico,

notas: determinação $\Delta(T) \rightarrow$ determinação energia das quasipartículas $E_{\vec{k}} = \sqrt{3\vec{k}^2 + \Delta^2(T)}$;

a entropia do sistema eletrônico é determinada através da expressão da entropia de um gás de férmions:

$$S = -2k_B \sum_{\vec{k}} f_{\vec{k}} \ln f_{\vec{k}} + (1 - f_{\vec{k}}) \ln(1 - f_{\vec{k}})$$

onde

$$f_{\vec{k}} = \frac{1}{e^{\beta E_{\vec{k}}} + 1}$$

(351.1)

a partir de S , é possível determinar o calor específico:

$$C = T \frac{\partial S}{\partial T} = -\beta \frac{\partial S}{\partial \beta}$$

(351.2)

temos que:

$$\begin{aligned} C &= 2\beta k_B \sum_{\vec{k}} (\partial_{\beta} f_{\vec{k}}) \ln f_{\vec{k}} + f_{\vec{k}} \cdot \frac{1}{f_{\vec{k}}} \partial_{\beta} f_{\vec{k}} + \\ &+ (-\partial_{\beta} f_{\vec{k}}) \ln(1 - f_{\vec{k}}) + (1 - f_{\vec{k}}) \cdot \frac{1}{1 - f_{\vec{k}}} (-1) \partial_{\beta} f_{\vec{k}} \\ &= 2\beta k_B \sum_{\vec{k}} \partial_{\beta} f_{\vec{k}} \ln \frac{f_{\vec{k}}}{1 - f_{\vec{k}}} = -2\beta^2 k_B \sum_{\vec{k}} E_{\vec{k}} \partial_{\beta} f_{\vec{k}} \\ &\quad \frac{1}{e^{-\beta E_{\vec{k}}}} \end{aligned}$$

$$\text{Como: } \frac{\partial}{\partial \beta} = \frac{\partial(\beta E)}{\partial \beta} \frac{\partial}{\partial(\beta E)} = (E + \beta \partial_{\beta} E) \frac{\partial}{\partial(\beta E)}$$

temos que:

$$C = -2\beta^2 V_B \sum_{\vec{k}} (E_{\vec{k}}^2 + \beta E_{\vec{k}} \partial_{\beta} E_{\vec{k}}) \frac{\partial f_{\vec{k}}}{\partial(\beta E_{\vec{k}})}$$

$$\frac{1}{\beta} \frac{\partial f_{\vec{k}}}{\partial E_{\vec{k}}}$$

$$\hookrightarrow C = 2\beta V_B \sum_{\vec{k}} \left(\frac{-\partial f_{\vec{k}}}{\partial E_{\vec{k}}} \right) \left(E_{\vec{k}}^2 + \beta E_{\vec{k}} \partial_{\beta} E_{\vec{k}} \right)$$

$$\underbrace{E_{\vec{k}} \cdot \frac{1}{2 E_{\vec{k}}} \cdot 2 \Delta(T) \partial_{\beta} \Delta(T)}_{\frac{1}{2} \partial_{\beta} \Delta^2(T)}$$

$$\hookrightarrow C_s = 2\beta V_B \sum_{\vec{k}} \left(\frac{-\partial f_{\vec{k}}}{\partial E_{\vec{k}}} \right) \left(E_{\vec{k}}^2 + \frac{1}{2} \beta \partial_{\beta} \Delta^2(T) \right) : \quad (352.1)$$

: calor específico SC BCS!

2 casos limites:

(1) : $T = T_c$;

1º termo Eq. (352.1) : como $\Delta(T_c = 0)$, temos $E_{\vec{k}} = |\hbar \vec{k}|$

↳ " " " " : calor específico metal normal;

verifique-se que (veja pg. 352.1) :

$$C_N = 2\beta V_B \sum_{\vec{k}} \frac{3}{2} \frac{\vec{k}^2}{|\hbar \vec{k}|} \left(\frac{-\partial f_{\vec{k}}}{\partial |\hbar \vec{k}|} \right) = \gamma T = \frac{2\pi^2}{3} N(0) V_B T \quad (352.2)$$

↳ a descontinuidade do calor específico a $T = T_c$

(veja Fig. (302.2)) é dada por:

$$\Delta C = (C_s - C_N)_{T=T_c} = 2\beta_c V_B \sum_{\vec{k}} \left(\frac{-\partial f_{\vec{k}}}{\partial E_{\vec{k}}} \right) \cdot \frac{1}{2} \beta_c \frac{\partial \Delta^2}{\partial \beta} \Big|_{\beta_c}$$

• sobre a Eq. (352.2):

