Disentangling superconducting and magnetic orders in NaFe$_{1-x}$Ni$_x$As using muon spin rotation


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Muon spin rotation and relaxation studies have been performed on a “111” family of iron-based superconductors, NaFe$_{1-x}$Ni$_x$As, using single crystalline samples with Ni concentrations $x = 0, 0.4, 0.6, 1.0, 1.3, and 1.5\%$. Static magnetic order was characterized by obtaining the temperature and doping dependences of the local ordered moment size and the volume fraction of the magnetically ordered regions. For $x = 0$ and 0.4\%, a transition to a nearly-homogeneous long range magnetically ordered state is observed, while for $x \gtrsim 0.4\%$ magnetic order becomes more disordered and is completely suppressed for $x = 1.5\%$. The magnetic volume fraction continuously decreases with increasing $x$. Development of superconductivity in the full volume is inferred from Meissner shielding results for $x \gtrsim 0.4\%$. The combination of magnetic and superconducting volumes implies that a spatially-overlapping coexistence of magnetism and superconductivity spans a large region of the $T$-$x$ phase diagram for NaFe$_{1-x}$Ni$_x$As. A strong reduction of both the ordered moment size and the volume fraction is observed below the superconducting $T_C$ for $x = 0.6, 1.0, and 1.3\%$, in contrast to other iron pnictides in which one of these two parameters exhibits a reduction below $T_C$, but not both. The suppression of magnetic order is further enhanced with increased Ni doping, leading to a reentrant nonmagnetic state below $T_C$ for $x = 1.3\%$. The reentrant behavior indicates an interplay between antiferromagnetism and superconductivity involving competition for the same electrons. These observations are consistent with the sign-changing $s^\pm$ superconducting state, which is expected to appear on the verge of microscopic coexistence and phase separation with magnetism. We also present a universal linear relationship between the local ordered moment size and the antiferromagnetic ordering temperature $T_N$ across a variety of iron-based superconductors. We argue that this linear relationship is consistent with an itinerant-electron approach, in which Fermi surface nesting drives antiferromagnetic ordering. In studies of superconducting properties, we find that the $T = 0$ limit of superfluid density follows the linear trend observed in underdoped cuprates when plotted against $T_C$. This paper also includes a detailed theoretical prediction of the muon stopping sites and provides comparisons with experimental results.

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I. INTRODUCTION

Iron-based high temperature superconductors (Fe-HTS) are materials exhibiting unconventional superconductivity that arise from parent compounds with static antiferromagnetic (AFM) order [1–3]. One of the grand challenges in understanding the behavior of these systems is determining the physical mechanism responsible for superconductivity. Essential information on the nature of superconductivity in strongly correlated electron systems can be deduced by investigating their phase diagrams as well as the superconducting (SC) gap structure. In the parent compound of many Fe-HTS, a spin density wave forms with spins ordered antiparallel to each other along one Fe-Fe axis and parallel to each other along the orthog-
nal Fe-Fe bond direction [1,2,4]. Carrier doping, isovalent chemical substitution, or application of pressure to the parent system suppresses magnetic order and begets a SC dome [5]. In addition to magnetism and superconductivity, Fe-HTS exhibit a tetragonal-to-orthorhombic structural distortion at a temperature $T_d$ that precedes or occurs concurrently with the magnetic phase transition at temperature $T_N$ [1,3,4,6,7]. The prominent in-plane anisotropy in resistivity along orthogonal axes in the paramagnetic (PM) orthorhombic state is associated with an electronic nematic order parameter that triggers the orthorhombic distortion of the crystal [8,9]. The aforementioned orders are found in close proximity with each other. AFM and SC orders homogeneously coexist in several Fe-HTS, such as in BaFe$_{2-x}$Co$_x$As$_2$ [10,11], BaFe$_{2-x}$Ni$_x$As$_2$ [12], Ba$_{1-x}$K$_x$Fe$_2$As$_2$ [13], and NaFe$_{1-x}$Co$_x$As [14]. In these systems, the ordered magnetic moment size and nematic order parameter smoothly decrease as the temperature is lowered below $T_C$, corroborating the fact that superconductivity and magnetic long range order compete for the same electrons [15]. However, other studies [16,17] have detected the mutual exclusion of these two order parameters, i.e., they exhibit macroscopic phase separation in different parts of the sample. Characterizing common features of the complex interplay among magnetic, nematic, and SC orders in various Fe-HTS is essential for elucidating the microscopic pairing mechanism in Fe-HTS and other unconventional superconductors.

One of the major experimental challenges in teasing apart AFM and SC orders is that individual experimental probes have limited ranges of sensitivity to magnetism and/or superconductivity. For instance, neutron scattering and magnetic susceptibility measurements can only reveal volume-integrated information about the magnetic and SC features of the specimens. At present, no individual experimental probe can unambiguously address the issue of whether the coexistence of AFM and SC orders directly overlap in real space or if the specimen undergoes macroscopic phase separation between two phases. In an attempt to clear this experimental hurdle, a detailed multiple-probe investigation was recently conducted on BaFe$_{2-x}$Ni$_x$As$_2$ involving muon spin rotation ($\mu$SR), scanning tunneling microscopy (STM), Mössbauer spectroscopy, neutron scattering, and specific heat measurements [12]. The results from this study offer convincing evidence that the AFM and SC phases in BaFe$_{2-x}$Ni$_x$As$_2$ almost completely overlap in real space and both phases compete for the same electrons. The question of whether a similar style of phase coexistence exists in other families of Fe-HTS remains unclear. In this paper, we present a detailed $\mu$SR investigation in context with recent susceptibility and neutron scattering measurements to gain a deeper understanding of the interplay between AFM and SC orders in NaFe$_{1-x}$Ni$_x$As, a member of the “111” family of Fe-HTS.

Recent neutron scattering experiments on NaFe$_{1-x}$Ni$_x$As show that the neutron magnetic order parameter is diminished below $T_C$ [18], which was interpreted as the reduction of the magnetic moment below $T_C$. Using the volume sensitive $\mu$SR technique, we demonstrate in single crystalline samples of NaFe$_{1-x}$Ni$_x$As, with $x = 0.6, 1.0, 1.3$, and $1.5\%$, that the reduction of magnetic intensity is due to a strong reduction of both the ordered moment and the magnetic volume fraction below $T_C$. The debilitating effect of superconductivity on magnetism intensifies as the doping level $x$ increases, leading to a reentrant non-AFM state below $T_C$ for $x = 1.3\%$. These results suggest an interesting scenario, in which the degree of competition between AFM and SC may be itself intrinsically inhomogeneous, varying as a function of position in the sample. Moreover, we establish a robust linear dependence between the ordered moment and the AFM ordering temperature $T_N$ for various Fe-HTS, which is consistent with a model of itinerant magnetism in Fe-HTS.

This paper is organized as follows: Section II describes the preparation and handling of the specimens, $\mu$SR experimental setup. Experimental zero-field $\mu$SR results are shown in Sec. III A and compared with neutron scattering results in Sec. III B. A discussion of these results is presented in Sec. IV. Section IV introduces a muon stopping site simulation performed to account for the multiple internal magnetic fields observed in the zero-field $\mu$SR spectra. Knowledge of the muon site locations enables the ordered moment size to be determined from the observed precession frequency. Section V describes a universal linear relation between the ordered moment size and $T_N$. A theoretical discussion of this result using a model of antiferromagnetism in Fe-HTS parent compounds based on an itinerant electron picture is also presented in this section. Section VI shows $\mu$SR measurements under a transverse external field on superconducting NaFe$_{1-x}$Ni$_x$As and demonstrates that a linear relationship between the superfluid density and $T_C$ is observed in NaFe$_{1-x}$Ni$_x$As and other high-$T_C$ cuprate superconductors. These results are summarized in the concluding Sec. VII. Appendix A describes detailed methods and results of the internal field simulation. A calculation for the universal scaling of the ordered moment size and ordering temperature based on a two-band model is presented in Appendix B. Finally, Appendix C provides a derivation of the superconducting gap symmetry from the temperature dependence of the penetration depth.

