Chapter 3. Plasmas

In Chapter 2, we introduced the idea of a *plasma* as a partially ionized gas consisting of equal numbers of positive and negative charges, and a different number of un-ionized neutral molecules. As we progress through this chapter, further requirements will be made of the gas in order to qualify it as a plasma.

In the type of plasmas discussed in this book, the degree of ionization is typically only 10^{-4}, so the gas consists mostly of neutrals. Although the Coulomb interaction between charges is both strong and long-range, it is possible to assume for an undisturbed plasma that the charges move around as free particles, since the sum of all the interactions tends to cancel, analogous to the role of a conduction electron in a solid. But also, to pursue the analogy, there are situations where the Coulomb interaction becomes dominant, as for example when the plasma is perturbed.

ELECTRON AND ION TEMPERATURES

To simplify, assume that the charged particles are singly charged positive ions and electrons. In addition, descriptions of the plasma will be made in the context of the glow discharge processes being considered. The essential mechanisms in the plasma are excitation and relaxation, ionization and recombination. To maintain a steady state of electron and ion densities, the recombination process must be balanced by an ionization process, i.e. an external energy source is required. In practice, that energy source is an electric field, which can act directly on the charged particles only. Let m_e and m_i be the masses of the electron and the ion respectively. Consider an electric field $\mathcal{E}$ acting on an initially stationary ion. The work done by the electric field, and hence the energy transferred to the ion, will be $\mathcal{E}ex$ where x is the distance travelled in time t. But:

$$x = \tfrac{1}{2}ft^2$$

where f is the acceleration due to the field (Figure 3-1), given by

$$\mathcal{E}e = m_i f$$

hence:

$$\text{Work done} = \mathcal{E}ex = \mathcal{E}e\frac{1}{2}\frac{\mathcal{E}e}{m_i}t^2 = \frac{(\mathcal{E}et)^2}{2m_i}$$

A similar relationship holds for the electrons, but since $m_i \gg m_e$, the action of the field is primarily to give energy to the electrons. The argument above ignored collisions, and we can always choose t to be short enough that this is so. But we have seen that, in general, collisions abound in plasmas. Electrons collide with neutral atoms and ions, but only a very small energy transfer to the heavy particle can take place; this is what the energy transfer function (q.v.) was about. In turn, the neutral atoms and ions share their energy efficiently in collision processes and likewise lose energy to the walls of the chamber. The net result is that electrons can have a high average kinetic energy, which might typically be 2 - 8 eV. The ions, which can absorb just a little energy directly from the electric field, have an average energy not much higher than that of the neutral molecules, which gain energy above the ambient only by collisions with ions (effectively) and electrons (ineffectively) and remain essentially at room temperature. We saw earlier that for the neutral gas atoms:

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$

The average energy is characterized by the kT term and although this would conventionally be measured in ergs, it is more convenient here to work in electron volts. It is useful to remember that kT has a value of 1/40 eV at 290 K,

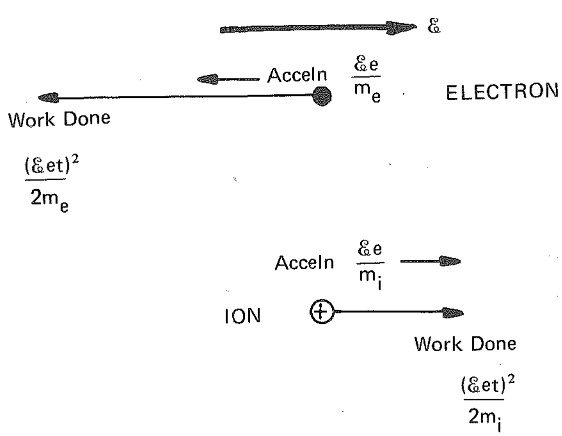


Figure 3-1. Energy transfer from the field to the electrons and ions

i.e. about room temperature. The concept of temperature applies to a random, i.e. Maxwell-Boltzmann, distribution. Can we apply this concept, then, to the energetic electrons? Based on an expectation of a large number of electron-electron collisions and other interactions, and very efficient energy sharing amongst the electrons because the energy transfer function takes all values between 0 and 1 for equal mass particles as the impact angle θ varies, a Maxwell-Boltzmann distribution seems quite reasonable. We assume this now and consider it again later. Since

$$\frac{1}{2}m_e\overline{c_e^2} = \frac{3}{2}kT_e$$

applies to electrons too, we can associate an effective temperature T_e with the electron motion. Measurements on glow discharge plasmas yield average electron energies around 2eV, which corresponds to an electron temperature of 23200 K! That doesn't mean that the containing vessel will melt, and that is because the heat capacity of the electrons is too small; we just have to think more carefully about the temperature concept. Since the ions are able to receive some energy from the external electric field, their temperature is somewhat above ambient; 500 K is representative.

PLASMA POTENTIAL

So three sets of particles exist in the plasma – ions, electrons, and neutrals – varying by mass and temperature. In addition, we saw in Chapter 1 that $\bar{c} = (8kT/\pi m)^{1/2}$, as in indicated for the typical parameters shown in Figure 3-2 based on argon. The *electron density* and *ion density* are equal (on average); this number, which is much less than the density of neutrals, is often known as the *plasma density*. The average speed of the electrons is enormous compared with those of the ions and neutrals, due to both the high temperature and low mass of the electrons.

Suppose we suspend a small electrically isolated substrate into the plasma. Initially it will be struck by electrons and ions with charge fluxes, i.e. current densities, predicted in Chapter 1 to be (Figure 3-3):

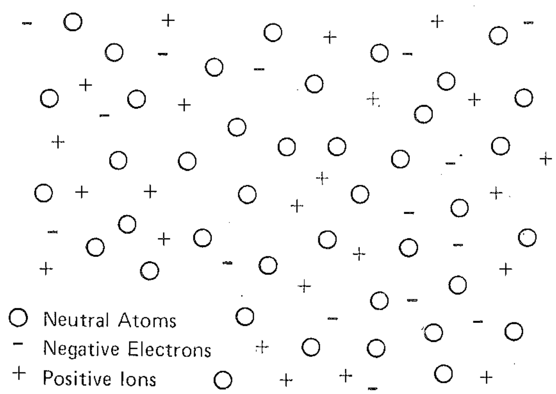
$$j_e = \frac{e n_e \overline{c_e}}{4}$$

$$j_i = \frac{e n_i \overline{c_i}}{4}$$

But $\overline{c_e}$ is much larger than $\overline{c_i}$. For the values shown in Figure 3-2,