Lembrar expansão de Sommerfeld, veja Eq. (6.28), Kittelson:

$$I = \int_0^{\infty} dE F(E) f(E) \approx \int_0^{\mu} dE F(E) + \frac{\pi^2}{6} (k_B T)^2 F'(\mu) +$$

$$+ \frac{7\pi^4}{360} (k_B T)^4 F''(\mu) + \dots, \quad T \ll T_F \quad (352.3)$$

onde $f(E) = \frac{1}{e^{\beta(E-\mu)} + 1}$

$$\text{Eq. (352.2)}: C_N = 2\beta k_B \int_{-\infty}^{+\infty} d\tau N(\tau) \tau^2 \left(-\frac{\partial f}{\partial \tau} \right)$$

= $-\partial f / \partial \tau$, pois $\partial_{\tau} f(\tau)$ é uma função par!

$$\hookrightarrow C_N = 2\beta k_B \int_{-\infty}^{+\infty} d\tau \partial_{\tau} (\tau^2 N(\tau)) f(\tau) - 2\beta k_B (\tau^2 N(\tau) f(\tau)) \Big|_{-\infty}^{+\infty}$$

$$2\tau N(\tau) + \tau^2 N'(\tau) = 0, \text{ pois } f(\tau \rightarrow +\infty) \rightarrow 0$$

$$\text{e } N(\tau \rightarrow -\infty) \rightarrow 0$$

⊕ Eq. (352.3):

$$C_N \approx 2\beta k_B \left(\underbrace{\int_{-\infty}^0 d\tau \frac{\partial}{\partial \tau} (\tau^2 N(\tau))}_{=0} + \frac{\pi^2}{6} (k_B T)^2 + \right.$$

$$\left. + \left(2N(\tau) + 2\tau N'(\tau) + 2\tau N(\tau) + \tau^2 N'(\tau) \right) \Big|_{\tau=0} + \dots \right)$$

$$2N(0)$$

$$\hookrightarrow C_N \approx \frac{2k_B}{k_B T} \cdot \frac{\pi^2}{3} (k_B T)^2 N(0)$$

temos que:

$$\Delta C = \beta_c^2 \kappa_B \left. \frac{\partial \Delta^2}{\partial \beta} \right|_{\beta_c} \sum_{\vec{u}} \left(\frac{-\partial f_{\vec{u}}}{\partial T} \right)$$

notan: 131 \rightarrow 3,
veja pg. 352.1

$$\frac{-\partial \Delta^2}{\partial T} \Big|_{T_c} \int_{-\infty}^{\infty} d\tau N(\tau) \left(\frac{-\partial f_{\vec{u}}}{\partial T} \right) \approx N(0) \int_{-\infty}^{\infty} d\tau \left(\frac{-\partial f_{\vec{u}}}{\partial T} \right) = 1$$

$$\hookrightarrow \Delta C = -N(0) \left. \frac{\partial \Delta^2}{\partial T} \right|_{T_c} \quad (353.1)$$

Lembran Eq. (350.3): $\Delta(T) \approx 1,74 \Delta(0) \left(1 - \frac{T}{T_c} \right)^{1/2}$, p/ $T \approx T_c$

$$\hookrightarrow \frac{\partial \Delta^2}{\partial T} = - (1,74 \Delta(0))^2 \frac{1}{T_c} = (-1,74 \cdot 1,764)^2 \kappa_B^2 T_c$$

Eq. (350.2)

$$\hookrightarrow \Delta C = 9,4 N(0) \kappa_B^2 T_c \quad (353.2)$$

$$\text{Eqs. (352.3) e (353.2)} \rightarrow \frac{\Delta C}{C_N(T=T_c)} = \frac{9,4}{2\pi^2/3} = 1,43$$

(2) $T \ll T_c$;

$$\text{como } f_{\vec{u}} = \frac{1}{e^{\beta E_{\vec{u}}} + 1} \quad : \text{ Eq. (351.1)}$$

$$\hookrightarrow \frac{-\partial f_{\vec{u}}}{\partial E_{\vec{u}}} = \frac{1}{(e^{\beta E_{\vec{u}}} + 1)^2} \cdot \beta \cdot e^{-\beta E_{\vec{u}}} = \beta f_{\vec{u}} (1 - f_{\vec{u}})$$

$$\hookrightarrow \text{Eq. (352.1)}: C_s = 2\beta^2 \kappa_B \sum_{\vec{u}} f_{\vec{u}} (1 - f_{\vec{u}}) (E_{\vec{u}}^2 + \beta \Delta(T) \partial_{\beta} \Delta(T))$$

$$L> C_s \approx 2\beta^2 k_B N(0) \int_{-\infty}^{+\infty} d\epsilon f(\epsilon) (1 - f(\epsilon)) (\epsilon^2 + \beta \Delta_0 \beta \Delta)$$

$$\text{onde } \epsilon^2 = \zeta^2 + \Delta^2(\tau)$$

pr $T \ll T_c$, temos que:

$$f(\epsilon) \approx e^{-\beta \epsilon} \ll 1 \rightarrow 1 - f(\epsilon) \approx 1$$