II. EXPERIMENTAL METHODS

Pristine single-crystal specimens of NaFe$_{1-x}$Ni$_x$As with $x = 0.0, 0.4, 0.6, 0.8, 1.0, 1.3$, and $1.5\%$ were prepared using the self-flux technique in accordance with Ref. [21], with each crystallite measuring about $1 \times 1 \times 0.2$ mm$^3$. Zero-field cooling DC susceptibility measurements were performed on these samples in an applied field of 5 Oe in the basal $a\bar{b}$ plane down to 3 K and the results are shown in Fig. 1(a). These measurements indicate that NaFe$_{1-x}$Ni$_x$As exhibits bulk superconductivity with full SC shielding fraction for the range $x = 0.6 \sim 1.5\%$, with a maximum $T_C \sim 17$ K achieved for $x = 1.5\%$. A phase diagram summarizing the structural, magnetic, and SC transitions is shown in Fig. 1(b), which is reminiscent of the electronic phase diagrams of NaFe$_{1-x}$Co$_x$As [22,23] and NaFe$_{1-x}$Cu$_x$As [24]. For clarity, collinear AFM spin structure of the undoped compound NaFeAs is also shown in Fig. 1(c).

Since NaFe$_{1-x}$Ni$_x$As is highly air and moisture sensitive, the crystallites were tightly encased in packets of Kapton film inside an Ar-filled glovebox. Each crystallite was aligned so that the crystallographic $c$ axis was oriented normal to the film packet, without any preferred alignment of the basal $ab$ plane. For each doping concentration, packets containing a few large
crystal specimens were mounted on an ultralow background sample holder using aluminum tape.

In a $\mu$SR experiment, positive muons implanted into a specimen serve as extremely sensitive local probes to simultaneously measure small internal magnetic fields and ordered magnetic volume fractions. Therefore, we can ascertain the temperature and doping evolution of the magnetic volume fraction and ordered moment separately, unlike reciprocal-space techniques such as neutron scattering. Time differential $\mu$SR measurements were performed using the Los Alamos Meson Physics Facility (LAMPF) spectrometer with a helium gas-flow cryostat at the M20 surface muon beamline (500 MeV) of TRIUMF in Vancouver, Canada and using the general purpose surface-muon instrument (GPS) with a standard low-background veto setup at the $\pi M3$ beam line of the Paul Scherrer Institute in Villigen, Switzerland. A continuous beam of 100% spin polarized muons was implanted into the sample and the time dependence of the ensemble muon polarization was collected at temperatures between 2 K and 70 K. The muon beam momentum was parallel to the crystal $c$ axis. By applying magnetic fields to the muon beam before the sample, the ensemble muon spin prior to implantation can be oriented parallel or perpendicular to the beam direction. See Refs. [25–27] for further details on the $\mu$SR experimental technique. The $\mu$SR spectra were analyzed in the time domain using least-squares optimization routines from the MUSRFIT software suite [28].

III. MAGNETISM IN NaFe$_{1-x}$Ni$_x$As

A. Zero field $\mu$SR results

The observed $\mu$SR time spectra (muon ensemble polarization) of $x = 0, 0.4, 0.6, 1.0, 1.3$, and 1.5% in zero applied field (ZF-$\mu$SR) are shown in Fig. 2. In these measurements, the initial muon spin polarization is in the $a$-$b$ plane of the crystals, and the time spectra were obtained using up and down positron counters. At high temperatures, only a very faint depolarization of the $\mu$SR signal is observed. This weak relaxation mostly originates from the interaction of the muon spin with randomly oriented nuclear magnetic moments. Upon cooling, the relaxation of the $\mu$SR signal increases due to the proliferation of Fe-spin correlations.

For $x = 0$ and 0.4% samples, three distinct precession frequencies occur in the $\mu$SR spectra, which emanate from three magnetically inequivalent muon stopping sites in NaFe$_{1-x}$Ni$_x$As, in agreement with our stopping site calculations presented in Sec. IV. No coherent oscillations are present in the $x \geq 0.6$% spectra shown in Figs. 2(e)–2(f), even at the lowest measured temperature, as only a rapidly relaxing signal is observed. The fast depolarization of the $\mu$SR signal (without oscillations) arises from a broad distribution of static internal magnetic fields, which has been confirmed using longitudinal field (LF)-$\mu$SR experiments. These measurements reveal that the muon spin relaxation is substantially suppressed at modest longitudinal external fields between 25 and 50 mT (of the order of internal quasistatic fields), suggesting an inhomogeneous magnetic state in the samples with $x = 0.6, 1.0$, and 1.3%. The ZF-$\mu$SR time spectra for the $x = 1.3$% compound shown in Fig. 2(g) demonstrate magnetic ordering between 14 K and 17 K. Below 14 K, magnetic order vanishes and the specimen only exhibits bulk superconductivity. Interestingly, a similar reentrance to a nonmagnetic state was observed in BaFe$_{1-x}$Co$_x$As$_2$ by neutron diffraction [15]. In the following, we present how the magnetic properties of NaFe$_{1-x}$Ni$_x$As evolve with temperature and doping.
All of the ZF-μSR spectra were fit to the following phenomenological model:

\[
P_{ZF}(t) = F \left( \sum_{j=1}^{3} \left( f_j \cos(2\pi \nu_j t + \phi) e^{-\lambda_j t} \right) + f_L e^{-\lambda_L t} \right)
+ (1-F) \left( \frac{1}{3} + \frac{2}{3} (1-\lambda t - (\sigma t)^2) e^{-\lambda t - \frac{1}{2}(\sigma t)^2} \right).
\]  

(1)

The model in (1) consists of an anisotropic magnetic contribution characterized by an oscillating “transverse” component and a slowly relaxing “longitudinal” component. The longitudinal component arises due to the parallel orientation of the muon spin polarization and the local magnetic field. In polycrystalline samples with randomly oriented fields, this leads to a so-called “one-third tail” with \(f_L = \frac{1}{3}\). For single crystals, \(f_j\) varies between zero and unity as the orientation between field and polarization changes from being parallel to perpendicular. In addition to the magnetically ordered contribution, there is a PM signal component characterized by the densely distributed network of nuclear dipolar moments \(\sigma\) and dilute electronic moments with random orientations \(\lambda\) [29]. The temperature-dependent magnetic ordering fraction \(0 \leq F \leq 1\) governs the trade-off between magnetically-ordered and PM behaviors.

Shown in Figs. 3(a) and 3(b) are the temperature dependences of the precession frequencies observed in the \(x = 0\) and 0.4% samples. For the undoped and \(x = 0.4\%\) systems, there are three distinct frequencies that share the same relationship \(\nu(T) = \nu(0)(1 - \left(\frac{T}{T_N}\right)^{\alpha})\), which are indicated by solid lines. In the parent system, a sharp step-like increase of \(\nu(T)\) is observed below \(T_N \approx 42\) K, which may be a signature of a first-order phase transition, although further experiments are needed to establish the character of the transition. This feature is absent in the \(x = 0.4\%\) sample, which could be due to disorder effects introduced by Ni impurities [30]. Similar ZF-μSR experiments were also performed by using positron counters located in the forward and backward directions with respect to the muon beam direction. Interestingly, the two fast frequencies are absent in the non-spin-rotated spectra for \(x = 0\) and 0.4%. If we associate each frequency to a different muon stopping site, these results suggest that the magnetic field directions at the high-field stopping sites are oriented along the \(c\) axis of the crystal. This feature is consistent with dipolar field simulations on muon stopping sites presented in Sec. IV.

We define the static magnetic order parameter \(M = \sqrt{(2\pi \nu)^2 + \lambda_T^2}\) to track the temperature and doping dependence of magnetism, where \(\nu\) is the maximum precession frequency and \(\lambda_T\) is the relaxation rate corresponding to \(\nu\). As defined, \(M\) takes into account both homogeneous (well-defined precession frequency \(\nu\)) and inhomogeneous contributions (rapid early-time relaxation \(\lambda_T\)) to the signal. Therefore, the magnetic transition temperature \(T_N\) corresponds to the onset of \(M\).