$$j_e \sim 38\ \text{mA/cm}^2$$

$$j_i \sim 21\ \mu\text{A/cm}^2$$



Neutrals $m = 6.6\ 10^{-23}$g

$T = 20°C = 293K \equiv 1/40eV$

$\bar{c} = 4.0\ 10^{4}$ cm/sec

Ions $m_i = 6.6\ 10^{-23}$g

$T_i = 500K \equiv 0.04eV$

$\bar{c}_i = 5.2\ 10^{4}$ cm/sec

Electrons $m_e = 9.1\ 10^{-28}$g

$T_e = 23\ 200K \equiv 2eV$

$\bar{c}_e = 9.5\ 10^{7}$ cm/sec

$\bar{c} = \left(\frac{8kT}{\pi m}\right)^{1/2}$

Figure 3-2. Typical parameter values for a glow discharge plasma

Since $j_e \gg j_i$, then the substrate immediately starts to build a negative charge and hence negative potential with respect to the plasma. Immediately the quasi-random motions of the ions and electrons in the region of our object, are disturbed. Since the substrate charges negatively, electrons are repelled and ions are attracted. Thus the electron flux decreases, but the object continues to charge negatively until the electron flux is reduced by repulsion just enough to balance the ion flux. We shall show shortly ("Debye Shielding") that the plasma is virtually electric field free, except around perturbations such as above, and so is equipotential. Let's call this potential the *plasma potential* V_p, also sometimes known as the *space potential*. Similarly, we can associate a *floating potential* V_f with the isolated substrate. [In the case of a plasma container having insulating walls, these walls also require zero steady state net flux, so that *wall potential* and floating potential are related terms.] Since V_f is such as to repel electrons, then $V_f < V_p$. In the absence of a reference, only the potential difference $V_p - V_f$ is meaningful. Because of the charging of the substrate, it is as though a potential energy 'hill' develops in front of the substrate (Figure 3-4). However, it is a downhill journey for ions from the plasma to the substrate, but uphill for the electrons, so that only those electrons with enough initial kinetic energy make it to the 'top', i.e. the substrate.

SHEATH FORMATION AT A FLOATING SUBSTRATE

Since electrons are repelled by the potential difference $V_p - V_f$, it follows that the isolated substrate (assumed planar for simplicity in Figure 3-3) will acquire a net positive charge around it. This is generally known as a *space charge* and, in the context of glow discharge plasmas, forms a *sheath*. The sheath has a certain density of charges, known as the *space charge density* ρ. Poisson's equation relates variation of potential V with distance x across regions of net space charge:

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon_0}$$

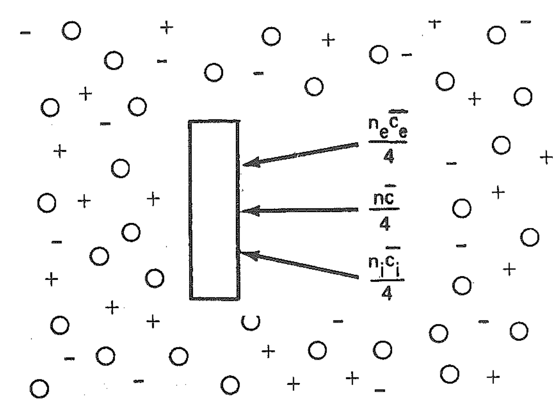


Figure 3-3. Initial particle fluxes at the substrate

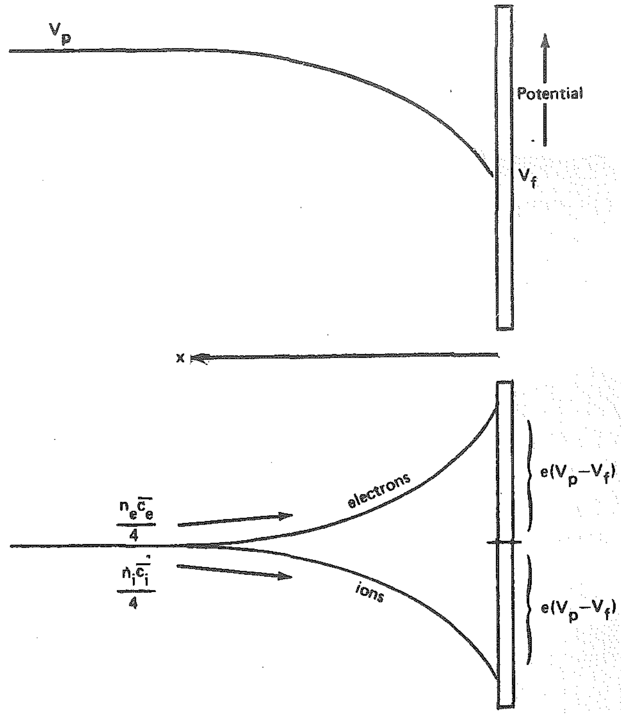


Figure 3-4. Variation of the electrical potential (upper) and of the potential energies of electrons and positive ions (lower), in the vicinity of an electrically floating substrate

This is the one-dimensional form for MKS units, where ϵ_0 is the permittivity of free space. Since electric field $\mathcal{E}$ is given by

$$\mathcal{E} = -\frac{dV}{dx}$$

then

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_0}$$

and this just says that the electric field across a gap changes as we go through regions of net charge, consistent with experience.

If the sheath acquires a net positive charge, it follows that the electron density decreases in the sheath – we shall obtain a quantitative expression for the decrease below. But one of the obvious features of a discharge is that it glows, and as we have already seen, this is due to the relaxation of atoms excited by

electron impact. So the glow intensity depends on the number density and energy of the exciting electrons. Since the electron density is lower in the sheath, it doesn't glow as much. So we can actually see the sheath as an area of lower luminosity than the glow itself – the substrate is surrounded by a (comparatively) *dark space*, a feature common to the sheaths formed around all objects in contact with the plasma, even though the sheath thicknesses may vary greatly.

Let us now try to get an idea of the magnitude of $V_p - V_f$, which represents a barrier to electrons. To surmount this barrier, an electron must acquire $e(V_p - V_f)$ of potential energy (Figure 3-5). Hence, only electrons that enter the sheath from the plasma with kinetic energies in excess of $e(V_p - V_f)$, will reach the substrate. The Maxwell-Boltzmann distribution function tells us that the fraction n_e'/n_e that can do this is:

$$\frac{n_e'}{n_e} = \exp -\frac{e(V_p - V_f)}{kT_e}$$

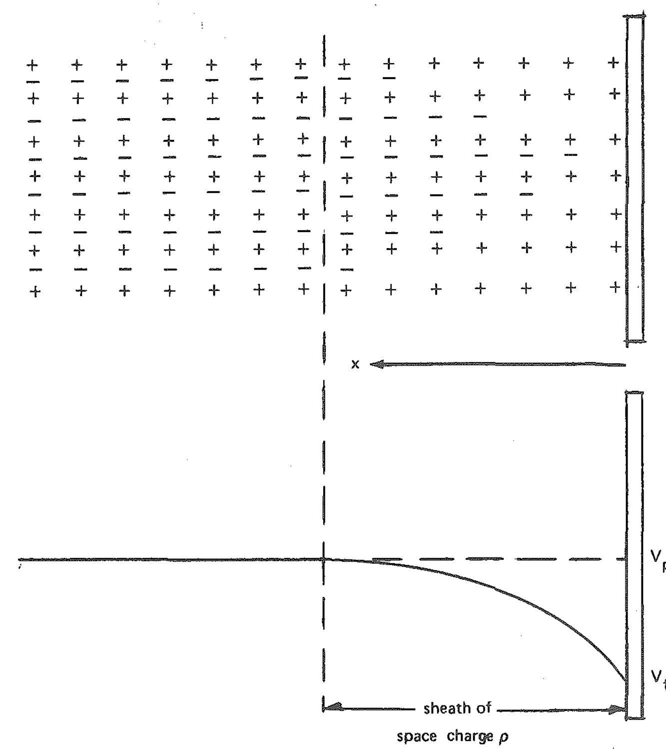


Figure 3-5. A space charge sheath develops in front of a floating substrate (upper), and establishes a sheath voltage (lower)