$\Delta_0 \beta \Delta \approx 0$: veja Fig. (349.1)

$$\epsilon = \underbrace{(\zeta^2 + \Delta^2(\tau=0))}^{\Delta_0}{}^{1/2} = \Delta_0 \left(1 + \frac{\zeta^2}{\Delta_0^2}\right)^{1/2} \approx \Delta_0 \left(1 + \frac{\zeta^2}{2\Delta_0^2}\right)$$

$$L> C_s \approx 2\beta^2 k_B N(0) \Delta_0^2 e^{-\beta \Delta_0} \int_{-\infty}^{+\infty} d\zeta \exp\left(-\frac{\beta \zeta^2}{2\Delta_0}\right)$$

$$\sqrt{\frac{2\pi \Delta_0}{\beta}}$$

$$L> C_s = \sqrt{8\pi} N(0) \Delta_0 k_B \left(\frac{\Delta_0}{k_B T}\right)^{3/2} \exp\left(-\frac{\Delta_0}{k_B T}\right)$$

⊕ Eq. (352.2):

(353.2)

$$L> \frac{C_s}{\gamma T_c} = 1,34 \left(\frac{\Delta_0}{k_B T}\right)^{3/2} \exp\left(-\frac{\Delta_0}{k_B T}\right) :$$

: noton $C_s \propto e^{-\Delta_0/k_B T}$: ou \exists energia de gap finita no espectro de excitação

Teoria BCS e funções de Green.

Lembrar definição (244.1): função de Green de Matsubara:

$$G_{\alpha\beta}(\vec{n}, \tau; \vec{n}', \tau') = - \langle T_{\tau} (\psi_{\alpha}(\vec{n}, \tau) \psi_{\beta}^{\dagger}(\vec{n}', \tau')) \rangle$$

a teoria BCS pode ser formulada em termos de função de Green de Matsubara; nesse caso, é interessante definir:

$$G_{\alpha\alpha}(\vec{n}, \tau) = - \langle T_{\tau} (C_{\vec{n}\alpha}(\tau) C_{\vec{n}\alpha}^{\dagger}(0)) \rangle :$$

: função de Green normal

(354.1)

$$F(\vec{n}, \tau) = - \langle T_{\tau} (C_{\vec{n}\uparrow}^{\dagger}(\tau) C_{\vec{n}\uparrow}(0)) \rangle :$$

: " " " anômala

é interessante determinar as eqs. de movimento de

$$G_{\alpha\alpha}(\vec{n}, \tau) \text{ e } F_{\uparrow\uparrow}(\vec{n}, \tau);$$

$$G_{\uparrow\uparrow}(\vec{n}, \tau) = - \theta(\tau) \langle C_{\vec{n}\uparrow}(\tau) C_{\vec{n}\uparrow}^{\dagger}(0) \rangle + \theta(-\tau) \langle C_{\vec{n}\uparrow}^{\dagger}(0) C_{\vec{n}\uparrow}(\tau) \rangle$$

$$\hookrightarrow \hbar \partial_{\tau} G_{\uparrow\uparrow}(\vec{n}, \tau) =$$

$$= - \delta(\tau) \langle C_{\vec{n}\uparrow}(\tau) C_{\vec{n}\uparrow}^{\dagger}(0) + C_{\vec{n}\uparrow}^{\dagger}(0) C_{\vec{n}\uparrow}(\tau) \rangle$$

$$\langle \{ C_{\vec{n}\uparrow}(0); C_{\vec{n}\uparrow}^{\dagger}(0) \} \rangle = 1$$

$$= - \langle T_{\tau} [(\hbar \partial_{\tau} C_{\vec{n}\uparrow}(\tau)) C_{\vec{n}\uparrow}^{\dagger}(0)] \rangle \tag{354.2}$$

$$e^{k\tau/\hbar} [k, C_{\vec{n}\uparrow}] e^{-k\tau/\hbar} = (1)$$

• sobre a Eq. (354.2):

$$\text{como } C_{\vec{k}\alpha}(\tau) = e^{k\tau/\hbar} C_{\vec{k}\alpha} e^{-k\tau/\hbar}$$

$$\hookrightarrow \hbar \partial_\tau C_{\vec{k}\alpha}(\tau) = e^{k\tau/\hbar} (k C_{\vec{k}\alpha} - C_{\vec{k}\alpha} k) e^{-k\tau/\hbar}$$