The temperature and doping evolution of the magnetic fraction \(F\) and magnetic order parameter \(M\) are shown in Fig. 4. The relative decrease in \(M\) below \(T_c\) is more pronounced with increased doping, as seen in Fig. 4(b). Indeed, the \(x = 1.3\%\) sample exhibits reentrant behavior in which the low-temperature state becomes non-AFM below \(T_c\) within experimental uncertainty. The temperature evolution of the magnetically ordered fraction \(F\) is shown in Fig. 4(a). \(F\) shows a sharp increase below \(T_N\) while the onset of SC causes \(F\) to decrease when cooled below \(T_c\). With higher doping, a stronger reduction of \(F\) is observed below \(T_c\). For the \(x = 1.3\%\) system, magnetic order is completely destroyed and the system loses long-range AFM order below 14 K. The evolution of magnetic and SC volume fractions across temperature and doping is presented in Fig. 5.

**B. Comparison with elastic neutron scattering**

Muon spin rotation and neutron scattering are two powerful experimental techniques for studying magnetic materials. As described in Ref. [31], these methods have distinct features...
that make them complementary probes of magnetism. For instance, both techniques are sensitive to different time scales of dynamic fluctuations. However, since we are probing static magnetic order in NaFe$_{1-x}$Ni$_x$As, both μSR and elastic neutron scattering should yield consistent results. In addition, as a volume-integrating probe in reciprocal space, neutron scattering is sensitive to both the ordered moment and its volume fraction, but these two contributions cannot be separated from the measured scattered intensity. However, μSR enables independent measurements of both the volume fraction and the ordered moment size, unlike neutron scattering and other bulk probes.

As described in Sec. I, elastic neutron scattering experiments on NaFe$_{1-x}$Ni$_x$As reveal that the neutron magnetic order parameter is diminished below $T_C$ [18], which was interpreted as the reduction of the magnetic moment below $T_C$. However, the suppression of magnetic order below $T_C$ observed in neutron diffraction cannot be unambiguously attributed to a reduction of the magnetic moment, as pointed out in the previous paragraph. A comparison between the neutron magnetic order parameter and the magnetic strength $\mathcal{M}^2 F$ from our μSR studies is shown in Fig. 6 for the $x = 1.0$ and 1.3% systems. From our ZF-μSR results in Fig. 4, we conclude that the suppression of magnetic ordering is due to a decrease in both the ordered volume fraction and the moment size.

C. Discussion

Our results offer strong evidence that both the ordered moment and fraction are partially or fully suppressed below $T_C$. The strong suppression of the magnetism below the onset of superconductivity was also observed in the “122” and “11” families of Fe-HTS: BaFe$_{2-x}$Co$_x$As$_2$ [10,11] (where reentrance of
measurements presented in Fig. indicating a reentrant non-AFM phase. (b) Doping and temperature evolution of the magnetic fraction from ZF-µSR. Orange diamonds indicate $T_N$ and the dashed curve is a guide to the eye. Observe the bend in the curve near $x \approx 0.013$ indicating a reentrant non-AFM phase. (b) Doping and temperature evolution of the SC volume fraction from magnetic susceptibility measurements presented in Fig. 1(a). Green circles indicate $T_c$ and the dashed curve is a guide to the eye. (c) Summary of magnetic and SC volume fractions. $V_{SC}$ is the SC volume fraction, $V_{Mag, Max}$ is the maximum value of $F$ for each doping, and $V_{Mag}(T \ll T_c)$ is $F$ at temperatures much less than $T_c$.

FIG. 5. Magnetic and SC volume fractions in NaFe$_{1-x}$Ni$_x$As. (a) Doping and temperature evolution of the magnetic fraction $V_{Mag} = F$ from ZF-µSR. Orange diamonds indicate $T_N$ and the dashed curve is a guide to the eye. Observe the bend in the curve near $x \approx 0.013$ indicating a reentrant non-AFM phase. (b) Doping and temperature evolution of the SC volume fraction from magnetic susceptibility measurements presented in Fig. 1(a). Green circles indicate $T_c$ and the dashed curve is a guide to the eye. (c) Summary of magnetic and SC volume fractions. $V_{SC}$ is the SC volume fraction, $V_{Mag, Max}$ is the maximum value of $F$ for each doping, and $V_{Mag}(T \ll T_c)$ is $F$ at temperatures much less than $T_c$.

The magnetic strength $M^2F$ from µSR measurements is plotted in red and the Bragg peak intensity from neutron scattering is shown in blue.

FIG. 6. Comparison of µSR and elastic neutron scattering measurements [18] for the (a) $x = 1.0\%$ and (b) $x = 1.3\%$ compounds. The magnetic strength $M^2F$ from µSR measurements is plotted in red and the Bragg peak intensity from neutron scattering is shown in blue.

Conducted [14]. In NaFe$_{1-x}$Co$_x$As, only the ordered moment was diminished below $T_c$. This reduction is much smaller than that of NaFe$_{1-x}$Ni$_x$As. We also noticed that the observed muon precession frequencies in NaFe$_{1-x}$Ni$_x$As and NaFe$_{1-x}$Co$_x$As follow different patterns. We have found two high frequencies and one low frequency for NaFe$_{1-x}$Ni$_x$As, while one high frequency and two low frequencies were reported for NaFe$_{1-x}$Co$_x$As. Although a definitive explanation for these differences requires further studies, it is clear that Ni and Co dopants perturb the electronic structure of NaFeAs in different ways.

Both the ordered moment and magnetic fraction decrease below $T_c$ for FeSe (which becomes magnetic under hydrostatic pressure). Results in the present investigation of NaFe$_{1-x}$Ni$_x$As are similar to the FeSe case. Itinerant AFM and SC orders are generally expected to compete strongly for the same electronic states; this behavior can be captured within a simple Ginzburg-Landau free energy for the AFM and SC order parameters, $M$ and $\Delta$, respectively (in the context of Fe-HTS, see for instance Refs. [15,33–35]):

$$F = \frac{a_m}{2} M^2 + \frac{a_m}{4} M^4 + \frac{a_s}{2} \Delta^2 + \frac{a_s}{4} \Delta^4 + \frac{g}{2} M^2 \Delta^2$$

The degree of competition between these two orders is encoded in the combination of coefficients $g = \gamma/\sqrt{a_m a_s}$. If the
competition is too strong \((g > 1)\), these two orders are macroscopically phase separated and do not coexist microscopically. On the other hand, if the competition is weak \((g < 1)\), they can establish a coexistence phase in which both order parameters are simultaneously nonzero at the same position. In a homogeneous system, the first scenario is manifested by a reduction of the AFM volume fraction \(F\) below \(T_C\) without a change in the size of the magnetic moment. Conversely, the second scenario is manifested by a reduction of the magnetic moment below \(T_C\) without any variation in the volume fraction. Interestingly, we observe both signatures in NaFe\(_{1-x}\)Ni\(_x\)As. Although a detailed theoretical analysis is beyond the scope of this paper, this suggests that the parameter \(g\) itself may be inhomogeneous and change as a function of the position in the sample. If Cooper pairs were to form an unconventional sign-changing \(s^\pm\) state \([36,37]\), it was argued \([15]\) that the system would be at the verge of phase separation and microscopic coexistence, i.e., \(g \approx 1\). In this case, local inhomogeneity could locally alter the value of \(g\) in a significant manner \([38]\).

IV. INTERNAL FIELD SIMULATIONS AT MUON STOPPING SITES

To investigate the effect of Ni dopants on the magnetism in NaFe\(_{1-x}\)Ni\(_x\)As, we numerically simulate the behavior of the muon in the magnetic environment of NaFe\(_{1-x}\)Ni\(_x\)As. In low-temperature \(\mu\)SR experiments, the incident muons thermalize with the lattice and are implanted at interstitial locations referred to as stopping sites. Muon implantation sites in Fe-HTS have been successfully identified using a succession of increasingly accurate theoretical calculations. Early studies were based on the analysis of the local minima of the unperturbed electrostatic potential within either the simple Thomas Fermi or a full density functional theory (DFT) approach. This strategy was specifically followed for the “1111” \([17,39–41]\) and the “11” \([32,42]\) classes of Fe-HTS. In addition, similar calculations were performed on selected “122” \([43,44]\) materials and other systems \([45,46]\). Recently, and exclusively for the “1111” family of Fe-HTS, the effect of the muon on the lattice was captured within a supercell DFT impurity calculation by considering force and energy relaxations of possible muon implantation sites \([47,48]\).