If the density n_e' just achieves charge flux balance at the object, then

$$\frac{n_e'\overline{c_e'}}{4} = \frac{n_i\overline{c_i}}{4}$$

One might at first think that the n_e' electrons close to the substrate would have a lower mean speed $\overline{c_e'}$ than the n_e electrons in the plasma, since the n_e' electrons suffer an $e(V_p - V_f)$ loss of kinetic energy in crossing the sheath. However, one must also bear in mind that the n_e' electrons that reach the substrate were not 'average' electrons, but had energies greater than average. In fact, the average energies of the n_e and n_e' groups of electrons are the same, i.e. they are at the same temperature. This can be shown from the Maxwell-Boltzmann distribution which, in a region of potential energy $e\phi$, becomes:

$$dn_e' = 4\pi n_e\left(\frac{m_e}{2\pi kT_e}\right)^{3/2} c_e^2 \exp -\frac{(\frac{1}{2}m_ec_e^2 + e\phi)}{kT_e} \quad dc_e$$

$$= \exp\left(-\frac{e\phi}{kT_e}\right)dn_e$$

$$\therefore \quad \overline{c_e'^2} = \frac{\int c_e^2 \exp\left(-\frac{e\phi}{kT_e}\right)dn_e}{\int \exp\left(-\frac{e\phi}{kT_e}\right)dn_e} = \overline{c_e^2}$$

Furthermore, by integration

$$n_e' = n_e \exp\left(-\frac{e\phi}{kT_e}\right) = n_e \exp -\frac{e(V_p - V_f)}{kT_e}$$

Returning to the charge flux balance equation, and substituting for n_e' and $\overline{c_e'}$, then

$$n_e \exp -\left(\frac{e(V_p - V_f)}{kT_e}\right)\frac{\overline{c_e}}{4} = \frac{n_i\overline{c_i}}{4}$$

But $n_e = n_i$ and $\overline{c} = \left(\frac{8kT}{\pi m}\right)^{1/2}$ (Chapter 1, "Mean Speed $\overline{c}$"), and so charge balance requires

$$V_p - V_f = \frac{kT_e}{e} \ln \frac{\overline{c_e}}{\overline{c_i}}$$

$$= \frac{kT_e}{2e} \ln\left(\frac{m_iT_e}{m_eT_i}\right)$$

(When we have learned a little more about sheath formation, in "Sheath Formation and The Bohm Criterion", we shall need to modify this result slightly.)

In our example (Figure 3-2), $(V_p - V_f)$ should have a value of +15 volts, which is of the right order to agree with observation. Note the polarity, which is to make the plasma positive with respect to the floating object, and indeed, positive with respect to almost everything. The rapid motion of the electrons, relative to the ions, means they can easily move away from the plasma. But in doing so they lease the plasma more positive which hinders the escape of the negative electrons and makes the process self-limiting.

Since the charging of the floating substrate serves to repel electrons, it also attracts positive ions. This does not increase the flux of ions, which is limited by the random arrival of ions at the sheath-plasma interface – in terms of the model in Figure 3-4, it doesn't matter how steep or high the hill is (this isn't quite true, as will be discussed later). However, the voltage across the sheath does directly influence the energy with which the ion strikes the substrate. The ion enters the sheath with very low energy. It is then accelerated by the sheath voltage, and, in the absence of collisions in the sheath, would strike the substrate with a kinetic energy equivalent to the sheath voltage.

In practice, the sheath above an electrically isolated substrate varies from one or two volts upwards. The resulting kinetic energies must be compared with interatomic binding energies in a thin film or substrate of typically 1 - 10 eV, so that it is easy to imagine that a growing thin film or an etching process on an electrically isolated surface in the plasma might be much affected by such impact.

DEBYE SHIELDING

If the numbers of ions and electrons in the plasma are equal and very large, then it is not surprising that their net Coulomb interaction with a particular charge sums to zero. But although this must be true on the average, we might expect that the instantaneous potential at a point due to some disturbance is both non-zero and time dependent. Let's consider this case (in 1 dimension, for simplicity) by assuming that the potential at $x = 0$ is ΔV_o (measured relative to the plasma), and then see how the potential $\Delta V(x)$ varies with x (Figure 3-6). In thinking about the problem assume that ΔV_o is less than V_p, i.e. more negative. Then a net positive space charge will form in front of the charged surface, since only energetic electrons can enter, as in the previous example. To a first approximation, the ion density in the sheath will be n_i as in the undisturbed plasma, since the ions are too massive to react rapidly to the space charge. This would not be true if the potential ΔV_o were maintained for a long time, but the random fluctuations that cause ΔV_o often happen on a very short time scale. And even when the potential perturbation is semi-permanent, this only serves to make n_i dependent on x, changing the argument in detail only.

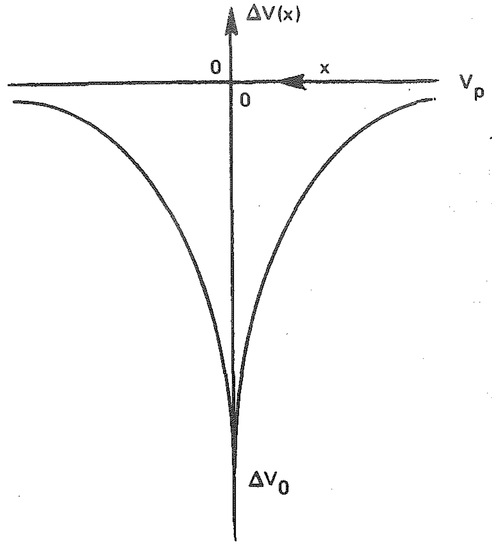


Figure 3-6. Variation of potential around a perturbation

If the electron density varies as $n_e(x)$, then Poisson's equation becomes:

$$\frac{d^2 V}{dx^2} = - \frac{e}{\epsilon_o} (n_i - n_e(x))$$

This is actually the MKS units version of Poisson's equation, where ϵ_o is the permittivity of free space, having the value $1/(36\pi\ 10^9)$ farads/metre. The purpose of using the MKS form is to avoid the cgs units of charge for the electron, and to avoid confusing myself; I always find the wrong answers when I use other units for this calculation!