$$= e^{k\tau/\hbar} [k, C_{\vec{k}\alpha}] e^{-k\tau/\hbar} \quad (354.3)$$

Eq. (334.3):

$$K_{BCS}^{MF} = E_0 + \sum_{\vec{k}} \gamma_{\vec{k}} (C_{\vec{k}\uparrow}^\dagger C_{\vec{k}\uparrow} + C_{-\vec{k}\downarrow}^\dagger C_{-\vec{k}\downarrow}) + \\ - \Delta_{\vec{k}} C_{\vec{k}\uparrow}^\dagger C_{-\vec{k}\downarrow}^\dagger - \Delta_{\vec{k}}^\dagger C_{-\vec{k}\downarrow} C_{\vec{k}\uparrow}$$

como (identidade): $[AB, C] = A\{B, C\} - \{A, C\}B$, vemos que:

$$[k, C_{\vec{k}\uparrow}] = -\gamma_{\vec{k}} C_{\vec{k}\uparrow}^\dagger + \Delta_{\vec{k}} C_{-\vec{k}\downarrow}^\dagger$$

(354.4)

$$[k, C_{-\vec{k}\downarrow}^\dagger] = +\gamma_{\vec{k}} C_{-\vec{k}\downarrow}^\dagger + \Delta_{\vec{k}}^\dagger C_{\vec{k}\uparrow}$$

pt $\kappa = \kappa_{BCS}^{MF}$, temos que, veja Eq. (354.4):

$$(I) = -\sum_{\vec{u}} C_{\vec{u}\uparrow}^\dagger(\tau) + \Delta_{\vec{u}} C_{-\vec{u}\downarrow}^\dagger(\tau)$$

$$\hookrightarrow \hbar \partial_\tau G_{\uparrow\uparrow}(\vec{u}, \tau) = -\delta(\tau) - \sum_{\vec{u}} G_{\uparrow\uparrow}(\vec{u}, \tau) + \Delta_{\vec{u}} F(\vec{u}, \tau) \quad (355.1)$$

similar:

$$F(\vec{u}, \tau) = -\theta(\tau) \langle C_{-\vec{u}\downarrow}^\dagger(\tau) C_{\vec{u}\uparrow}^\dagger(0) \rangle + \theta(-\tau) \langle C_{\vec{u}\uparrow}^\dagger(0) C_{-\vec{u}\downarrow}^\dagger(\tau) \rangle$$

$$\hookrightarrow \hbar \partial_\tau F(\vec{u}, \tau) = -\delta(\tau) \langle \{ C_{-\vec{u}\downarrow}^\dagger(\tau); C_{\vec{u}\uparrow}^\dagger(0) \} \rangle +$$

$$= 0$$

$$- \langle T_\tau [(\hbar \partial_\tau C_{-\vec{u}\downarrow}^\dagger(\tau)) C_{\vec{u}\uparrow}^\dagger(0)] \rangle$$

Eq. (354.4)

$$= \sum_{\vec{u}} C_{-\vec{u}\downarrow}^\dagger(\tau) + \Delta_{\vec{u}} C_{\vec{u}\uparrow}^\dagger(\tau)$$

$$\hookrightarrow \hbar \partial_\tau F(\vec{u}, \tau) = \sum_{\vec{u}} F(\vec{u}, \tau) + \Delta_{\vec{u}} G_{\uparrow\uparrow}(\vec{u}, \tau) \quad (355.2)$$

como:

$$G_{\alpha\alpha}(\vec{u}, \tau) = \frac{1}{\beta\hbar} \sum_n e^{-i\omega_n \tau} G_{\alpha\alpha}(\vec{u}, \omega_n) \quad ; \quad \text{Eq. (246.2)}$$

$$\stackrel{e}{=} \delta(\tau) = \frac{1}{\beta\hbar} \sum_n e^{-i\omega_n \tau}$$

temos que Eqs. (355.1) e (355.2) assumem a forma:

$$-i\hbar\omega_n G_{\uparrow\uparrow}(\vec{u}, \omega_n) = -1 - \sum_{\vec{u}} G_{\uparrow\uparrow}(\vec{u}, \omega_n) + \Delta_{\vec{u}} F(\vec{u}, \omega_n)$$

$$-i\hbar\omega_n F(\vec{u}, \omega_n) = \sum_{\vec{u}} F(\vec{u}, \omega_n) + \Delta_{\vec{u}} G_{\uparrow\uparrow}(\vec{u}, \omega_n)$$