The \textit{ab initio} search often identifies clusters of sites. This is true also in the simple unperturbed potential method that fails in insulators such as fluorides \([49]\) but yields a correct first approximation in the metallic pnictides owing to the electron screening of the muon charge. In this case, the clusters are defined as the portion of the unit cell volume enclosed by the isosurface corresponding to the muon ground state energy. More accurate stopping site determination would require an impurity DFT approach. Under this methodology, clusters of candidate muon sites are generally found with smaller \textit{intra}cluster and larger \textit{inter}cluster energy barriers.

Since muon localization is a metastable epithehal kinetic process that cannot be described by a mere minimum energy criterion, all of these methods uncover clusters of candidate locations that may not directly correspond to observed muon sites. In principle, the true muon fate could be simulated by robust \textit{ab initio} path integral molecular dynamics \([50,51]\). At present, these techniques are computationally prohibitive for impurity calculations on complex structures such as Fe-HTS. Therefore, the most feasible method for muon site determination in Fe-HTS involves comparing the experimental and calculated local field values at candidate sites.

A. Candidate muon stopping sites in “111” systems

To determine plausible muon implantation sites in the “111” family of Fe-HTS, we employed DFT methods that account for local crystal deformations and electronic band structure perturbations due to the implanted muons. In particular, muon stopping site calculations in NaFeAs and NaFe\(_{1-x}\)Ni\(_x\)As were performed using spin-polarized DFT with plane wave expansions of the Kohn-Sham orbitals at both atomic and interstitial sites. The generalized gradient approximation (GGA) was applied for the exchange correlation functional within the Perdew-Burke-Ernzerhof (PBE) formalism \([52,53]\). Finally, the core wave function was approximated using the projector augmented wave (PAW) method \([54]\). The plane wave and charge density cutoffs were chosen to be 120 and 1080 Ry, respectively. More details on the muon site determination procedure are found in Appendices A 1 and A 2.

This initial search with DFT methods uncovered five plausible muon sites in NaFeAs, which are also assumed to be valid for lowly doped NaFe\(_{1-x}\)Ni\(_x\)As. The five candidate sites were grouped into two clusters based on relative calculated energies. Since the muon is treated as a classical particle within DFT, corrections due to its light mass can be included by taking into account the spread of the muon wave function in the double Born-Oppenheimer (DBO) approximation method \([45]\). As described in Appendix A 2, we invoked the DBO approximation to examine the relative stability of the five candidate sites. From our stability checks, we concluded that only two of the three muon sites in the low-energy cluster proved to be stable. In addition, both muon sites in the high-energy cluster relax into each other, suggesting that the muon is likely delocalized between these two sites, which are also in close proximity to each other. As a result, we have determined three plausible muon stopping locations (two stopping sites and a delocalized high-energy stopping position) in NaFeAs, which are listed in Table I.

B. Dipolar internal field simulations on NaFe\(_{1-x}\)Ni, As

With the muon stopping sites determined, magnetic dipolar field simulations were performed by simulating the NaFeAs as an array of localized magnetic dipoles. The two dipolar contributions considered in the internal field simulation are localized electronic moments from AFM-ordered Fe atoms and random nuclear dipolar moments from all atoms. Nonmagnetic nickel impurities were randomly substituted into the Fe sites on the host NaFeAs lattice to generate NaFe\(_{1-x}\)Ni\(_x\)As. By performing a vector sum of the array of (static) electronic and random nuclear dipolar moments, the internal field distribution was numerically simulated for all points in the crystal.

To capture the stochastic fluctuations in the random nuclear moment directions and Ni site substitutions, the internal field distribution was simulated by performing 10 000 independent trials of dipolar sums for each muon site as a function of Ni concentration \(x\). Although simulated results can be
TABLE I. Summary of dipolar field simulations in NaFeAs using muon stopping site positions obtained from DFT methods. Similar stopping sites are expected for lightly-doped NaFe\textsubscript{1\textminus\textsubscript{x}}, Ni, As. Stopping site positions are given in fractional coordinates. The highest frequency from the dipolar field simulations is in close agreement with the experimental results assuming a static ordered Fe moment of 0.175(3)\textmu B.

<table>
<thead>
<tr>
<th>Site</th>
<th>Site Position</th>
<th>Simulated (\nu) (MHz)</th>
<th>Experimental (\nu) (MHz)</th>
<th>Simulated (\theta) (°)</th>
<th>Experimental (\theta) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.000, 0.875, 0.100)</td>
<td>10.987(49)</td>
<td>10.981(27)</td>
<td>42.1(5)</td>
<td>0(10)</td>
</tr>
<tr>
<td>2</td>
<td>(0.100, 0.750, 0.100)</td>
<td>7.839(30)</td>
<td>10.685(57)</td>
<td>31.1(6)</td>
<td>0(10)</td>
</tr>
<tr>
<td>3\textsuperscript{a}</td>
<td>(0.500, 0.250, 0.600)</td>
<td>2.090(21)</td>
<td>0.864(06)</td>
<td>0.6(4)</td>
<td>18(10)</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Stopping site positions given in fractional coordinates.
\textsuperscript{b}Muon precession frequency from dipolar field simulations.
\textsuperscript{c}Average acute angle between the simulated field direction and the \(c\) axis.
\textsuperscript{d}Average acute angle between the local field direction and the \(c\) axis. The local field direction was estimated from ZF-\(\mu\)SR measurements with the muon spins rotated in orthogonal directions.
\textsuperscript{e}From stability analysis of calculated muon sites, the third frequency is likely attributed to a stopping site delocalized across sites D and E. See Appendix A 2 for more details. For simplicity, we list here the simulated results calculated for site class E from Table IV in Appendix A 2.

C. Discussion of computational results

The main results of our computational investigation are summarized in Table I. Our stopping site calculations and stability analysis reveal three plausible muon stopping sites in NaFeAs. This is consistent with the observation that there are three precession frequencies in the ZF-\(\mu\)SR spectra in lightly-doped NaFe\textsubscript{1\textminus\textsubscript{x}}, Ni, As. The calculated precession frequencies are listed in Table I, along with the extrapolated frequencies from power law fits of the frequencies from \(\mu\)SR found in Fig. 3(a). Moreover, our simulations show that the mean local fields at the two high-field sites make an acute angle of approximately 36° with the crystal \(c\) axis, implying that the strong fields at these sites are preferentially aligned with the \(c\) axis. This is consistent with our experimental observation that the high frequency oscillations have noticeable amplitudes when the initial muon spin is not aligned with the \(c\) axis (i.e., in the spin-rotated configuration), as shown in Fig. 2(a). Differences in the simulated and experimentally-obtained angles \(\theta\) suggest that the true muon sites are likely a small displacement from the ones listed in Table I.

The doping evolution of the simulated precession frequencies are shown in Fig. 3(c). Comparisons of the simulated and observed frequencies for Site 1 in our dipolar field simulations enabled us to estimate the ordered moment size of the Fe atoms in NaFeAs to be \(\mu_{Fe} = 0.175(3)\textmu B\). The difference between the simulated and experimental frequencies for the second and third sites suggests that quantum correlations (e.g., contact hyperfine fields) contribute to the internal field, which are not included in the dipolar model. In addition, the presence of Ni dopants can perturb the ordering of Fe moments, which was not included in the simulation. Nonetheless, our computational investigation provides a physical interpretation of the frequencies observed in the ZF-\(\mu\)SR spectra and corroborates the model for the magnetic ordering in (1).

The ordered moment size in a variety of Fe-HTS has been explored experimentally using \(\mu\)SR, neutron scattering, and Mössbauer measurements [55]. The reported variations of the ordered Fe moments of the same specimen is a testament to the differences in sensitivity across these three probes of the local moment. Table II shows a comparison of the ordered moment size of representative systems from the various classes of Fe-HTS. The estimate from our present investigation in NaFeAs, \(\mu_{Fe} = 0.175(3)\textmu B\), is consistent with the moment sizes reported from neutron scattering [56] and Mössbauer spectroscopy [57].

V. LINEAR RELATIONSHIP BETWEEN ORDERED MOMENT AND \(T_N\)

Despite the notable differences in the experimentally-measured ordered moment sizes across different Fe-HTS [55], there are some notable relationships between the ordered moment and other material parameters. The observation of a linear relationship between the muon precession frequency \(\nu\) and the magnetic ordering temperature \(T_N\) was initially noted in Fe-HTS [58].