Using the Boltzmann relation again:

$$\frac{n_e(x)}{n_e} = \exp - \frac{e\,\Delta V(x)}{kT_e}$$

Substituting into Poisson's equation, and remembering that $n_e = n_i$ in the undisturbed plasma, we obtain

$$\frac{d^2 V}{dx^2} = - \frac{en_i}{\epsilon_o}\left(1 - \exp - \frac{e\Delta V(x)}{kT_e}\right)$$

There is a difficulty here in that this equation can be simplified and solved when $\Delta V(x) \ll kT_e$. But in many cases that arise, this inequality does not hold for all x. However, if one solves the equation more exactly (Mitchner and Kruger 1973, p. 133), and then numerically compares the resulting solution with that obtained using the inequality above, then apparently there is good agreement; this is because the major variation of $\Delta V(x)$ is near $x = 0$, where it varies very rapidly. So using the inequality $\Delta V(x) \ll kT_e$ to expand the exponential, then

$$\frac{d^2 V}{dx^2} \simeq \frac{e^2 n_i}{kT_e\,\epsilon_o}\,\Delta V(x)$$

This approximate differential equation has a solution (also approximate):

$$\Delta V(x) = \Delta V_o \exp - \frac{|x|}{\lambda_D}$$

where

$$\lambda_D = \left(\frac{kT_e\,\epsilon_o}{n_e e^2}\right)^{1/2}$$

This quantity λ_D has the dimensions of a length, and is known as the *Debye length.* The spatial dependence of $\Delta V(x)$ tells us that if the potential in the plasma is perturbed, then the plasma reacts to oppose that change. The Debye length tells us how rapidly the potential perturbation is attenuated in the plasma; over a distance λ_D the perturbation is reduced to 0.37 (1/e) of its initial value. For the example we have chosen ($n_i = n_e = 10^{10}/cm^3$ and $kT_e = 2$ eV), λ_D has the value $1.05\ 10^{-2}$ cm, or 105 μm.

Another way of regarding the Debye length concept is to say that, from the perspective of a particular charge at a particular point in the plasma, we need to consider the sum of the individual interactions with all of the other charged particles contained within a sphere centered on the particular point having a radius of 1 or 2 Debye lengths. Outside of this sphere, the detailed nature of the interaction becomes immaterial and the net interaction is zero. Hence, the unperturbed plasma is equipotential except for small fluctuating voltages which are attenuated over distances of the order of the Debye length.

One of the requirements for a collection of charged particles to be considered a plasma is that the range of these microfields must be very small on the scale of the total dimension of the plasma, i.e. $\lambda_D \ll d$ where d is the characteristic diameter of the discharge.

A similar argument to that used above could be made for the case where ΔV_o is imposed on a conducting element in the plasma, by an external source, e.g. a battery. If ΔV_o is dc or low frequency ac, then the ions around the object do have an opportunity to respond to the applied field, and n_i becomes a function of x. Nevertheless, the basis of the argument is the same, and we again come to the conclusion that the plasma attenuates voltage perturbations by forming a sheath, leaving the undisturbed region, i.e. the plasma itself, equipotential.

These screening phenomena also have a bearing on our initial assumptions about treating the plasma as a collection of three quasi-independent ideal gases. Since the plasma is equipotential, then it is also electric-field-free, so none of the constituent charged particles is subject to any externally imposed fields, except to the extent that the plasma will respond to any further applied voltages by forming a screening sheath around the relevant electrode. So the charge assembly does exhibit *collective behaviour*, a necessary criterion for its classification as a plasma. And even within the plasma, the individual charged particle interactions are important over the range of a few Debye lengths, and there is certainly a considerable and continuous energy interchange amongst the gas species; hence the gases are only quasi-independent.

Finally, we must note that the Debye shielding effect is not complete. A screening charge cloud forms around a voltage perturbation, but the resulting electric field becomes weak towards the edge of the cloud. As soon as the electrostatic potential reduces to the thermal energy of the electrons and ions, then they can escape from the charge cloud. So we come to the conclusion that the edge of the cloud is where $\Delta V \sim kT_e$, and that voltages $\sim kT_e/e$ can penetrate into the plasma. We shall see an effect of this later, in "Sheath Formation and The Bohm Criterion". Note that assuming the edge of the shielding charge cloud is where $\Delta V \sim kT_e$, contradicts the earlier assumption made in the derivation of Debye length that $\Delta V \ll kT_e$, which means that one has to be cautious in using the Debye screening length concept.

PROBE CHARACTERISTICS

Let us return to the simple plasma of Figure 3-2. Previously we considered what would happen to an electrically isolated probe placed in the plasma. Now let us pursue further what happens when that probe is maintained at a potential V set by an external power source (Figure 3-7). To make the situation more realistic, introduce a conducting wall at ground potential (0 V) to act as a reference voltage and as a return current path. The plasma potential V_p is then defined with respect to ground. The random fluxes in the plasma are $n_e\overline{c_e}/4$, and $n_i\overline{c_i}/4$ for electrons and ions respectively. We have already seen that the net flux, and hence net current, would be zero when the probe acquires a potential V_f, the floating potential. So we can begin to plot a curve of probe current density versus probe voltage (Figure 3-8). By biasing the probe negatively with respect to V_p, some electrons are prevented from reaching the probe, but the ion current density j_i remains at a value dictated by the arrival rate of ions at the edge of the sheath, and this is limited to the random flux in the discharge, i.e. $n_i\overline{c_i}/4$. If V is made very negative with respect to V_p, then the electron current would be completely suppressed. The saturation current density for negative V is then

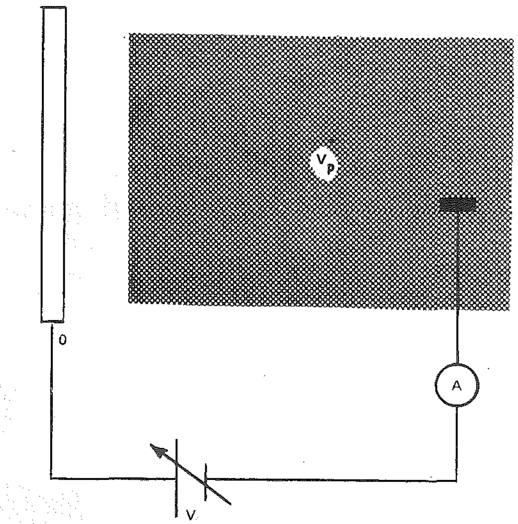


Figure 3-7. Schematic for probe measurements in a plasma

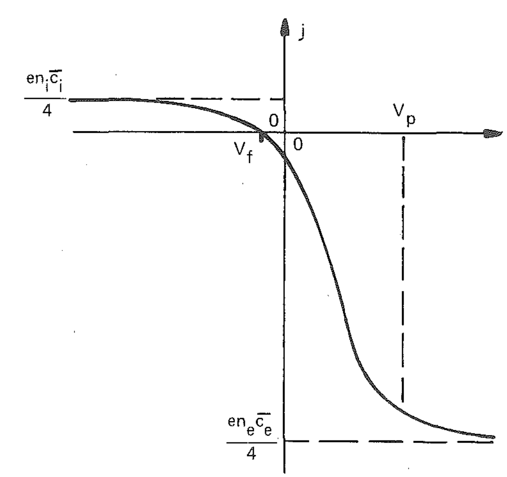


Figure 3-8. Current density-voltage characteristic of a probe

just $en_i\bar{c_i}/4$. From our earlier considerations ("Sheath Formation at a Floating Substrate"), the electron current density j_e to the probe at voltage V should follow the form:

$$j_e = \frac{en_e\bar{c_e}}{4} \exp\left(-\frac{e(V_p - V)}{kT_e}\right) .$$

and hence

$$\ell n\, j_e = \ell n \frac{en_e\bar{c_e}}{4} - \frac{e(V_p - V)}{kT_e}$$

This expression is derived on the assumption that the electrons have a Maxwellian energy distribution, and it predicts that $\ell n\, j_e$ is linearly dependent on $(V_p - V)$. This prediction is substantiated by experimental results (Figure 3-9), adding credence to our initial assumption ("Electron and Ion Temperatures") that the electrons do indeed have a Maxwellian energy distribution.