(355.3)

verifica-se que (exercício):

$$G_{\uparrow\uparrow}(\vec{u}, \omega_n) = \frac{i\omega_n + \gamma\vec{u}}{(i\omega_n)^2 - (\gamma\vec{u}^2 + |\Delta\vec{u}|^2)}$$

: nota (356.1)

$$\underline{\underline{e}} \quad F(\vec{u}, \omega_n) = \frac{-\Delta\vec{u}^*}{(i\omega_n)^2 - (\gamma\vec{u}^2 + |\Delta\vec{u}|^2)}$$

poços!

de modo análogo, verifica-se que (exercício):

$$G_{\uparrow\uparrow}^{\dagger}(-\vec{u}, \omega) = -\langle T_{\tau} (C_{-\vec{u}\uparrow}^{\dagger}(\tau) C_{-\vec{u}\uparrow}(0)) \rangle =$$

$$= \frac{i\omega_n - \gamma\vec{u}}{(i\omega_n)^2 - E_{\vec{u}}^2}$$

(356.2)

$$\underline{\underline{e}} \quad \bar{F}(\vec{u}, \omega_n) = -\langle T_{\tau} (C_{\vec{u}\uparrow}(\tau) C_{-\vec{u}\uparrow}(0)) \rangle =$$

$$= \frac{-\Delta\vec{u}}{(i\omega_n)^2 - E_{\vec{u}}^2}$$

$$\text{onde } E_{\vec{u}} = \sqrt{\gamma\vec{u}^2 + |\Delta\vec{u}|^2}$$

vimos que, dada a estrutura de K_{BCS}^{HF} [Eq. (334.3)], é necessário introduzir as funções de Green anômalas $F(\vec{u}, \tau)$ e $\bar{F}(\vec{u}, \tau)$; nesse caso, é interessante considerar o spinon de Nambu (335.2) e introduzir uma função de Green matricial:

$$\text{como } \hat{\Psi}_{\vec{u}}^{\dagger}(\tau) = \begin{pmatrix} C_{\vec{u}\uparrow}(\tau) \\ C_{-\vec{u}\uparrow}^{\dagger}(\tau) \end{pmatrix} \quad : \text{ Eq. (334.3)}$$

$$\underline{\underline{e}} \quad \hat{\Psi}_{\vec{u}}^{\dagger}(0) = \begin{pmatrix} C_{\vec{u}\uparrow}(0) \\ C_{-\vec{u}\uparrow}^{\dagger}(0) \end{pmatrix}$$

$$\hookrightarrow \hat{G}(\vec{n}, \tau) = - \langle T_{\tau} (\hat{\psi}_{\vec{n}}(\tau) \hat{\psi}_{\vec{n}}^{\dagger}(0)) \rangle$$

$$= \begin{bmatrix} - \langle T_{\tau} (C_{\vec{n}\uparrow}(\tau) C_{\vec{n}\uparrow}^{\dagger}(0)) \rangle & - \langle T_{\tau} (C_{\vec{n}\uparrow}(\tau) C_{-\vec{n}\downarrow}(0)) \rangle \\ - \langle T_{\tau} (C_{-\vec{n}\downarrow}^{\dagger}(\tau) C_{\vec{n}\uparrow}^{\dagger}(0)) \rangle & - \langle T_{\tau} (C_{-\vec{n}\downarrow}^{\dagger}(\tau) C_{-\vec{n}\downarrow}(0)) \rangle \end{bmatrix}$$

(357.1)

⊕ Eqs. (356.1) e (356.2):

$$\hat{G}(\vec{n}, \omega_n) = \frac{1}{(i\omega_n)^2 - E_{\vec{n}}^2} \begin{pmatrix} i\omega_n + \gamma_{\vec{n}} & -\Delta_{\vec{n}} \\ -\Delta_{\vec{n}}^{\dagger} & i\omega_n - \gamma_{\vec{n}} \end{pmatrix} \quad (357.2)$$

• Eqs. (357.1) e (357.2): funções de Green de Nambu.

• Efeito Meissner,

Lembrar Eq. (300.2): campo magnético interior SC = 0:
diamagnetismo perfeito ~ 1 corrente SC na superfície da amostra;