TABLE II. Comparison of the low-temperature Fe ordered magnetic moments in selected Fe-HTS. All magnetic moments are given in units of \(\textmu_B\).

<table>
<thead>
<tr>
<th>Fe-HTS</th>
<th>(\mu)SR</th>
<th>Neutron Scattering</th>
<th>Mössbauer\textsuperscript{a}</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaFeAs</td>
<td>0.175(3)</td>
<td>0.17(2) [56]</td>
<td>0.158(2) [57]</td>
</tr>
<tr>
<td>BaFe\textsubscript{2}As\textsubscript{2}</td>
<td>0.75(5) [59]</td>
<td>0.87(3) [60]</td>
<td>0.36(4) [61]</td>
</tr>
<tr>
<td>LaFeAsO\textsubscript{2}\textsuperscript{b}</td>
<td>0.68(2) [39]</td>
<td>0.63(1) [62]</td>
<td>0.34(1) [63]</td>
</tr>
<tr>
<td>FeSe\textsubscript{0.98}</td>
<td>0.20(5)\textsuperscript{b}</td>
<td>Undetected\textsuperscript{d}</td>
<td>0.18(1)\textsuperscript{f}</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Ordered moment extrapolated from measured low-temperature hyperfine field using the scaling relation 15 T/\(\mu\) [55,58].
\textsuperscript{b}Measured at \(T \approx 25\) K, above the magnetic ordering temperature of La.
\textsuperscript{c}Taken under pressure \(p = 2.4\) GPa, from Ref. [32].
\textsuperscript{d}No magnetic Bragg peaks observed under pressure according to Ref. [32].
\textsuperscript{e}FeSe under pressure \(p = 2.5\) GPa, from Ref. [64].
by Uemura in Ref. [65] for the “122” and “1111” classes of Fe-HTS. 

Separate linear trends in “122” and “1111” classes of Fe-HTS were discovered from Mössbauer spectroscopy relating the internal hyperfine field and the orthorhombic lattice distortion [10]. The different proportionality constants between the two classes of Fe-HTS have been ascribed to the critical dynamics of the structural and magnetic transitions [66, 67]. In this section, we make use of muon stopping site calculations to extend the investigation of the linear trend between the ordered moment size and $T_N$ from μSR results.

A. Linear trends from μSR results

Figure 7 depicts the correlation between the low temperature precession frequency $\nu(T \rightarrow 0)$ and the magnetic ordering temperature $T_N$ for a variety of Fe-HTS, including NaFe$_{1-x}$Ni$_x$As from the present investigation. Note that for the SC samples, the values of the precession frequencies, extrapolated to $T = 0$ are taken. Since the precession frequency $\nu$ is proportional to the local magnetic field at the muon site, $\nu$ is proportional to the ordered moment size, and therefore the magnetization. $\nu$ also depends on the distance between the muon stopping site and the dominant ordered moment (Fe atoms). Remarkably, despite the differences in chemical composition and crystal structure across the various main families of Fe-HTS, (which influence the number and location of the muon stopping sites) a linear trend between $\nu(T \rightarrow 0)$ and $T_N$ appears to persist. This suggests that the mechanism responsible for driving the magnetic ordering may be similar across different crystal structures and dopant atoms. Under this linear scaling relationship, there is an increase of 0.244(3) MHz/K between the $\nu(T \rightarrow 0)$ and $T_N$. Since muon stopping sites have been calculated for a variety of Fe-HTS, we can compare the ordered Fe moment sizes directly. The ordered magnetic moment on the Fe atom can be calculated as the scaling factor necessary for matching the precession frequencies from dipolar field simulations against experimental results. Shown in Fig. 8 is a comparison between the ordered magnetic moment of the Fe atoms and $T_N$. A linear model was fit to the data, revealing that the magnetic moment $\mu_{Fe}$ scales with $T_N$ as 0.0062(6) $\mu_B$/K across these families of Fe-HTS.

B. Discussion

The universal linear relationship between the $T = 0$ sublattice magnetization $M$ and the experimentally observed $T_N$ provides important insight into the nature of the magnetic state. Within an itinerant mean-field approach, in which AFM is driven by Fermi surface nesting, $M \propto T_N$ follows naturally whenever the Fermi surfaces are perfectly nested; this is the same relationship between the SC gap function and $T_C$ that appears in BCS theory. Since the Fermi pockets of the iron pnictides are not perfectly nested, it is important to verify whether $M \propto T_N$ applies more generally in itinerant antiferromagnets. To investigate this issue, we consider a widely studied toy model consisting of one circular hole pocket located at the center of the Brillouin zone and one elliptical electron pocket shifted from the center by the AFM ordering vector [15,33–35,73,74]. The mismatch between the Fermi pockets is tuned by two parameters: $\delta_3$, which characterizes the ellipticity of the electron pocket, and $\delta_6$, which describes the difference between the areas of the Fermi pockets (and is therefore indirectly
VI. SUPERCONDUCTIVITY IN NaFe$_{1-x}$Ni,As

$\mu$SR experiments performed with an applied field transverse to the initial muon ensemble spin, called TF-$\mu$SR, allow determination of the magnetic field penetration depth $\lambda$, which is one of the fundamental parameters of a superconductor [16]. (Recall that $\lambda$ is related to the superconducting carrier density $n$, through $\lambda^{-2} = \mu_0e^2n_e/m^*$, where $m^*$ is the effective mass and $\mu_0$ is the vacuum permeability). Most importantly, the temperature dependence of $\lambda$ is particularly sensitive to the presence of SC nodes. In a fully gapped superconductor, $\Delta \lambda^{-2}(T) = \lambda^{-2}(0) - \lambda^{-2}(T)$ vanishes exponentially at low $T$ and decays as a power of $T$ in a nodal SC. As a result, the $\mu$SR technique is a powerful tool to measure $\lambda$ in type II superconductors. Specifically, $\mu$SR experiments in the vortex state of a type II superconductor allow the determination of $\lambda$ in the bulk of the sample, in contrast to many techniques that probe $\lambda$ only near the surface.

To understand the temperature evolution of $\lambda$, it is informative to study the symmetry and structure of the SC gap. Significant experimental and theoretical efforts have focused on this issue in Fe-HTS [1,2]. However, there is no consensus on a universal gap structure and the relevance for the particular gap symmetry for Fe-HTS, which are the first noncuprate materials exhibiting superconductivity at relatively high temperatures.

In contrast to cuprates, where the SC gap symmetry is universal, the gap symmetry and/or structure of the Fe-HTS varies across different systems. For instance, nodeless isotropic gap distributions were observed in optimally doped Ba$_{1-x}$K$_x$Fe$_2$As$_2$ [75,76], Ba$_{1-x}$Rb$_x$Fe$_2$As$_2$ [29], and BaFe$_{2−x}$Ni$_x$As$_2$ [77] as well as in BaFe$_{2−x}$Co$_x$As$_2$ [78], K$_x$Fe$_{2−x}$Se$_2$ [79], and FeTe$_{1−x}$Se$_x$ [80,81]. Signatures of nodal SC gaps were reported in LaFePO [82], LiFeP [83], KFe$_2$As$_2$ [84], BaFe$_2$(As$_{1−x}$P$_x$)$_2$ [85–88], BaFe$_{2−x}$Ru$_x$As$_2$ [89], and FeSe [90] as well as in overdoped Ba$_{1-x}$K$_x$Fe$_2$As$_2$ [91] and in optimally doped Ba$_{1-x}$Rb$_x$Fe$_2$As$_2$ under pressure [92]. Therefore, it is fruitful to extend the study of the SC gap symmetry to other Fe-based materials, specifically the “111” family of Fe-HTS. In this section, we present and discuss TF-$\mu$SR results on the $x = 1.3\%$ sample in the superconducting state.

