F 415 – Mecânica Geral II – Lista 2

Center of mass and conservation theorems:

01. P.9.3, Marion:
Find the center of mass of a uniformly solid cone of base diameter $2a$ and height h and a solid hemisphere of radius a where the two bases are touching.

02. P.9.4, Marion:
Find the center of mass of a uniform wire that subtends an arc θ if the radius of the circular arc is a, as shown in Figure 9-A.

03. P.9.5, Marion:
The center of gravity of a system of particles is the point about which external gravitational forces exert no net torque. For a uniform gravitational force, show that the center of gravity is identical to the center of mass for the system of particles.

04. P.9.9, Marion:
A projectile is fired at an angle of $45°$ with initial kinetic energy E_0. At the top of its trajectory, the projectile explodes with additional energy E_0, into two fragments. One fragment of mass m_1 travels straight down. What is the velocity (magnitude and direction) of the second fragment of mass m_2 and the velocity of the first? What is the ratio of m_1/m_2 when m_1 is a maximum?

P.05. P.9.13 and P.9.26, Marion:

a) Even though the total force on a system of particles (Equation 9.9) is zero, the net torque may not be zero. Show that the net torque has the same value in any coordinate system.

b) The force of attraction between two particles is given by

$$\mathbf{f}_{12} = k\left(\mathbf{r}_2 - \mathbf{r}_1\right) - \frac{kr}{v_0}\left(\dot{\mathbf{r}}_2 - \dot{\mathbf{r}}_1\right)$$

where k is a constant, v_0 is a constant velocity, and $r = |\mathbf{r}_2 - \mathbf{r}_1|$. Calculate the internal torque for the system; why does this quantity not vanish? Is the system conservative?

Elastic and inelastic collisions:

06. P.9.23, Marion:
A particle of mass m_1 and velocity u_1 collides with a particle of mass m_2 at rest. The two particles stick together. What fraction of the original kinetic energy is lost in the collision?

07. P.9.28, Marion and P.4.15, Symon:
A particle of mass m_1 elastically collides with a particle of mass m_2 at rest. What is the maximum fraction of kinetic energy loss for m_1? Describe the reaction.

P.08. P.9.32, Marion:
A particle of mass m and velocity u_1 makes a head-on collision with another particle of mass $2m$ at rest. If the coefficient of restitution is such to make the loss of total kinetic energy a maximum, what are the velocities v_1 and v_2 after the collision?

09. P.9.34, Marion:
A billiard ball of initial velocity u_1 collides with another billiard ball (same mass) initially at rest. The first ball moves off at $\psi = 45°$. For an elastic collision, what are the velocities of both balls after the collision? At what LAB angle does the second ball emerge?

10. P.9.36, Marion:
In an elastic collision of two particles with masses m_1 and m_2, the initial velocities are u_1 and $u_2 = \alpha u_1$. If the initial kinetic energies of the two particles are equal, find the conditions on u_1/u_2 and m_1/m_2 such that m_1 is at rest after the collision. Examine both cases for the sign of α.

P.11. P.9.40, Marion:
A particle of mass m_1 and velocity u_1 strikes head-on a particle of mass m_2 at rest. The coefficient of restitution is ϵ. Particle m_2 is tied to a point a distance a away as shown in Figure 9-H. Find the velocity (magnitude and direction) of m_1 and m_2 after the collision.

12. P.9.41, Marion:
A rubber ball is dropped from rest onto a linoleum floor a distance h_1 away. The rubber ball bounces up to a height h_2. What is the coefficient of restitution? What fraction of the original kinetic energy is lost in terms of ϵ?

13. P.9.43, Marion:
A proton (mass m) of kinetic energy T_0 collides with a helium nucleus (mass $4m$) at rest. Find the recoil angle of the helium if $\psi = 45°$ and the inelastic collision has $Q = -T_0/6$.

P.14. P.4.16, Symon:
A cloud-chamber picture shows the track of an incident particle which makes a collision and is scattered through an angle ψ_1. The track of the target particle makes an angle ψ_2 with the direction of the incident particle. Assuming that the collision was elastic and that the target particle was initially at rest, find the ratio m_1/m_2 of the two masses. (Assume small velocities so that the classical expressions for energy and momentum may be used.)

15. P.4.17, Symon:
A proton of mass m_1 collides elastically with an unknown nucleus in a bubble chamber and is scattered through an angle ψ_1. The ratio P_{1f}/P_{1i} is determined from the curvature of its initial and final tracks. Find the mass m_2 of the target nucleus. How might it be possible to determine whether the collision was indeed elastic?

16. P.4.21, Symon:
A particle of mass m_1, momentum p_{1i} collides elastically with a particle of mass m_2, momentum p_{2i} going in the opposite direction. If m_1 leaves the collision at an angle ψ_1 with its original course, find its final momentum.