Lembrar: op. densidade de corrente:

$$\vec{J}(\vec{r}) = \frac{1}{2} \sum_{i=1}^N \vec{v}_i \delta(\vec{r} - \vec{r}_i) + \delta(\vec{r} - \vec{r}_i) \vec{v}_i \quad ; \text{ Eq. (9.1)}$$

onde $\vec{v}_i = \vec{p}_i/m$

ou em termos ops. de campo $\hat{\psi}(\vec{r})$ e $\hat{\psi}^\dagger(\vec{r})$:

$$\vec{J}(\vec{r}) = \frac{\hbar}{2mi} \left(\hat{\psi}^\dagger(\vec{r}) \vec{\nabla} \hat{\psi}(\vec{r}) - (\vec{\nabla} \hat{\psi}^\dagger(\vec{r})) \hat{\psi}(\vec{r}) \right) \quad ; \text{ Eq. (38.2)}$$

na presença campo EM:

$$\vec{v} = \frac{1}{m} (\vec{p} - q\vec{A}) \quad ; \quad (358.1)$$

nesse caso, verifica-se que (exercício):

$$\vec{J}(\vec{r}) = \frac{\hbar}{2mi} \left(\hat{\psi}^\dagger(\vec{r}) \vec{\nabla} \hat{\psi}(\vec{r}) - (\vec{\nabla} \hat{\psi}^\dagger(\vec{r})) \hat{\psi}(\vec{r}) \right) - q \frac{1}{m} \vec{A}(\vec{r}) \hat{\psi}^\dagger(\vec{r}) \hat{\psi}(\vec{r})$$

$$\equiv \underbrace{\vec{J}^D(\vec{r})}_{\text{paramagnética}} \quad \equiv \underbrace{\vec{J}^A(\vec{r})}_{\text{diamagnética}}$$

(358.2)

pr sistema eletrônico, a densidade de corrente elétrica:

$$\vec{J}_e = -e\vec{J} \quad ; \quad q = -e; \quad e > 0 \quad (358.3)$$

temos que:

$$\vec{J}_e(\vec{n}, \vec{A}) = -e \vec{J}^\nabla(\vec{n}, \vec{A}) - \frac{e^2}{m} \rho(\vec{n}) \vec{A}(\vec{n}) \quad (359.1)$$

$$\equiv \underbrace{\vec{J}_e^\nabla(\vec{n}, \vec{A})}_{\text{}} + \underbrace{\vec{J}_e^A(\vec{n}, \vec{A})}_{\text{}}$$

metal normal: $J_e^\nabla \ll J_e^A \rightarrow$ pequena resposta diamagnética

SC: hipótese London and London:

$$\vec{J}_e^\nabla(\vec{n}, \vec{A}) = \vec{J}_e^\nabla(\vec{n}, 0) = 0 \sim \text{rigidez função de onda SC} \quad (359.2)$$

$$\hookrightarrow \vec{J}_e(\vec{n}, \vec{A}) = -\frac{e^2}{m} \rho(\vec{n}) \vec{A}(\vec{n});$$

de fato $\rho(\vec{n}) \rightarrow \rho_s(\vec{n})$: densidade elétrons ~ SC;
c/ $\rho_s < \rho$

$$\hookrightarrow \vec{J}_e(\vec{n}, \vec{A}) = -\frac{e^2}{m} \rho_s(\vec{n}) \vec{A}(\vec{n}) : \text{eq. de London} \quad (359.3)$$

considerar: gauge de Coulomb: $\vec{\nabla} \cdot \vec{A} = 0$;

$$\text{Eq. de Maxwell: } \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_e$$

$$\hookrightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = -\nabla^2 \vec{A} = \mu_0 \vec{J}_e$$

hipótese: $\rho_s(\vec{n}) = \rho_s = \text{cte}$; temos que:

$$\nabla^2 \vec{A} = \frac{\mu_0 e^2 \rho_s \vec{A}}{m} \equiv \frac{1}{\lambda_L^2} \vec{A}; \quad \lambda_L = \sqrt{\frac{m}{\mu_0 e^2 \rho_s}} : \quad (359.4)$$

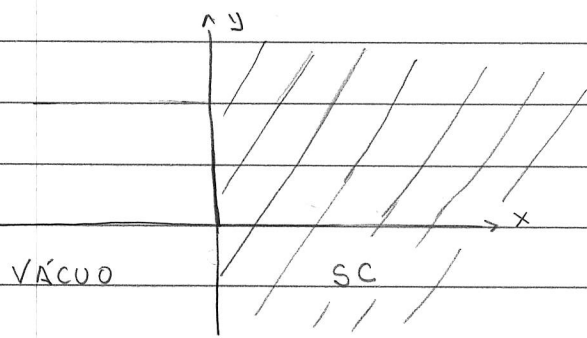
: comprimento de London



· Considerar sistema:

$$\textcircled{+} \vec{B} = B(x) \hat{z}$$

$$= \Theta(-x) B \hat{z}$$



nesse caso, verifica-se que solução Eq. (359.1)
(veja P. 18.9; Bnuus):

$$B(x) = \begin{cases} B, & x < 0 \\ B e^{-x/\lambda_L}, & x > 0 \end{cases} \quad (360.1)$$

Como $\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B}$

$$\hookrightarrow \vec{J}(x) = + \frac{1}{\mu_0 \lambda_L} B e^{-x/\lambda_L} \Theta(x) \hat{y} : \text{ corrente SC}$$

na superfície

Obs.: verifica-se que $\lambda_L(T=0) \sim 500 - 10^4 \text{ \AA}$.