A. TF-$\mu$SR results

Shown in Fig. 10 are the TF-$\mu$SR time spectra on the $x = 1.3\%$ system, measured in an applied field of 300 Oe above (40 K) and below (2 K) $T_C \approx 15$ K. Above $T_C$, the oscillations show a small relaxation due to random local fields from nuclear magnetic moments. As the sample is field cooled below $T_C$, the relaxation steadily increases due to the presence of a nonuniform local field distribution as a result of the formation of a flux-line lattice (FLL) in the SC state. The TF-$\mu$SR spectra were analyzed using the following functional form:

$$P_{TF}(t) = F_{imn} \cos(2\pi vt + \phi) \times \exp\left\{-\frac{1}{2}\left(\sigma_{imn}^2 + \sigma_{SC}^2\right)t^2\right\}e^{-\lambda_m t}.$$

The defining parameters in (2) are the precession frequency $v$, the relaxation rates $\sigma_{imn}$ and $\sigma_{SC}$ characterizing the damping due to the formation of FLL in the SC state and the nuclear magnetic dipolar contribution, respectively, and an exponential relaxation rate for field-induced magnetism $\lambda_m$ [93]. The model in (2) has been previously used [29,94] for Fe-HTS in the presence of dilute or fast fluctuating electronic moments related to doping). The case $\delta_0 = \delta_1 = 0$ corresponds to perfect nesting, giving $T_N = (\frac{\lambda}{\lambda_0})M_0 \approx 0.567 M_0$.

Following Refs. [15,33,34], we compute not only $T_N$ as a function of the parameters $\delta_0$ and $\delta_1$, but also the magnetization $M$ at $T = 0$. We focus on the regime in which the AFM transition is second order (see Appendix B for details). As shown in Fig. 9, $M$ monotonically increases with increasing $T_N$. Each curve corresponds to a fixed value of $\delta_1$ and continuously changing values of $\delta_0$. Interestingly, when $\delta_0$ is not too large, $T_N$ and $M$ follow a nearly linear relationship over a wide parameter range, which is consistent with previous works [15,33]. Although a quantitative comparison with experimental findings must account for band structure details of different compounds, the results of this simple model are qualitatively consistent with the experimental observations, suggesting that nesting plays an important role in driving the AFM instability.

FIG. 9. Theoretical calculations of the ordered moment $M$ at $T = 0$ versus the AFM critical temperature $T_N$. The ellipticity of the electron band, given by $\delta_1$, is fixed for each curve, whereas the parameter $\delta_0$, corresponding to doping, is varied continuously. $M_0$ is the AFM moment $M$ at $T = 0$ when the hole and electron pockets are perfectly nested.

FIG. 10. TF-$\mu$SR polarization on field-cooled $x = 1.3\%$ in an applied field of 300 G. Time spectra in the $x = 1.3\%$ system with an applied transverse field at $T = 40$ K (PM) and $T = 2$ K (SC).
such as vortex lattice disorder consistent with field-induced magnetism. Other phenomena to elucidate the origin of the PM shift, however the effects are ascribed to the SC and magnetic orders, respectively. The solid line in field of 300 G. (a) Temperature dependence of the field shift from 40 K in Fig. 11(a), which is different from the expected diamagnetic shift imposed by the SC state [29,94,95]. It is difficult to elucidate the origin of the PM shift, however the effects are consistent with field-induced magnetism. Other phenomena such as vortex lattice disorder [93] or a Yosida-like decrease of the spin susceptibility [96] may also contribute to this behavior and can be investigated further. The SC and magnetic relaxation rates, $\sigma_{SC}$ and $\lambda_{Mag}$, respectively, are shown in Figs. 11(b) and 11(c), demonstrating an additional effect of a weak contribution of static magnetism to the SC state. We also observe the nonmonotonic temperature dependence of $\lambda_{Mag}$, which may be caused by the interplay between magnetism and superconductivity [15,97]. As the sample is cooled in an external transverse field below $T_c \approx 15$ K, $\sigma_{SC}$ begins to rise from 0 due to the FLL formation. $\sigma_{SC}$ saturates upon further cooling, which resembles the behavior of an isotropic nodeless superconductor.

We found that an isotropic two-band $(s + s)$-wave SC model describes the temperature dependence of the measured $\sigma_{SC}$ remarkably well [see Figure 11(b)], yielding a large gap $\Delta_1 \simeq 4.5(6)$ meV and a small gap $\Delta_2 \simeq 1.8(5)$ meV. Refer to Appendix C for details on the SC gap symmetry analysis. A two-gap scenario is also consistent with the generally accepted view of multigap superconductivity in Fe-HTS [1,36]. The magnitudes of the large $2\Delta_1/k_B T_c \simeq 6.9(5)$ and the small $2\Delta_2/k_B T_c \simeq 2.8(5)$ gap for NaFe$_{1-x}$Ni$_x$As ($x = 0.013$) are in good agreement with previous work [98]. There it was pointed out that most Fe-HTS exhibit a two-gap SC behavior, characterized by a large gap with magnitude $2\Delta/k_B T_c \simeq 7(2)$ and a small gap with 2.5(1.5).

![FIG. 11. TF-µSR results on field cooled $x = 1.3\%$ in an applied field of 300 G. (a) Temperature dependence of the field shift from 40 K (normal state). (b),(c) Temperature dependences of the relaxation rates ascribed to the SC and magnetic orders, respectively. The solid line in (b) represents an isotropic two-band SC model fit to the temperature evolution of $\sigma_{SC}$. The peak in $\lambda_{Mag}$ in (c) is close to $T_c$, indicating that the onset of SC order affects the dilute electronic moment distribution.](image1)

![FIG. 12. Uemura plot for hole and electron doped Fe-HTS (see Ref. [99] and references therein). The linear relation observed for underdoped cuprates is shown as a blue colored solid line for hole doping [100,101] and as a red colored dashed line for electron doped systems [102]. The points for conventional BCS superconductors are also shown. The orange star marker shows the data point for NaFe$_{1-x}$Ni$_x$As obtained in this work.](image2)

**B. Connection with other unconventional superconductors**

An interesting result of $\mu$SR investigations in Fe-HTS is the observed proportionality between $T_c$ and the zero-temperature relaxation rate $\sigma(0) \propto \lambda^{-2}(0)$, known as the Uemura plot [100,101]. This relation, which seems to be generic for various families of cuprate HTS, has the features that upon increasing the charge carrier doping $T_c$ first increases linearly in the underdoped region (blue line in Fig. 12), then saturates, and finally is suppressed for high carrier doping. The initial linear trend of the Uemura relation indicates that for these unconventional HTS, the ratio $T_c/E_F$ ($E_F$ is the Fermi energy)
is much larger than that of conventional BCS superconductors. Figure 12 shows \( T_c \) plotted against \( x^2 \) for various hole- and electron-doped Fe-HTS (see Ref. [99] and references therein), including the current results on NaFe\textsubscript{1-x}Ni\textsubscript{x}As. The linear relation observed for underdoped cuprates is also shown as a solid line for hole doped cuprates [100,101] and as a dashed line for electron doped cuprates [102]. The present data for NaFe\textsubscript{1-x}Ni\textsubscript{x}As in the Uemura plot is in close proximity to the line observed in hole-doped cuprates and other Fe-HTS. This connection contrasts with LiFeAs, which shows behavior following electron-doped cuprates. The observation of superconductivity in NaFe\textsubscript{1-x}Ni\textsubscript{x}As as compared to LiFeAs presents a new challenge for theoretical explanations.

### V. CONCLUSION

In conclusion, the magnetic and SC properties of NaFe\textsubscript{1-x}Ni\textsubscript{x}As were studied as a function of Ni-content \( x \) by DC magnetization and \( \mu \)SR techniques. The long range magnetic order is observed for \( x = 0 \) and 0.4% samples, while for \( x > 0.4\% \) magnetic order becomes inhomogeneous and is completely suppressed for \( x = 1.5\% \). The magnetic volume fraction continuously decreases with increasing \( x \). Furthermore, superconductivity acquires its full volume for samples with \( x \gtrsim 0.4\% \). This implies that there is a coexistence of magnetism and superconductivity in NaFe\textsubscript{1-x}Ni\textsubscript{x}As. Both the ordered moment and the magnetic volume fraction decrease below \( T_c \), showing that magnetism, which develops at higher temperatures, becomes partially (or even fully) suppressed by the onset of superconductivity. These results indicate that the competition between the SC and magnetic order parameters in NaFe\textsubscript{1-x}Ni\textsubscript{x}As develop in an intrinsically inhomogeneous environment, providing important insight for theoretical modeling. A linear relationship between the \( T \) = 0 ordered moment and the AFM ordering temperature \( T_\text{AFM} \) for various Fe-HTS is noted, which is consistent with a mean-field approach for itinerant electrons, in which antiferromagnetism is driven by Fermi surface nesting. From TF-\( \mu \)SR measurements, the temperature evolution of the penetration depth in NaFe\textsubscript{1-x}Ni\textsubscript{x}As is consistent with an isotropic two-gap (s + s')-wave model for superconductivity.