P.17. P.4.25, Symon:
A billiard ball sliding on a frictionless table strikes an identical stationary ball. The balls leave the collision at angles $\pm\psi$ with the original direction of motion. Show that after the collision the balls must have a rotational energy equal to $1 - 0.5\cos^{-2}\psi$ of the initial kinetic energy, assuming that no energy is dissipated in friction.

Central force: scattering:

P.18. P.9.46, Marion:
With the aid of Eq. (9.123), calculate the differential cross section $\sigma(\theta)$ and the total cross section σ_t, for the elastic scattering of a particle from an impenetrable sphere; the potential is given by

$$U(r) = \begin{cases} 0, & r > a \\ \infty, & r < a. \end{cases}$$

19. P.9.49, Marion:
Consider the case of Rutherford scattering in the event that $m_1 \ll m_2$. Obtain an expression of the differential cross section in the CM system that is correct to first order in the quantity m_1/m_2. Compare this result with Eq. (9.140).

P.20. P.3.71, Symon and P.9.50, Marion:
Show that for a repulsive central force

$$F(r) = \frac{k}{r^3}, \qquad k > 0,$$

the possible orbits have the form discuss in problem P.3.50, Symon. Determine β in terms of k, the energy E, angular momentum l, and the mass m of the incident particle. Show that the differential cross–section $\sigma(\theta)$ is given by

$$\sigma(\theta) = \frac{k\pi^2(\pi - \theta)}{mu_0^2\theta^2(2\pi - \theta)^2 \sin\theta},$$

where u_0 is the initial velocity of the particle.

21. P.9.51, Marion:.
It is found experimentally that in the elastic scattering of neutrons by protons ($m_n \approx m_p$) at relatively low energies, the energy distribution of the recoiling protons in the LAB system is constant up to a maximum energy, which is the energy of the incident neutrons. What is the angular distribution of the scattering in the CM system?
Hint: Energy distribution $= dN/dT_i$.

22. P.3.34, Goldstein:
Consider a truncated repulsive Coulomb potemial defined as

$$U(r) = \begin{cases} k/a, & r \le a \\ k/r, & r > a. \end{cases}$$

For a particle of total energy $E > k/a$, obtain expressions for the scattering angle θ as a function of s/s_0, where s_0 is the impact parameter for which the periapsis occurs at the point $r = a$. (The formulas can be given in closed form bm they are not simple!) Make a numerical plot of θ versus s/s_0

for the special case $E = 2k/a$. What can you deduce about the angular scattering cross section from the dependence of θ on s/s_0 for this particular case?

23. Calculate the differential cross–section $\sigma(\theta)$ and the total cross–section σ_t for the elastic scattering of a particle from the repulsive potential ("soft–sphere")

$$U(r) = \begin{cases} U_0, & r < a \\ 0, & r > a \end{cases} \qquad\qquad U_0 > 0.$$

Consider the cases $E < U_0$ and $E > U_0$.

24. P.1.16 Fetter and Walecka:
A uniform beam of particles with energy E is scattered by a repulsive central potential $V(r) = \gamma/r^2$. Derive the differential elastic cross section

$$\frac{d\sigma}{d\Omega} = \frac{\gamma\pi^2}{E\sin\theta}\frac{\pi-\theta}{\theta^2\,(2\pi-\theta)^2}.$$

Sketch carefully the angular dependence. Discuss the total cross section. What happens if the potential is attractive, that is, $\gamma < 0$?

P.25. P.1.18 Fetter and Walecka:
A particle with large impact parameter b is slightly deflected from a uniform trajectory by a central potential $V(r)$.

a) In the impulse approximation, the (small) integrated deflecting force is evaluated along the original straight-line trajectory. Use this approximation to derive the expression

$$\theta \approx \frac{2b}{mv_\infty^2}\left|\int_b^\infty \frac{dr}{\sqrt{r^2-b^2}}\frac{dV}{dr}\right|$$

for the (small) deflection angle.

b) If $V(r) = \gamma/r^n$ with positive n and γ, find the differential cross section for small-angle scattering and discuss its behavior as $\theta \to 0$. Show that the answer reproduces the known results for $n = 1$ (Sec. 5) and 2 (Prob. 1.16). Is σ_T defined for any n?

c) If $V(r) = \gamma\exp(-\lambda r)$, show that b varies approximately like $(1/\lambda)\ln(1/\theta)$. Hence obtain the approximate form of the differential cross section. Is σ_T well defined?

d) In quantum mechanics, the small-angle part of σ_T is finite whenever $r^2V(r) \to 0$ as $r \to 0$. Discuss briefly why the classical behavior is different.