Obs.: veja Sec. 32.2, Kittelson p/ procedimento alternativo
 \hookrightarrow eq. de London:

notas: London and London:

- teoria fenomenológica;
- baseada na hipótese (359.2);

\hookrightarrow próxima etapa: verificar Eq. (359.2)
(p/ detalhes, veja Sec. 18.5.3, Bnuus);

· em resposta linear, a densidade de corrente elétrica
e o campo elétrico aplicado estão relacionados via o
(tensor) de condutividade:

$$J_e^\alpha(\vec{n}, t) = \int_{\beta} \int d\vec{n}' dt' \sigma^{\alpha\beta}(\vec{n}, t; \vec{n}', t') E^\beta(\vec{n}', t'); \alpha, \beta = x, y, z \quad (361.1)$$

considerando $\sigma^{\alpha\beta}(\vec{n}, t; \vec{n}', t') = \sigma^{\alpha\beta}(\vec{n}, \vec{n}', t-t')$, em resposta linear, verifico-se que (veja Sec. 6.2, Bruns):

$$\sigma^{\alpha\beta}(\vec{n}, \vec{n}', \omega) = \frac{ie^2}{\omega} D_{\alpha\beta}^R(\vec{n}, \vec{n}', \omega) + \frac{e^2}{i\omega m} \rho(\vec{n}) \delta(\vec{n}-\vec{n}') \delta_{\alpha\beta} \quad (361.2)$$

onde $D_{\alpha\beta}^R(\vec{n}, \vec{n}', t-t') = -i\theta(t-t') \langle [J_0^\alpha(\vec{n}, t); J_0^\beta(\vec{n}', t')]] \rangle \quad (361.3)$

$$\vec{J}_0(\vec{n}, t) = \vec{J}^\nabla(\vec{n}, t) + \frac{e}{m} \vec{A}_0(\vec{n}) \rho(\vec{n})$$

$$\vec{A}(\vec{n}) = \vec{A}_0(\vec{n}) + \vec{A}_{ext}(\vec{n})$$

↑ potenciale velon p/ sistema em equilibrio

p/ sistema homogêneo, a função de correlação corrente-corrente correspondente a (361.3) e dada por:

$$D_{\alpha\beta}(\vec{q}, \nu_n) = -\frac{1}{v_\beta} \langle J_0^\alpha(\vec{q}, \nu_n) J_0^\beta(-\vec{q}, -\nu_n) \rangle \quad (361.4)$$

considerar: $\vec{A}_0(\vec{n}) = 0$;
como

$$\vec{J}^\nabla(\vec{q}) = \vec{J}(\vec{q}) = \frac{1}{2m} \sum_{\sigma} \sum_{\vec{u}} (2\vec{u} + \vec{q}) C_{\vec{u}\sigma}^{\dagger} C_{\vec{u}+\vec{q},\sigma} \quad (361.5)$$

$$\hookrightarrow \vec{J}(\vec{q}, \nu_n) = \frac{1}{2m v} \sum_{\sigma} \sum_{\vec{u}} \int_0^{\beta} d\tau e^{i\nu_n \tau} (2\vec{u} + \vec{q}) C_{\vec{u}\sigma}^{\dagger}(\tau) C_{\vec{u}+\vec{q},\sigma}(\tau)$$

notar: determinação de função de correlação corrente-corrente (361.4) é similar

" " " " " densidade-densidade (224.5)!

↳ Eq. (361.4) pode ser escrita em termos $g(\vec{q}, \omega)$!

notar: op. densidade de corrente (361.5) pode ser escrito como:

$$\vec{J}(\vec{q}) = \frac{1}{2m} \sum_{\vec{u}} (2\vec{u} + \vec{q}) (C_{\vec{u}+\vec{q}\uparrow}^\dagger + C_{\vec{u}+\vec{q}\downarrow}^\dagger + C_{\vec{u}\uparrow} + C_{\vec{u}\downarrow})$$

$$- C_{-\vec{u}-\vec{q}\uparrow}^\dagger - C_{-\vec{u}\downarrow}$$

$$C_{-\vec{u}\uparrow} + C_{-\vec{u}-\vec{q}\downarrow}^\dagger - \delta_{\vec{q},0}$$

$$= \frac{1}{2m} \sum_{\vec{u}} (2\vec{u} + \vec{q}) (C_{\vec{u}\uparrow}^\dagger + C_{\vec{u}\downarrow}^\dagger + C_{-\vec{u}\uparrow} + C_{-\vec{u}-\vec{q}\downarrow}^\dagger) - \frac{1}{2m} \delta_{\vec{q},0} \sum_{\vec{u}} 2\vec{u}$$

