CRYOGENICS

MICHAEL McCLINTOCK

Riehle Testing Machines
A Division of Ametek, Inc.

formerly with
Cryogenic Engineering Laboratory
National Bureau of Standards

New York
REINHOLD PUBLISHING CORPORATION
Chapman & Hall, Ltd., London

ELECTRICAL CONDUCTIVITY

Certainly one of the most outstanding properties of metals, in fact a property often used in the definition of a metal, is electrical conductivity. When an electric field is applied to a metal, its quasi-free conduction electrons are accelerated, resulting in the transport of electrical energy. If we adopt once again the point of view that this transport process by electron "particles" is limited by processes of collision, we can quickly derive a general expression for the electrical resistivity.

The mean time spent by an electron in free travel is

$$\tau = L/u$$

where L is again the mean free path, and u is the random mean thermal velocity. During this time the electron is given an acceleration in the direction of the field. From Newton's second law this (constant) acceleration, a, is

$$a = eE/m$$

where e is the charge on the electron, m is its mass, and E is the electric field strength. The electron therefore goes from zero velocity to $(eE/m)(L/u)$ in the direction of the field in this time, and its average velocity in this direction is therefore

$$u_d = \tfrac{1}{2}(eE/m)(L/u) \qquad (7.19)$$

This "drift velocity" is superposed on the randomly directed thermal velocity of the electrons.

If there are n participating electrons per unit volume, then the current density, j, is given by

$$j = neu_d$$

and Ohm's law allows us to arrive at the resistivity, ρ, by substitution:

$$\rho = E/j = 2mu/(ne^2L) \qquad (7.20)$$

Here n is not the total number of free electrons per unit volume, but only those capable of exchanging energy with the applied field or the lattice in collision processes. We have already seen that this number is only a fraction of the total—those within about kT of the Fermi level.

The values of m and e are, of course, constant, and for a given metal n is also constant. Furthermore, u is only slightly dependent on temperature because of the degeneracy of the conduction electrons, a fact we are already familiar with from the section on thermal conductivity in this chapter. To a good approximation, therefore, the problem of electrical resistivity reduces to one of considering the mechanisms limiting the mean free path.

In the previous section on thermal conductivity in metals we noted that the mean free path of electrons is limited by collisions with phonons and with imperfections in the crystal lattice. The first is a dynamic scattering

mechanism, the second, static. As in the case of thermal conductivity these scattering mechanisms can be considered independent to a very good approximation, and we can write for the electrical resistivity, ρ:

$$\rho = \rho_i + \rho_r \qquad (7.21)$$

Here ρ_i is the resistivity caused by phonon scattering in a perfect crystal lattice, i.e., the resistivity of an "ideal" crystal, and ρ_r is that caused by imperfections, i.e., the "residual" resistivity of a real crystal at 0°K when phonon scattering has become inoperative. The lattice vibrations depend on temperature, and lead to a term with a temperature dependence in the electrical resistivity. The number of impurities and defects are independent of temperature, and lead to a term with no temperature dependence. This additive expression is Matthiessen's rule, formulated empirically from early observations on metals of different purity.

At high temperatures, roughly greater than θ_D, scattering of electrons by phonons in pure metals predominates over imperfection scattering. Since the effectiveness of phonon scattering is directly proportional to temperature in this range, we expect a linear relationship between electrical resistivity and temperature.

At low temperatures, where phonon scattering becomes less and less important, the resistivity is governed by imperfection scattering, and since this is independent of temperature, the resistivity levels off at a constant value at low temperatures. The resulting curve for a relatively pure metal is illustrated in Figure 7.14. From a linear region at higher temperatures, the resistance becomes constant at low temperatures, the constant or residual value depending on the degree of perfection of the lattice.

From a theoretical analysis by Bloch and an independent semiempirical treatment of experimental observations by Grüneisen, the following expression for electrical resistivity due to phonon scattering was evolved:

$$\rho = K\frac{T^5}{\theta_R{}^6}\int_0^{\theta_D/T}\frac{x^5e^x dx}{(e^x-1)^2} = K\frac{T^5}{\theta_R{}^6}J_5\left(\frac{\theta_R}{T}\right) \qquad (7.22)$$

In this expression K is constant for a given metal, θ_R is a characteristic temperature for electrical resistance that compares conceptually with θ_D for thermal processes, and the other symbols are clear from Eq. (7.2). It is instructive, in fact, to compare these two equations for their similarity, and to use an analysis of Eq. (7.22) similar to that used before. For T/θ_R greater than about 0.5, Eq. (7.22) becomes very nearly

$$\rho = AT/\theta_R{}^2$$

which is the required linear relationship to temperature at high temperatures.