The net current density to the probe, for $V < V_p$, is just the sum of j_i and j_e:

$$j = \frac{en_i\bar{c_i}}{4} - \frac{en_e\bar{c_e}}{4} \exp - \left(\frac{e(V_p - V)}{kT_e}\right)$$

By a similar argument, one would expect for $V > V_p$ that

$$j = \frac{en_i\bar{c_i}}{4} \exp\left(-\frac{e(V - V_p)}{kT_i}\right) - \frac{en_e\bar{c_e}}{4}$$

and also, since $T_i \ll T_e$, that the ion current term would rapidly go to zero as soon as V exceeds V_p, leaving the electron saturation current and a fairly well-defined V_p at the knee of the curve.

In principle, this probe technique, which was introduced by Irving Langmuir and colleagues in the 20's (Langmuir 1923, Langmuir and Mott-Smith 1924) and carries his name, should be able to give us quite simply all of the parameters of the plasma that we need to know – electron and ion temperatures, plasma density and plasma potential. But ....

Practical Complications

Unfortunately, the situation with real probe measurements is much more complex, for a variety of reasons. The effective current-collecting area of the probe is not its geometric surface area, but rather the area of the interface between the plasma and the sheath around the probe (Figure 3-10); and the thickness of the sheath, for a given plasma, is a function of the probe potential. This would not matter for a plane probe except that such a probe has ends where the problem

arises again; and the relative contribution of the problem is increased because of the requirement that the probe be small, so that the probe current does not constitute a significant drain on the plasma. With a cylindrical probe, the varying sheath thickness is an even larger effect.

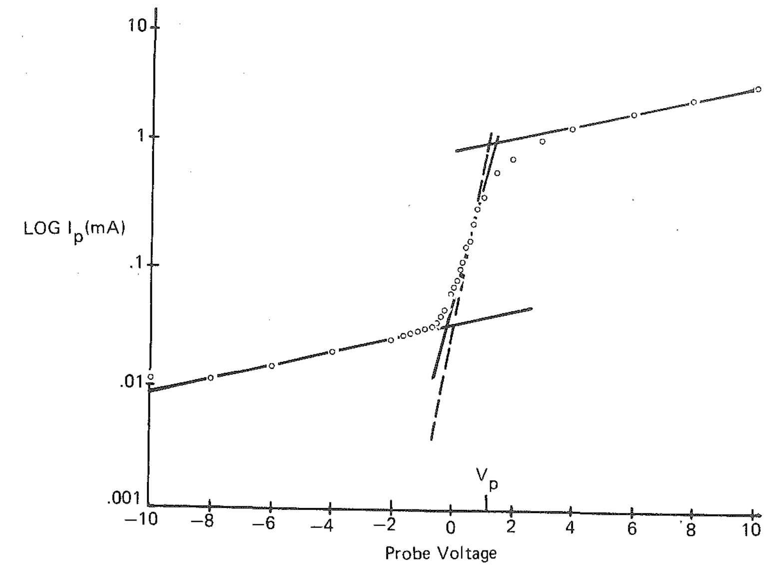


Figure 3-9. Typical probe characteristic showing a quasi-linear region where $\log j \propto V - V_s$ (Ball 1972). Tantalum target, 1000 cm². Argon discharge at 10 mtorr, 3kV and 59 mA.

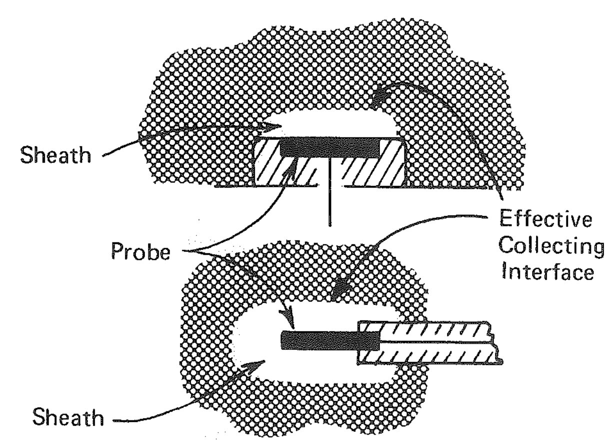


Figure 3-10. Effective current-collecting areas of probes

Two more complications are associated with additional charge generation. Secondary electrons (Chapter 4, "Secondary Electron Emission") may be generated at the probe due directly to the impact of ions, electrons, and photons or the the heating effects caused by such impact, giving rise to additional current flow; electron impact ionization may occur in the sheath, again enhancing current flow.

Yet one more problem concerns the tendency of charged particles to take up orbital paths around the probe, further influencing the probe characteristics. Even our assumption that the ion current density at the edge of the sheath is equal to the random density $en_i\bar{c}_i/4$, turns out to be incorrect, as we shall see later. And in the glow discharges used in sputtering and plasma etching, there are additional difficulties due to directed high energy electrons which flow through the plasma.

All of these effects, and others that exist, add considerable complexity to proper interpretation of probe data. The reader is referred to one of the many reviews of probe techniques, such as those of Chen (1965), Laframboise (1960), Swift and Schwar (1970), and Loeb (1961), or more recently to articles referring specifically to sputtering discharges by Clements (1978), Thornton (1978), and Eser et al. (1978).

Positively Biased Probes

Another probe effect is quite difficult to deal with: as soon as the probe potential approaches the plasma potential, the electron current density to the probe should approach the saturation value, $en_e\bar{c}_e/4$. But even with a tiny probe, the actual current drain can easily become a serious drain on the plasma, causing a significant perturbation, at least for glow discharge processes, which are of rather low density. This current drain can be limited by minimizing the size of the probe, but the following example shows that a *very* small probe is required. Use the typical plasma parameters shown in Figure 3-2 and a total current of 10 mA. Let us estimate a tolerable electron current drain of 1 mA. Since the random electron current density is 38 mA/cm^2 ("Plasma Potential"), 1 mA would be drawn by a collection area of 2.6 10^{-2} cm^2. Imagine a thin cylindrical wire probe 0.25 cm in length; such a collection area would correspond to a cylinder radius of 166 μm. But this radius corresponds to the sum of the probe and sheath radii (Figure 3-10) and the sheath itself is going to be ~ 1 Debye length, which alone is 105 μm for our example ("Debye Shielding")!