= 0

$$(C_{\vec{u}\uparrow}^\dagger \quad C_{-\vec{u}\downarrow}) \begin{pmatrix} C_{\vec{u}\downarrow}^\dagger \\ C_{-\vec{u}-\vec{q}\uparrow}^\dagger \end{pmatrix}$$

$$= \hat{\Psi}_{\vec{u}}^\dagger \hat{\Psi}_{\vec{u}+\vec{q}} : \text{spinones de Nambu (334.3)}$$

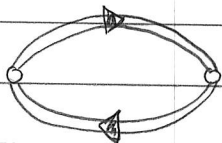
$$\text{↳ } \vec{J}(\vec{q}) = \frac{1}{2m} \sum_{\vec{u}} (2\vec{u} + \vec{q}) \hat{\Psi}_{\vec{u}}^\dagger \hat{\Psi}_{\vec{u}+\vec{q}} \quad (362.1)$$

notar similaridade entre Eqs. (361.5) e (362.1)

↳ determinação (361.4) p/ spinones $\hat{\Psi}_{\vec{u}}$ é similar

" " " " férmions $C_{\vec{u}\sigma}$!

em particular, em menor ordem, temos que (veja Eq. (36.13), Bnuss):

$$D_{\alpha\beta}(\vec{q}, \nu_n) = \text{Diagram} \quad (363.1)$$


onde Diagram : função de Green de Nambu (357.2) $\hat{G}(\vec{u}, \omega_n)$;

temos que:

$$D_{\alpha\beta}(\vec{q}, \nu_n) = \frac{1}{v\beta} \sum_{\vec{u}} \sum_m \frac{1}{2m} (2\vec{u} + \vec{q})_\alpha \frac{1}{2m} (2\vec{u} + \vec{q})_\beta +$$

$$+ T_n \left(\hat{G}(\vec{u}, \omega_m) \hat{G}(\vec{u} + \vec{q}, \omega_m + \nu_n) \right) :$$

↑
sob índices internos

(363.2)

: comparan c/ Eq. (283.1)

· p/ campos EM estáticos e no limite $\vec{q} \rightarrow 0$, verifica-se que:

$$D_{\alpha\beta}(0,0) = -\frac{2}{3} \frac{1}{v} \sum_{\vec{u}} \sum_{\sigma} \sum_{\vec{r}} \left(-\frac{\partial f_{\vec{u}}}{\partial E_{\vec{u}}} \right) \delta_{\alpha\beta}$$

(363.3)

onde $f_{\vec{u}} = \frac{1}{e^{\beta E_{\vec{u}}} + 1}$

incluindo o termo diamagnético, temos que:

$$\vec{J}_e = -\frac{e^2}{m} (\rho - \delta\rho) \vec{\Lambda},$$

(363.4)

onde $\delta\rho \equiv \frac{2}{3} \frac{1}{v} \sum_{\vec{u}} \sum_{\sigma} \sum_{\vec{r}} \left(-\frac{\partial f_{\vec{u}}}{\partial E_{\vec{u}}} \right)$: densidade

quasipartículas \notin condensado $\rightarrow \vec{J}_e^{\nabla} \neq 0!$

• Eqs. (359.3) e (363.4) $\rightarrow p_s = p - \delta p$!

• c'interessante determinar δp p/ dois casos:

(1) $T = 0$,

$$\text{como: } \delta p = -\frac{2}{3} \int_{-\infty}^{+\infty} d\tau N(\tau) \tau N'(\tau) \left(\sqrt{\tau^2 + \Lambda^2} \right)$$

e p/ $T = 0$:

$$N'(\tau) \left(\sqrt{\tau^2 + \Lambda^2} \right) = \delta \left(\sqrt{\tau^2 + \Lambda^2} \right) \rightarrow \delta p = 0 \rightarrow p_s(T=0) = p$$

(2) $T > T_c$,

temos que:

$$\delta p = \frac{2}{3} \int_{-\infty}^{+\infty} d\tau \underbrace{\partial \tau (\tau N(\tau))}_{N(\tau) + \tau N'(\tau)} f(\tau)$$

Eq. (352.3) ?

$$\approx \underbrace{\frac{2}{3} \int_{-\infty}^0 d\tau N(\tau) f(\tau)}_p + \underbrace{\frac{2}{3} \int_{-\infty}^0 d\tau \tau N'(\tau) f(\tau)}_{=0}$$

$\hookrightarrow p_s(T > T_c) = 0$.