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### APPENDIX A: INTERNAL FIELD SIMULATION

#### 1. Initialization of Crystal Properties

At low temperatures, NaFeAs crystallizes into the Cmme space group, with the following assumed lattice constants for the orthorhombic structure based on Ref. [19]: \( a = 5.6834 \, \text{Å}, \quad b = 5.6223 \, \text{Å}, \quad c = 6.9063 \, \text{Å} \). Stopping site calculations and subsequent dopolar field calculations were performed on NaFe\textsubscript{1-x}Ni\textsubscript{x}As with the atomic properties displayed in Table III. The sample was assumed to be in the low temperature ordered state with Fe spins aligned in the usual collinear AFM arrangement as depicted in Fig. 1(d).

#### 2. Muon Stopping Site Determination

The search for muon sites was initiated by sampling a \( 4 \times 4 \times 4 \) grid of possible interstitial positions in the NaFeAs lattice that are at least 1 \( \text{Å} \) away from lattice atoms. Symmetry-equivalent points in the search grid were removed with the space group symmetry of the lattice. The stability of a H atom in a \( 2 \times 2 \times 2 \) supercell consisting of 96 Na-Fe-As atoms was examined at each point in the grid. A \( 2 \times 2 \times 2 \) Monkhorst-Pack grid of \( k \) points was used for Brillouin zone sampling. DFT calculations were carried out assuming the usual collinear magnetic ordering of Fe atoms in NaFeAs.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Sym.</th>
<th>Position (^a)</th>
<th>Nuc. Mom. (^b)</th>
<th>Mag. Mom. (^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na</td>
<td>4g</td>
<td>(0.000, 0.250, 0.651)</td>
<td>2.217</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>4a</td>
<td>(0.250, 0.000, 0.000)</td>
<td>0.091</td>
<td>0.175</td>
</tr>
<tr>
<td>Ni</td>
<td></td>
<td>Fe-substitution</td>
<td>−0.750</td>
<td></td>
</tr>
<tr>
<td>As</td>
<td>4g</td>
<td>(0.000, 0.250, 0.198)</td>
<td>1.439</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Atomic positions given in fractional coordinates.
\(^b\) Nuclear moments given in units of \( \mu_B \).
\(^c\) Ordered magnetic moments given in units of \( \mu_B \).
The local field comes from the ordered Fe moments. The static ordered Fe moment was set to NaFe\textsubscript{1−x}Ni\textsubscript{x}As. The local magnetic field strength |\(\vec{B}\)| is nearly constant for each particular muon site position, confirming that the majority of the local field comes from the ordered Fe moments. The static ordered Fe moment was set to \(\mu_{Fe} = 0.175(3)\mu_B\) to match the high frequency in the experimental spectra.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Label</th>
<th>Symmetry</th>
<th>Site Position(\text{a})</th>
<th>(\Delta E) (meV)(\text{b})</th>
<th>Field</th>
<th>Frequency (\nu) (MHz)(\text{c})</th>
<th>Angle (\theta) (°)(\text{d})</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A</td>
<td>8n</td>
<td>(0.100, 0.750, 0.100)</td>
<td>0</td>
<td>578.5</td>
<td>7.839(30)</td>
<td>42.1(5)</td>
</tr>
<tr>
<td>I</td>
<td>B</td>
<td>8m</td>
<td>(0.000, 0.875, 0.100)</td>
<td>42</td>
<td>810.9</td>
<td>10.987(49)</td>
<td>31.1(6)</td>
</tr>
<tr>
<td>I</td>
<td>C</td>
<td>8l</td>
<td>(0.250, 0.500, 0.250)</td>
<td>183</td>
<td>488.3</td>
<td>6.616(56)</td>
<td>88.6(4)</td>
</tr>
<tr>
<td>II</td>
<td>D</td>
<td>4b</td>
<td>(0.750, 0.500, 0.500)</td>
<td>287</td>
<td>1.002</td>
<td>0.014(40)</td>
<td>0.2(3)</td>
</tr>
<tr>
<td>II</td>
<td>E</td>
<td>4g</td>
<td>(0.500, 0.250, 0.600)</td>
<td>436</td>
<td>154.2</td>
<td>2.090(21)</td>
<td>0.6(4)</td>
</tr>
</tbody>
</table>

\(\text{a}\)Candidate muon stopping site positions given in fractional coordinates.
\(\text{b}\)DFT total energy difference from stopping site A.
\(\text{c}\)Magnetic field at muon site from dipolar field simulations.
\(\text{d}\)Simulated muon precession frequency.

As shown in Fig. 1(d) [19]. To accommodate for structural relaxations, the forces were optimized till a threshold of \(10^{-3}\) atomic units and the energies till a threshold of \(10^{-4}\) atomic units. Table IV lists five candidate muon sites for NaFe\textsubscript{As} using this first-order search procedure. These sites are also assumed to be compatible for lowly doped NaFe\textsubscript{1−x}Ni\textsubscript{x}As.

We group the five candidate sites into two clusters based on stability checks using the double Born-Oppenheimer approximation method (DBO) [45], which takes into account the quantum description of the muon. Within this method, a potential exploration algorithm (PEA) is used to efficiently sample the \textit{a priori} unknown potential felt by the muon. With the sampling of the potential, site C is observed to be a local minimum in the muon potential. Site C relaxes towards sites A and B since site C has very low barrier less than 0.24 eV that is too small to bind the muon. Sites A and B are also close in proximity to each other and in energy difference. Consequently, we associate sites A, B, and C together as cluster I. Similarly, we also observe that sites E and D relax into each other, which together form cluster II. Our clustering also explains the observed frequencies—cluster I contains the low DFT energy sites that describe the high muon field observed from experiment, while cluster II contains sites that correspond to the low field.

Shown in Fig. 13 are the results of further analysis of the sites in cluster II. The energy profile extracted from the DBO potential map can be represented by the toy model shown in Fig. 13. This enables us to solve the Schrödinger equation of the muon, yielding a ground state energy of 0.17 eV (independent of the interpolation method and the boundary condition, to some extent) which is greater than the barrier seen in the potential map. These findings suggest that the muon wave function for the low field sites may be delocalized over positions between sites D and E (hereafter the D-E site). As a result, the low frequency detected from experiments may come from an averaging of the field at the two sites. Following analysis considering the quantum nature of the muon due to its light mass, we propose that sites A, B, and D-E are the possible implantation sites of the muon.

DBO would still predict a zero average at the D-E sites probed by the muon wave function due to the symmetry of the sites in the lattice. However a DFT mapping of the total energy and a separate solution of the muon Schrödinger equation may not give the final answer, since the muon quantum nature is ignored in the DFT assessment of the total energy. The actual muon site may still be slightly distorting the local environment, thus justifying the small but nonvanishing low precession frequency listed in Table I.

### 3. Low Temperature Dipolar Field Simulation

A \(9 \times 9 \times 9\) supercell of magnetic dipoles was used to model the internal field of NaFe\textsubscript{1−x}Ni\textsubscript{x}As. Dipole positions and strengths for the idealized crystal structure in NaFe\textsubscript{As} listed in Table III. Nuclear dipole moment directions are assumed to be random for all atoms while the spins on Fe are assumed to take on a collinear AFM striped pattern, common to other Fe-HTS. To simulate the effect of doping, the magnetic Fe atoms are randomly substituted with nonmagnetic Ni atoms to achieve the desired Ni concentration \(x\). The dipolar field at the muon site was obtained by summing over all dipoles in the NaFe\textsubscript{1−x}Ni\textsubscript{x}As supercell.