The effect of attempting to draw too much electron current from the plasma is illustrated in Figure 3-11 where the probe circuit of Figure 3-7 is redrawn along with the discharge circuit. The electron current to the probe is in addition to the electron current to the anode. So either the ion current to the cathode must increase or the electron current to the anode must decrease. Under normal circumstances where the probe circuit supplies very little power to the discharge, the latter dominates. A decrease in the electron current to the cathode is accomplished by an increase in the plasma potential, causing more electron retardation in the anode sheath. One arrives at the same result by arguing that the probe starts to drain the plasma of electrons, leaving it space charge positive so that the plasma potential has to rise; or by arguing that the probe becomes the new anode as soon as its potential exceeds that of the original anode and that the plasma potential is determined by the anode potential and the need to maintain current continuity in the circuit. Coburn and Kay (1972) have encountered just this difficulty of not being able to find a small enough probe for sputtering discharges, and, using an independent technique to determine plasma potential based on measuring the energy distribution of ions accelerated across a sheath, have found that application of positive probe voltages serves only to increase the plasma potential, in agreement with the above argument.

So we are left with the conclusion that, at least in the rather tenuous discharges of sputtering and plasma etching, the plasma potential will be the most positive potential in the system. This becomes increasingly true with increasing size of the perturbing electrode.

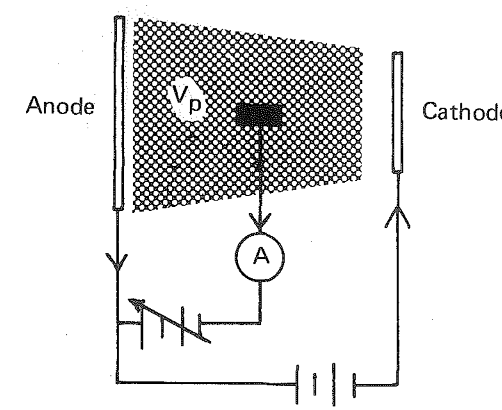


Figure 3-11. Schematic of probe and discharge circuits

SHEATH FORMATION AND THE BOHM CRITERION

Earlier in this chapter, in the section on "Plasma Potential", we calculated the random ion current density $n_i\bar{c}_i/4$ which flows in the plasma and found that it had a value of 21 $\mu A/cm^2$ for a typical plasma of density $10^{10}/cm^3$ and ion temperature 500K. We further reasoned, in "Sheath Formation at a Floating Substrate", that the ion current density to any object more negative than the plasma potential should be equal to the random ion current density. In that section, the substrate was electrically floating so that the net current flow was zero. However, it is a simple matter to extend the arguments given there to include the case where there is a net ion current to the object, and one would still expect to find a current density of 21 $\mu A/cm^2$. But if we measure the current density at the target in a dc sputtering glow discharge, as in Figure 4-1 of Chapter 4, we find that the current density is larger, of the order of a few tenths of a milliamp per square centimetre. Although we shall learn in the next chapter that some of this latter current is due to the emission of electrons from the target, there is apparently a discrepancy in these two values of current density. Although the ion temperature which we used to derive $\bar{c}_i$ was only an estimate, this estimate can't be far out, and anyway $\bar{c}_i$ varies as the square root of the ion temperature, which is rather a weak dependence. So the reason for the discrepancy must lie elsewhere.

The problem turns out to be due to an oversimplification of the model for the sheath. We had assumed that the sheath terminated at the plane where the ion and electron densities became equal, to become an undisturbed plasma again (Figure 3-5). In fact, between these two regions there is a quasi-neutral *transition region* of low electric field (Figure 3-12), and the effect of this region is to increase the velocity of ions entering the sheath proper. The existence of this velocity change was demonstrated by Bohm (1949) and the resulting criterion for sheath formation has come to be known as the *Bohm sheath criterion*, and is demonstrated as follows:

In Figure 3-12, we assume a monotonically decreasing potential V(x) as ions traverse the positive space charge sheath; x = 0 corresponds to the boundary between the two regions so that $n_i(0) = n_e(0)$, i.e. space charge neutrality at x = 0. We also assume that the sheath is collisionless and the consequent absence of ionization ensures that the ion current $e\,n_i(x)\,u(x)$ is constant.

Conservation of energy for the ions requires that

$$\tfrac{1}{2}m_i u(x)^2 = \tfrac{1}{2}m_i u(0)^2 - e[V(x) - V(0)]$$

$$\therefore \quad u(x) = \left(u(0)^2 - \frac{2e[V(x) - V(0)]}{m_i}\right)^{1/2}$$

and

$$n_i(x) = \frac{n_i(0)u(0)}{u(x)} = n_i(0)\left(1 - \frac{2e[V(x) - V(0)]}{m_i u(0)^2}\right)^{-1/2}$$

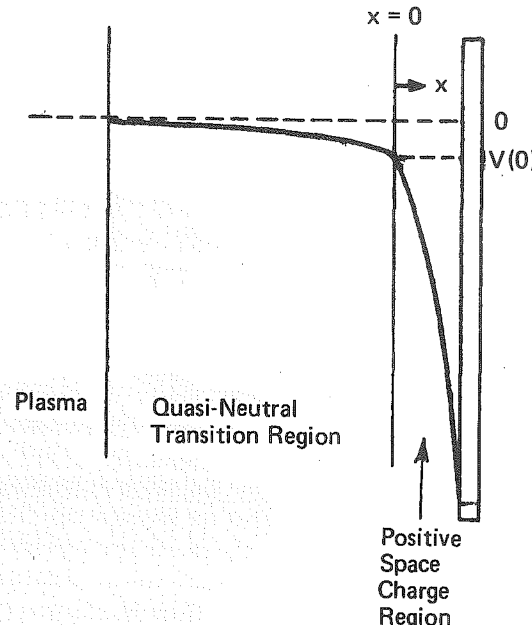


Figure 3-12. Potential variation near a negative electrode. Density $n_i(x)$ and potential V(x) at $x \geq 0$

By the Boltzmann relation for the electrons

$$n_e(x) = n_e(0) \exp \frac{e[V(x) - V(0)]}{kT_e}$$

Poisson's equation is then

$$\frac{d^2\phi}{dx^2} = \frac{e}{\epsilon_0}(n_e(x) - n_i(x))$$

$$= e n_e(0)\left(\exp \frac{e[V(x) - V(0)]}{kT_e} - \left(1 - \frac{2e[V(x) - V(0)]}{m_i u(0)^2}\right)^{-1/2}\right)$$

But if this is to be a positive space charge sheath, then d^2V/dx^2 must be negative for all x > 0 (and zero for x = 0)

i.e.
$$\left(1 - \frac{2e[V(x) - V(0)]}{m_i u(0)^2}\right)^{-1/2} > \exp \frac{e[V(x) - V(0)]}{kT_e}$$