For T/θ_R less than about 0.1, Eq. (7.22) becomes approximately

$$\rho = BT^5/\theta_R{}^6$$

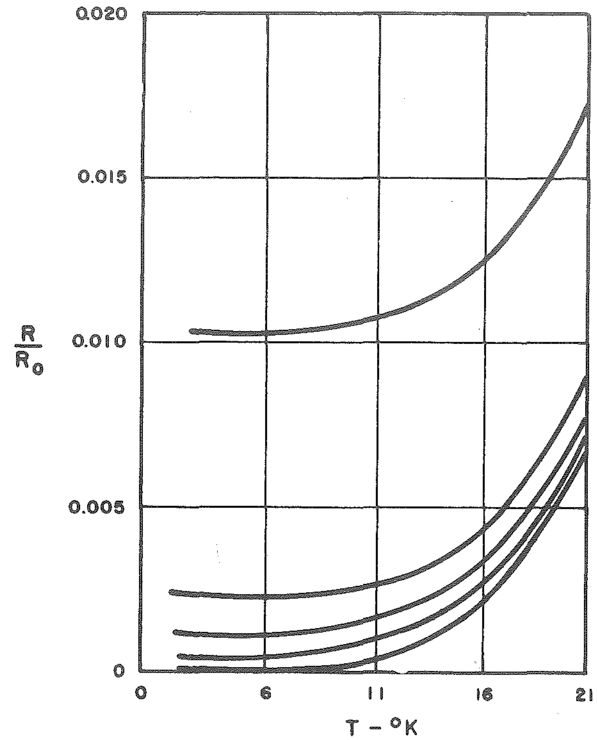

Figure 7.14. The electrical resistance of gold at low temperatures. Higher resistance curves are for gold of lesser purity. (Van den Berg, Thesis, Leiden, 1938)

and predicts that the resistance will have a T^5 dependence at very low temperatures. The validity of the Bloch-Grüneisen relation is illustrated in Figure 7.15, where the ratio of the resistance of five metals to their resistance at 0°C is plotted *vs* reduced temperature, which is the temperature divided by the appropriate θ_R for the metal. The relation has been observed to hold best for monovalent metals such as sodium, lithium, potassium, copper, silver, gold, etc., although to fit the data properly θ_R must be considered a function of temperature rather than a constant. This situation is similar to the procedure found necessary for θ_D when considering thermal processes on the basis of the Debye model. Other metals show a less satisfactory agreement with the Bloch-Grüneisen theory, and in fact some of the simpler metals in slightly impure form even show a resistance minimum at low temperatures.

The electrical resistance of alloys has a much different temperature de-

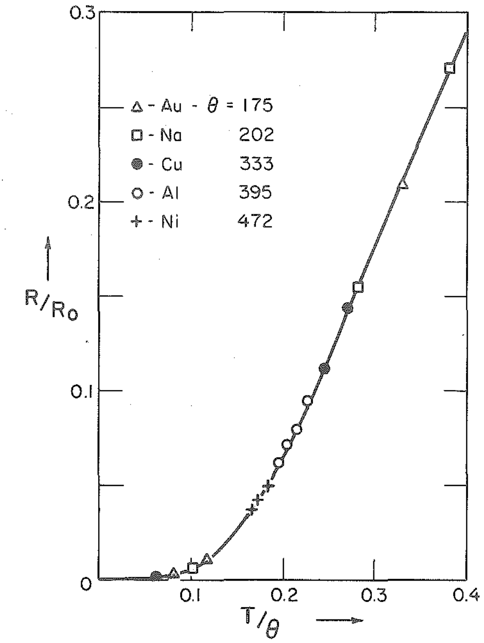

Figure 7.15. The reduced resistance of five metals at low reduced temperatures. [After Meissner, "Handbuch der Experimental Physik," Vol. 11, Pt. 2, p. 30, 1935]

pendence than that of relatively pure metals, but this presents no difficulty to the basic ideas of the theory we have given. Alloys and highly cold-worked metals represent materials in which the disorder of the lattice is so great that the scattering of electrons by imperfections overshadows that by phonons even at room temperature. The electron mean free path is therefore relatively constant, and consequently the electrical resistivity is roughly independent of temperature. As mentioned in Chapter 3, however, there are some alloys that show an unexplained decrease in resistivity with temperature, but the decrease is not more than about 20 per cent of the total in going from 300°K down to 4°K.

At present it is not possible to discuss a quantitative theory of imperfection scattering of electrons. Generally, severe cold working of a metal will affect its electrical resistance through the generation of dislocations and other lattice imperfections but a metal that already has a high resistance is not greatly affected by cold work. This is not true, however, of thermal conductivity, since the conduction of heat at low temperatures takes place largely by phonons, the scattering of which is more sensitive to extended defects in the lattice than to point defects. The increase of extended defects (dislocations, etc.) produced by cold work can therefore decrease the thermal conductivity of an alloy at low temperatures quite markedly while leaving its electrical resistance unaffected.

BIBLIOGRAPHY

Blackman, M., in "Handbuch der Physik," Vol. 7, Berlin, Springer-Verlag, 1955.
Carruthers, P., *Rev. Mod. Phys.*, **33**, 92 (1961).
Corruccini, R. J., *Chem. Eng. Progr.*, **53**, 262 (1957).
Keesom, P. H., and Pearlman, N., in "Handbuch der Physik," Vol. 14, Berlin, Springer-Verlag, 1956.
Kittel, C., "Solid State Physics," New York, John Wiley & Sons, Inc., 1957.
Klemens, P. G., in "Handbuch der Physik," Vol. 14, Berlin, Springer-Verlag, 1956.
MacDonald, D. K. C., in "Handbuch der Physik," Vol. 14, Berlin, Springer-Verlag, 1956.
MacDonald, D. K. C., "Progress in Metal Physics," Vol. 3, London, Pergamon Press, 1952.
Mendelssohn, K., "Cryophysics," New York, Interscience Publishers, Inc., 1960.
Parkinson, D. H., "Reports on Progress in Physics," Vol. 21, London, The Physical Society, 1958.
Scott, R. B., "Cryogenic Engineering," Princeton, New Jersey, D. Van Nostrand Company, 1959.
Squire, C. F., "Low Temperature Physics," New York, McGraw-Hill Book Company, Inc., 1953.
White, G. K., "Experimental Techniques in Low Temperature Physics," Oxford, Clarendon Press, 1959.
Zemansky, M. W., "Heat and Thermodynamics," New York, McGraw-Hill Book Company, Inc., 1957.