F 415 – Mecânica Geral II – Lista 3

01. P.10.1, Marion:
Calculate the centrifugal acceleration, due to Earth's rotation, on a particle on the surface of Earth at the equator. Compare this result with the gravitational acceleration. Compute also the centrifugal acceleration due to the motion of Earth about the Sun and justify the remark made in the text that this acceleration may be neglected compared with the acceleration caused by axial rotation.

02. P.10.2, Marion:
An automobile drag racer drives a car with acceleration a and instantaneous velocity v. The tires (of radius r_0) are not slipping. Find which point on the tire has the greatest acceleration relative to the ground. What is this acceleration?

P.03. P.10.8, Marion:
If a particle is projected vertically upward to a height h above a point on Earth's surface at a northern latitude λ, show that it strikes the ground at a point $(4\omega/3)(\cos\lambda)\sqrt{8h^3/g}$ to the west. (Neglect air resistance, and consider only small vertical heights.)

P.04. P.10.9 and P.10.10, Marion:
A projectile is fired due east from a point on the surface of Earth at a northern latitude λ with a velocity of magnitude V_0 and at an angle of inclination to the horizontal of α.

a) Show that the lateral deflection when the projectile strikes Earth is

$$d = \frac{4V_0^3}{g^2}\omega\sin\lambda\sin^2\alpha\cos\alpha,$$

where ω is the rotation frequency of Earth.

b) If the range of the projectile is R_0' for the case $\omega = 0$, show that the change of range due to the rotation of Earth is

$$\Delta R' = \sqrt{\frac{2R_0'^3}{g}}\omega\cos\lambda\left(\cot^{1/2}\alpha - \frac{1}{3}\tan^{3/2}\alpha\right).$$

05. P.10.12, Marion:
Show that the small angular deviation ϵ of a plumb line from the true vertical (i.e., toward the center of Earth) at a point on Earth's surface at a latitude λ is

$$\epsilon = \frac{R\omega^2 \sin\lambda\cos\lambda}{g_0 - R\omega^2\cos^2\lambda},$$

where R is the radius of Earth. What is the value (in seconds of arc) of the maximum deviation? Note that the entire denominator in the answer is actually the effective g and g_0 denotes the pure gravitational component.

06. P.10.15, Marion:
Consider a particle moving in a potential $U(r)$. Rewrite the Lagrangian in terms of a coordinate system in uniform rotation with respect to an inertial frame. Calculate the Hamiltonian and determine whether $H = E$. Is H a constant of the motion? If E is not a constant of motion, why isn't it? The expression for the Hamiltonian thus obtained is the standard formula $(1/2)mv^2 + U$ plus an additional term. Show that the extra term is the centrifugal potential energy. Use the Lagrangian you obtained to reproduce the equations of motion given in Equation 10.25 (without the second and third terms).

07. P.7.11, Symon:
A gyroscope consists of a wheel of radius r, all of whose mass is located on the rim. The gyroscope is rotating with angular velocity $\dot{\theta}$ about its axis, which is horizontal and is fixed relative to the earth's surface. We choose a coordinate system at rest relative to the earth whose z-axis coincides with the gyroscope axis and whose origin lies at the center of the wheel. The angular velocity o of the earth lies in the xz-plane, making an angle a with the gyroscope axis.
Find the $x-$, $y-$, and $z-$components of the torque $\mathbf{N}$ about the origin, due to the coriolis force in the xyz-coordinate system, acting on a mass m on the rim of the gyroscope wheel whose polar coordinates in the $x - y$ plane are r and θ. Use this result to show that the total coriolis torque on the gyroscope, if the wheel has a mass M, is

$$\mathbf{N} = -\frac{1}{2}Mr^2\omega\dot{\theta}\sin\alpha\hat{y}.$$

This equation is the basis for the operation of the gyrocompass.

P.08. P.7.13, Symon and P.9.23, Taylor:
A particle moves in the xy–plane under the action of a force

$$F(r) = -kr, \qquad k > 0,$$

directed toward the origin. Introduce a coordinate system rotating about the z–axis with angular velocity ω chosen so that the centrifugal force just cancels the force F. Find the possible motions of the particle by solving the equations of motion in this coordinate system. Show that the general solution is an ellipse.
Hint: $\eta = x + iy$.

09. P.7.14, Symon:
A ball of mass m slides without friction on a horizontal plane at the surface of the earth. Show that it moves like the bob of a Foucault pendulum provided it remains near the point of tangency. Find its frequency of oscillation. Assume the earth is a sphere.

P.10. P.9.20, Taylor:
Consider a frictionless puck on a horizontal turntable that is rotating counterclockwise with angular velocity Ω.

a) Write down Newton's second law for the coordinates x and y of the puck as seen by me standing on the turntable. (Be sure to include the centrifugal and Coriolis forces, but ignore the earth's rotation.)

b) Solve the two equations by the trick of writing $\eta = x + iy$ and guessing a solution of the form $\eta = e^{-i\alpha t}$. Write down the general solution.

c) At time $t = 0$, the position of the puck is $\mathbf{r}_0 = (x_0, 0)$ and its velocity $\mathbf{v}_0 = (v_{x0}, v_{y0})$. Show that

$$x(t) = +(x_0 + v_{x0}t)\cos\Omega t + (v_{y0} + \Omega x_0)t\sin\Omega t$$

$$y(t) = -(x_0 + v_{x0}t)\sin\Omega t + (v_{y0} + \Omega x_0)t\cos\Omega t.$$

11. P.9.22, Taylor:
If a negative charge $-q$ (an electron, for example) in an elliptical orbit around a fixed positive charge Q is subjected to a weak uniform magnetic field $\mathbf{B}$, the effect of B is to make the ellipse precess slowly - an effect known as Larmor precession. To prove this, write down the equation of motion of the negative charge in the field of Q and $\mathbf{B}$. Now rewrite it for a frame rotating with angular velocity $\vec{\omega}$. [Remember that this changes both d^2r/dt^2 and dr/dt.] Show that by suitable choice of $\vec{\omega}$ you can arrange that the terms involving $\mathbf{V}_r$ cancel out, but that you are left with one term involving $\mathbf{B} \times (\mathbf{B} \times \mathbf{r})$. If B is weak enough this term can certainly be neglected. Show that in this case the orbit in the rotating frame is an ellipse (or hyperbola). Describe the appearance of the ellipse as seen in the original nonrotating frame.

P.12. P.2.6, Fetter:
Extend the calculation of the falling particle on the rotating earth by solving the equations of motion to second order in ω. Show that the time to fall a vertical distance h is increased by $(1/2)\omega^2(2h/g)^{3/2}\sin^2\theta$ and that the particle is deflected toward the equator by an amount $(\omega^2 h^2/3g)\sin 2\theta$.