Squaring and inverting, then

$$\exp - \frac{2e[V(x) - V(0)]}{kT_e} > 1 - \frac{2e[V(x) - V(0)]}{m_i u(0)^2}$$

We now restrict out attention to the beginning of the space charge sheath where $V(x) - V(0)$ is very small compared to kT_e so that we can expand and approximate the exponential thus:

$$1 - \frac{2e[V(x) - V(0)]}{kT_e} > 1 - \frac{2e[V(x) - V(0)]}{m_i u(0)^2}$$

i.e. $$u(0) > \left(\frac{kT_e}{m_i}\right)^{1/2}$$

This says that the ion velocity on entering the sheath must be greater than $(kT_e/m_i)^{1/2}$, i.e. is determined by the electron temperature, which is a rather peculiar result and demonstrates how the ion and electron motions are coupled. Chen (1974) demonstrates (Figure 3-13) that the physical significance of the criterion is that the acceleration of ions in the sheath and repulsion of electrons there, both of which decrease the relevant particle volume densities, must be such that the ion density decreases less rapidly than the electron density across the sheath. This is equivalent to the requirement that d^2V/dx^2 is negative, and it is clear from Figure 3-13 that this requirement is most stringent at the beginning of the sheath where $V(x) - V(0)$ is very small, as we had assumed.

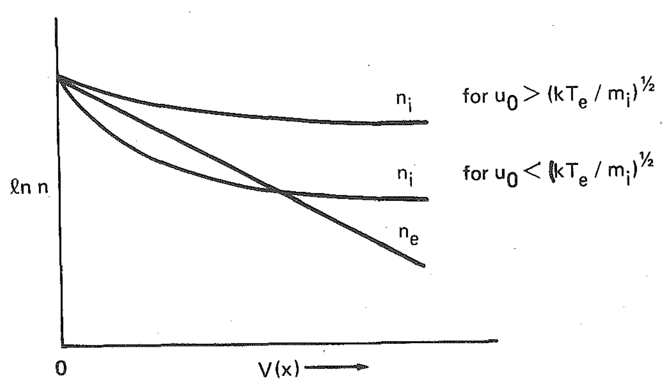


Figure 3-13. Variation of ion and electron density with potential V(x) in a sheath, for two cases: u_0 greater than and u_0 less than the critical velocity $(kT_e/m_i)^{1/2}$. From Chen 1974.

How do the ions acquire this velocity? There must be an electric field across the transition region so as to give the ions a directed velocity of u(0) towards the electrode. If we assume that the ion temperature is negligibly small so that the random motion of the ions can be neglected, then since the potential at the boundary is V(0) with respect to the plasma,

$$\tfrac{1}{2} m_i u(0)^2 = eV(0)$$

$$\therefore \quad V(0) = \frac{m_i u(0)^2}{2e} = \frac{m_i kT_e}{2e\, m_i} = \frac{kT_e}{2e}$$

The existence of a field in the transition region does not contradict our earlier claim that the plasma is equipotential, since that claim was qualified then to the extent that voltages of the order of kT_e/e could 'leak' into the plasma, and here we see an example of this.

We can pursue the exercise further to calculate the ion flux at the sheath boundary. Since the potential there is V(0) with respect to the plasma in which the electron density is n_e, then using the Boltzmann relation again,

$$\begin{aligned} n_e(0) &= n_e \exp - \frac{V(0)}{kT_e} \\ &= n_e \exp - \tfrac{1}{2} \\ &= 0.6\, n_e \end{aligned}$$

since $V(0) = kT_e/2$. But $n_e(0) = n_i(0)$, and so the ion flux is given by

$$n_i(0)\, u(0) = 0.6\, n_e \left(\frac{kT_e}{m_i}\right)^{1/2}.$$

Substituting in the values from Figure 3-2 again, we obtain an ion current density of 0.2 mA/cm^2, which is more like reality. However, this derivation is still not quite realistic since it assumes that the ion temperature is zero, which is never so; and that there are no collisions in the sheath, which is not true for the sheaths that form in front of our glow discharge cathodes, although it is reasonably true for the much thinner sheaths that form in front of low voltage anodes and probes. We also know that the cathode current depends on the cathode voltage in practice, and the ion current expression derived above does not explicitly include the electrode voltage except to the extent that the electrode voltage does control the electron temperature and plasma density, as we shall see when we explore how discharges are maintained, in the next chapter.

The Floating Potential — Again

The effect of the Bohm criterion is to increase the ion flux to any object negatively biased with respect to the plasma. In particular it will change the ion flux to a floating substrate. We had calculated the floating potential earlier in the chapter, and apparently we must now change this to allow for this changed ion flux. Using a similar derivation to before, the criterion for net zero current becomes:

$$\left(n_e \exp - \frac{e(V_p - V_f)}{kT_e}\right)\frac{c_e}{4} = n_i\, 0.6\left(\frac{kT_e}{m_i}\right)^{1/2}$$

$$\therefore \quad V_p - V_f = -\frac{kT_e}{e}\,\ell n\; 2.4\left(\frac{kT_e}{m_i}\right)^{1/2}\left(\frac{\pi m_e}{8kT_e}\right)^{1/2}$$

$$= \frac{kT_e}{2e}\,\ell n\left(\frac{m_i}{2.3\, m_e}\right)$$

In our example (Figure 3-2), $V_p - V_f$ should have a value of 10.4 V compared with 15 V as derived earlier. The larger ion flux requires a larger electron flux for current neutrality, and so a smaller electron retarding potential. The logarithmic dependence minimizes the change in potential due to the increased ion flux.

PLASMA OSCILLATIONS

One might at first think that in a dc plasma, all parameters would be time independent. This is not the case. Although the electrons and ions are in equilibrium as a whole, this is only the average result of the many detailed interactions. If a plasma, or even a small section of it, is perturbed from neutrality for any reason, then there will be large restoring forces striving to re-establish charge neutrality. Because of the large mass difference between ions and electrons, it will be the electrons which will first respond to the restoring forces. We shall find that these restoring forces are proportional to displacement, which is just the condition for *oscillations.*

Electron Oscillations

The frequency of oscillation can be found in the following way. Consider a slab of plasma of thickness L and density n (Figure 3-14a) and then suppose that all the electrons are displaced a distance Δ along the x axis by some external force (Figure 3-14b). The regions between x = O and x = L, and for all x < - Δ, will remain space charge neutral. However, the electrons between x = - Δ and x = O will give rise to a space charge there. By Poisson's equation, since the electron density is n,

$$\frac{d\mathcal{E}}{dx} = \frac{ne}{\epsilon_o}$$

Integrating,

$$\mathcal{E} = \frac{ne\Delta}{\epsilon_o}$$

But the action of the field $\mathcal{E}$ is to exert a restoring force on the electron:

$$m_e \frac{d\Delta}{dt} = -e\mathcal{E}$$

$$= -\frac{ne^2}{\epsilon_o}\,\Delta$$

When released, the inertia of the electrons will cause them to overshoot their original positions, and they will continue to describe motion determined by the same equation, where Δ now becomes a function of time.