![Toy model potential](https://example.com/toy-model.png)

FIG. 13. A toy model potential \(V(0, 0, z)\) (solid line) together with the ground state energy, \(E_0 = 0.17\) eV from solving the Schrödinger equation for a muon in a potential of the form \(V(x, y, z) = -a(x^2 + y^2) + b(c^2 - c^2 z^2 + d) + f\) with \(a = 2.44 \times 10^{-3}, b = 5.04 \times 10^{-4}, c = 3 \times 10^{-3}, d = 2.85 \times 10^{-3}, \) and \(f = 4.79 \times 10^{-3}\), all in Hartree atomic units. The green dots show the minimum energy profile map from the DBO for the symmetric site D to site E. These simulations imply that the muon is likely delocalized over the two sites in cluster II (sites D and E).
By comparing the simulated frequencies, shown in Table IV, with the experimental results, we can associate the two high frequencies \( v_1 \) and \( v_2 \) with sites B and A, respectively. The low frequency \( v_3 \) corresponds best with site E of cluster II. However, our stability analysis shows that the muon is likely delocalized over sites D and E. A comparison between simulated and experimental results is presented in Table I. Our simulations show that the experimentally observed frequency \( v_1 = 10.9 \) MHz in NaFeAs corresponds to an ordered Fe moment size of about \( \mu_{B} \approx 0.175(3) \mu_{B} \).

APPENDIX B: ORDERED MOMENT
SCALING CALCULATIONS

In this section, we present a description of the two-band model discussed in Sec. V and introduced in Refs. \([33,34]\). The effective free energy density of the model can be written as

\[
f = \frac{2M^2}{l} - \frac{T}{\nu} \sum_{\omega_n} \sum_{k} \ln \left[ (\omega_n^2 + E_{+,k}^2)(\omega_n^2 + E_{-,k}^2) \right],
\]

where \( M \) is the temperature dependent ordered AFM moment, \( l > 0 \) is the AFM interaction coupling constant, \( \omega_n = 2\pi T(n + 1/2) \) is a fermionic Matsubara frequency (\( n \in \mathbb{Z} \)), \( \nu \) is the volume of the system, and

\[
E_{\pm,k} = \sqrt{M^2 + \xi_k^2} \pm |\delta|.
\]

Here, \( \xi_k = k^2/2m - \epsilon_0 \) is a parabolic energy dispersion, \( \theta \) is the angle in the Fermi surface between the momentum \( k \) and the x axis, and \( \delta = \delta_0 + \delta_2 \cos(2\theta) \) describes deviations from the perfect nesting condition.

The momentum sum can be evaluated as

\[
\frac{M}{m} = \frac{T}{\nu} \sum_{0<\omega_n<\Lambda} \int \frac{d\theta}{2\pi} \text{Re} \frac{M}{\sqrt{M^2 + (\omega_n + i|\delta|)^2}}
\]

where \( \Lambda \) is a high-frequency cutoff. At \( T = 0 \), we integrated over frequencies to obtain

\[
\text{Re} \int \frac{d\theta}{2\pi} \ln \left( |i\delta| + \sqrt{M^2 - \delta^2} \right) = \ln M_0,
\]

where \( M_0 = M(T = 0) \) and \( M_0 = 2\Lambda e^{-2\pi/m} \) is the value of \( M \) for \( \delta_0 = 0 = \delta_2 \).

When the transition is second order, \( T_N \) is the temperature at which \( M(T \rightarrow T_N) \rightarrow 0 \). Setting \( M = 0 \) in (B2) and performing the Matsubara sum yields

\[
2 \ln \left( \frac{4\pi T_N}{M_0} \right) + \int \frac{d\theta}{2\pi} \left[ \psi \left( \frac{1}{2} + i\delta_0 \right) \right.
\]

\[
\left. + \psi \left( \frac{1}{2} - i\delta_0 \right) \right] = 0,
\]

where \( \psi(z) \equiv \frac{d}{dz} \ln \Gamma(z) \) is the digamma function.

FIG. 14. Summary of numerical results for the ordered moment \( M \) and \( T_N \). (a) Ordered moment \( M \) at \( T = 0 \) and (b) \( T_N \) as functions of the area difference parameter \( \delta_2 \) for fixed values of the ellipticity parameter \( \delta_2 \). Results are shown in the regime where the transition is second order. Colors represent different values of \( \delta_2/M_0 \) listed in Fig. 9.

We have numerically calculated the remaining angular integral in the self-consistent equations (B3) and (B4) to determine the behavior of \( M \) and \( T_N \) as functions of \( \delta_0 \) and \( \delta_2 \). For fixed \( \delta_2 \), the transition is second order at small \( \delta_0 \) but becomes first order at larger \( \delta_0 \). Moreover, there is no ordered AFM state \([34]\) for \( \delta_2 > M_0 \). On the other hand, the ordered magnetic moment at \( T = 0 \) generally increases with decreasing \( \delta_0 \) and \( \delta_2 \) and abruptly saturates at \( \delta_0 + \delta_2 \) \( = M_0 \). These results are presented in Fig. 14. More importantly, these results enable us to plot and examine the behavior of the ordered moment at \( T = 0 \) vs \( T_N \), which is shown in Fig. 9.

APPENDIX C: TF-\( \mu \)SR SC GAP ANALYSIS

To explore the SC gap symmetry, we recall that the penetration depth \( \lambda(T) \) (in an isotropic superconductor) is related to the quadratic relaxation rate \( \sigma_{SC}(T) \) through \( \sigma_{SC}(T) = k\gamma_{\mu}\Phi_0 \lambda^{2}(T) \), where \( \gamma_{\mu} \) is the muon gyromagnetic ratio, \( \Phi_0 \equiv \frac{h}{2e} \) is the quantum of magnetic flux, and \( k \approx 0.06091 \) is a geometric factor characterizing the FLL \([25,103]\). The temperature evolution of \( \lambda(T) \) can be modeled for a variety of SC gap symmetries and structures.

Within the local London limit of electrodynamics (where the penetration depth \( \lambda \) is much greater than the SC coherence length \( \xi \)), the \( \alpha \) model is a popular phenomenological
framework used to study multiband superconductivity [104–106]. The \( \alpha \) model assumes that the SC gaps in different bands are independent from each other (aside from sharing a common \( T_C \)) and that the normalized penetration depth \( \frac{\lambda(0,\Delta_j)}{\lambda(0,\Delta_s)} \) follows the same temperature dependence as in the single-band clean-limit BCS theory. A two-band \( \alpha \) model in which the superfluid densities from each band are added together was used to analyze the TF-\( \mu \)-SR results:

\[
\frac{\sigma_{SC}(T)}{\sigma_{SC}(0)} = \sum_{j=1}^{2} f_j \frac{\lambda^{-2}(T,\Delta_j)}{\lambda^{-2}(0,\Delta_j)}.
\] (C1)

where \( \Delta_j \) is the maximum value of the SC gap at \( T = 0 \) for each band (\( j = 1, 2 \)). The relative contributions from each band are imposed through the constraint \( \sum_{j=1}^{2} f_j = 1 \) in (C1).

Assuming that the Fermi velocity is constant in magnitude, the penetration depth is determined through the integral expression [107]:

\[
\frac{\lambda^{-2}(T,\Delta_j)}{\lambda^{-2}(0,\Delta_j)} = 1 + \frac{1}{\pi} \int_0^{\infty} d\varphi \int_{\Delta_j}^{\infty} E \, dE \times \cdots \cdot \frac{1}{\sqrt{E^2 - \Delta^2_j(T,\varphi)}}
\] (C2)

where \( f(E) = [1 + \exp(E/k_B T)]^{-1} \) is the Fermi function. The SC gap functions \( \Delta_j(T,\varphi) \) in (C2) are assumed to have the separable form: \( \Delta_j(T,\varphi) = \Delta_j g(\frac{\varphi}{T}) \). The temperature dependence of the gap is approximated by the function \( g(\varphi) = \tan[\alpha(\beta(t^{-1} - 1)^{\delta})] \), with \( \alpha = 1.82, \beta = 1.018, \) and \( \delta = 0.51 \) [107]. The SC gap symmetry is embedded in \( S(\varphi) \), which is defined to be 1 for \( s \)-wave and \( (s + s) \)-wave gaps and \( |\cos(2\varphi)| \) for \( d \)-wave gaps. The results of applying (C1) to the \( x = 1.3 \)% system are shown in Fig. 11(b), demonstrating that the two-band \( \alpha \) model with an \( (s + s) \)-wave SC gap is a feasible model for the data.