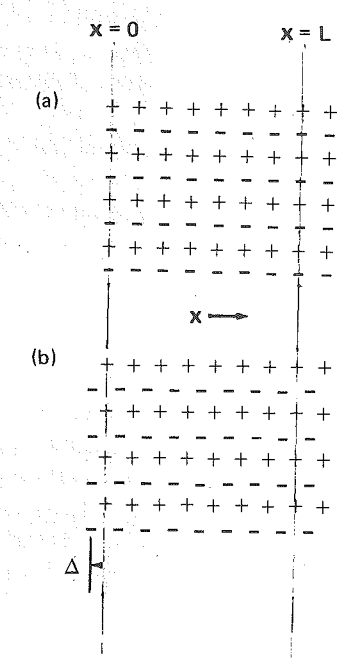


Figure 3-14. When a uniformly distributed plasma (a) is displaced (b), oscillations may result

This is just an equation of simple harmonic motion of angular frequency ω_e given by

$$\omega_e = \left(\frac{ne^2}{m_e\epsilon_o}\right)^{1/2}$$

which corresponds to a *plasma frequency* of 8.98 10^3 $n_e^{1/2}$ Hz, where n_e is the density per cm^3. So for a plasma density of $10^{10}/cm^3$, the plasma frequency is 9 10^8 Hz, which is very much higher than the 13.56 Mhz with which rf plasmas are usually driven (see Chapter 5).

The period of these oscillations, about 1 nS in our example, tells us the response time of the plasma to charge fluctuations. Since the frequency is determined by the interaction between the ions and the electrons, it is not too surprising to find that ω_e is related to the Debye length λ_D.

$$\lambda_D\,\omega_e = \left(\frac{\epsilon_o\,kT_e}{ne^2}\right)^{1/2}\left(\frac{mne^2}{m_e\,\epsilon_o}\right)^{1/2} = \left(\frac{kT_e}{m_e}\right)^{1/2} \simeq \overline{c_e}$$

This relationship enables us to give some more physical meaning to λ_D and ω_e. (Mitchner and Kruger 1973). In the derivation of the plasma frequency, the time required for the electron displacement Δ to build up would be about $\Delta/\overline{c_e}$. This displacement would be impeded if the response time $1/\omega_e$ of the electrons was shorter than $\Delta/\overline{c_e}$. Therefore regions of disturbance will be restricted to a distance Δ given by

$$\frac{\Delta}{\overline{c_e}} \simeq \frac{1}{\omega_e}$$

or

$$\Delta \simeq \frac{\overline{c_e}}{\omega_e} \simeq \lambda_D$$

This is consistent with our earlier picture of λ_D as the extent of deviation from charge neutrality.

Alternatively, the relationship $\lambda_D\,\omega_e \simeq \overline{c_e}$ says that the electrons can move a distance of about λ_D in a time of $1/\omega_e$. This means that if the plasma is disturbed by an electromagnetic wave of angular frequency ω, then the plasma electrons can respond fast enough to maintain neutrality if $\omega < \omega_e$. So ω_e is the minimum frequency for propagation of longitudinal waves in the plasma.

In our simple derivation, the resulting oscillation was stationary, but we ignored the thermal random motion of the electrons, and when this is taken into account it can be shown that disturbances can be propagated as waves. In fact a plasma is very rich in wave motion. This gives a means for propagating energy through the plasma. There can also be energy interchange between these plasma waves and fast electrons which travel through the plasma. We shall need to consider these energy exchanges in Chapter 4.

Ion Oscillations

Just as the electrons could oscillate in the plasma, so also can the ions. The mass of the ions ensures that their oscillations are so slow that the electrons can maintain thermal equilibrium. The ion frequency is more complex to find than the electron frequency; although we were able to ignore the ion motion when deriving the electron frequency, we cannot ignore the electron motion when deriving the ion frequency. However, in the case where T_e is large, the ion frequency simplifies to the same form $\omega_i = (ne^2/m_i\epsilon_o)^{1/2}$ as the electron plasma oscillation frequency. For our example, this would amount to 3.3 Mhz. In the more general case, these low frequency oscillations occur with frequencies between zero and a few megahertz. They can be observed (Pekarek and Krejci 1961) as *striations* in the positive column of dc glow discharge tubes (our glow discharge processes don't usually have positive columns – see Chapter 4). When the ion frequency is low enough, these striations can be observed with the naked eye as slow moving or even stationary regions of higher optical emission intensity.

AMBIPOLAR DIFFUSION

Finally, there is a topic which we won't be using much in this book, but it does play a significant role in plasmas and therefore we need to know of the concept.

Whenever there is a concentration gradient of particles, the random motion of the particles results in a net flow down the gradient. This is the phenomenon of *diffusion*. The resulting ion and electron current densities in the presence of a diffusion gradient dn/dx (assumed in one dimension for simplicity) can be written:

$$j_e = -e\,D_e\,\frac{dn_e}{dx}$$

$$j_i = -e\,D_i\,\frac{dn_i}{dx}$$

D_e and D_i are the diffusion coefficients of the electrons and ions respectively. It is possible to show that the diffusion coefficient and mobility μ (the drift velocity in unit electric field) are related by temperature:

$$\frac{D}{\mu} = \frac{kT}{e}$$

This is *Einstein's relation*. We already know that the mobility of the electrons is very much greater than that of the ions, and therefore the electron diffusion coefficient will be very much greater than the ion diffusion coefficient. One might expect as a result that, in a region of concentration gradient, the electrons would stream out very much faster than the ions. This is initially true, but the exodus of the electrons leaves the rest of the plasma more positive and sets up a restraining electric field $\mathcal{E}$ which grows large enough to equalize the diffusion rates of the ions and electrons.

In the presence of both the resulting electric field $\mathcal{E}$ and the diffusion gradient, the resulting ion and electron densities can be written as follows:

$$j_i = e\, n_i\, \mu_i\, \mathcal{E} - e\, D_i \frac{dn_i}{dx}$$

$$j_e = -e\, n_e\, \mu_e\, \mathcal{E} - e\, D_e \frac{dn_e}{dx}$$

where μ_i and μ_e are the mobilities of the ions and electrons respectively. The equalization of diffusion rates is achieved by putting $j_i = j_e$ in our current flow equations. Since n_i and n_e (and hence their concentration gradients) are closely equal throughout the main body of the plasma, equating the current densities yields the following result for $\mathcal{E}$:

$$\mathcal{E}\, n_e (\mu_i + \mu_e) = (D_i + D_e) \frac{dn_e}{dx}$$

Substituting this value of $\mathcal{E}$ back, we obtain the following expression for the current flow of ions and electrons:

$$j_i = j_e = \left(\frac{D_e \mu_i + D_i \mu_e}{\mu_i + \mu_e}\right) \frac{dn_e}{dx}$$

So the collective behaviour of the ions and electrons causes them to move with the *same* diffusion coefficient. This is the phenomenon of *ambipolar diffusion*, which will apply to all motion within the plasma.

The current density formulation that we have used in this derivation, is borrowed from solid state physics where mobilities, which actually imply collision-dominated motion, are relevant. Using mobilities in the present application is stretching things rather, but we can take care of this by making the mobilities dependent on the electric field and plasma conditions. But again we run into the problem that simple concepts become rather complex when the details are considered.

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