HALL EFFECT IN SEMICONDUCTORS

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"One day in the year of 1820, walking to his lecture at the University of Copenhagen, Oersted got an idea. If static electricity did not affect magnets in any way, maybe things would be different if one tried electricity moving through the wire connecting the two poles of the Volta pile. Arriving at the classroom filled with a crowd of young students, Oersted placed on the lecture table his Volta pile, connected the two opposite ends of it by a platinum wire, and placed a compass needle close to it. The needle, which was supposed to orient itself always in the north-south direction, turned around and came to rest in the direction perpendicular to the wire. The audience was not impressed but Oersted was."

George Gamow
Biography of Physics (Harper Brothers, 1961)

Magnetically operated Hall effect switches are based on the Hall effect in semiconductors. This magnetically operated position sensor is commercially available from Micro Switch (Honeywell).

Hall effect in a sample where there are both negative and positive charge carriers, e.g. electrons and holes in a semiconductor, involves not only the concentrations of electrons and holes, n and p respectively, but also the electron and hole drift mobilities, μ_e and μ_h. We first have to reinterpret the relationship between the drift velocity and the electric field, E.

If μ_e is the drift mobility and v_e the drift velocity of the electrons, then we have already shown that $v_e = \mu_e E$. This has been derived by considering the *net electrostatic force*, eE, acting on a single electron and the imparted acceleration $a = eE/m_e$. The drift is therefore due to the net force, $F_{net} = eE$, experienced by a conduction electron. If we were to keep eE as the *net force* F_{net} acting on a single electron then we would have found

$$v_e = \frac{\mu_e}{e} F_{net} \qquad (1)$$

Equation (1) emphasizes the fact that drift is due to a net force, F_{net}, acting on an electron. A similar expression would also apply to the drift of a hole in a semiconductor.

When both electrons and holes are present as in a semiconductor sample, both charge carriers experience a Lorentz force in the same direction since they would be drifting in the opposite directions as illustrated in Figure 1.
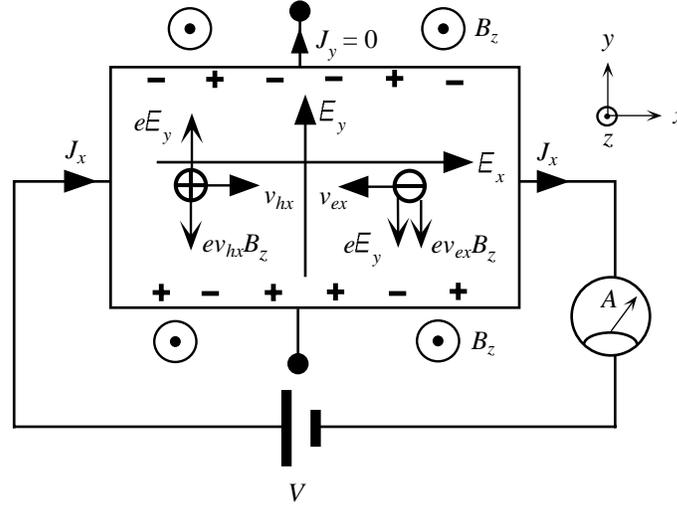
Hall effect for ambipolar conduction as in a semiconductor where there are both electrons and holes. The magnetic field B_z is out from the plane of the paper. Both electrons and holes are deflected toward the bottom surface of the conductor and consequently the Hall voltage depends on the relative mobilities and concentrations of electrons and holes.

Figure 1

Thus, both holes and electrons tend to pile near the bottom surface. The magnitude of the Lorentz force, however, will be different since the drift mobilities and hence drift velocities will be different. Once equilibrium is reached, there should be no current flowing in the y-direction as we have an open circuit. Let us suppose that more holes have accumulated near the bottom surface so that there is a built-in electric field E_y along y-direction as shown in Figure 1. Suppose that v_{ey} and v_{hy} are the *usual* electron and hole drift velocities in the $-y$ and $+y$ directions respectively (as if the electric field E_y existed alone in the $+y$ direction). In the y-direction there is no net current, therefore

$$J_y = J_h + J_e = epv_{hy} + env_{ey} = 0 \tag{2}$$

It is apparent that either the electron or the hole drift velocity must be reversed with respect to its usual direction to obtain a zero net current along y. (In Figure 1 this means holes are drifting in the opposite direction to E_y.) From Equation (2) we obtain

$$pv_{hy} = -nv_{ey} \tag{3}$$

We note that the net force acting on the charge carriers cannot be zero. This is impossible when two types of carriers are involved and that both carriers are drifting along y to give a net current J_y that is zero. This is what Equation (2) represents. We therefore conclude that, along y, both the electron and the hole must experience a driving force to drift them. The net force experienced by the carriers, as shown in Figure 1, is

$$F_{hy} = eE_y - ev_{hx}B_z \qquad \text{and} \qquad -F_{ey} = eE_y + ev_{ex}B_z \tag{4}$$

where v_{hx} and v_{ex} are the hole and electron drift velocities along x. We know that, in general, the drift velocity is determined by the net force acting on a charge carrier, that is, from Equation (1),

$$F_{hy} = ev_{hy}/\mu_h \qquad \text{and} \qquad -F_{ey} = ev_{ey}/\mu_e$$

so that Equation (4) becomes,

$$\frac{ev_{hy}}{\mu_h} = eE_y - ev_{hx}B_z \qquad\qquad \frac{ev_{ey}}{\mu_e} = eE_y + ev_{ex}B_z$$

where v_{hy} and v_{ey} are the hole and electron drift velocities along y. Substituting $v_{hx} = \mu_h E_x$ and $v_{ex} = \mu_e E_x$, these become

$$\frac{v_{hy}}{\mu_h} = E_y - \mu_h E_x B_z \qquad\qquad \frac{v_{ey}}{\mu_e} = E_y + \mu_e E_x B_z \qquad\qquad (5)$$

From Equation (5) we can substitute for v_{hy} and v_{ey} in Equation (3) to obtain

$$p\mu_h E_y - p\mu_h^2 E_x B_z = -n\mu_e E_y - n\mu_e^2 E_x B_z$$

or
$$E_y(p\mu_h + n\mu_e) = B_z E_x(p\mu_h^2 - n\mu_e^2) \qquad\qquad (6)$$

We now consider what happens along the x-direction. The total current density is finite and is given by the usual expression,

$$J_x = epv_{hx} + env_{ex} = (p\mu_h + n\mu_e)eE_x \qquad\qquad (7)$$

We can use Equation (7) to substitute for E_x in Equation (6), to obtain

$$eE_y(n\mu_e + p\mu_h)^2 = B_z J_x(p\mu_h^2 - n\mu_e^2)$$

The Hall coefficient, by definition, is $R_H = E_y/J_x B_z$ so that

$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e\left(p\mu_h + n\mu_e\right)^2} \qquad\qquad \text{\textit{Hall Effect for ambipolar conduction}} \qquad (8)$$

or
$$R_H = \frac{p - nb^2}{e\left(p + nb\right)^2} \qquad\qquad \text{\textit{Hall Effect for ambipolar conduction}} \qquad (9)$$

where $b = \mu_e/\mu_h$. It is clear that the Hall coefficient depends on both the drift mobility ratio and the concentrations of holes and electrons. For $p > nb^2$, R_H will be positive and for $p < nb^2$, it will be negative. We should note that when only one type of carrier is involved, e.g. electrons only, $J_y = 0$ requirement means that $J_y = env_{ey} = 0$, or $v_{ey} = 0$. The drift velocity along y can only be zero, if the net driving force, F_{ey}, along y is zero. This occurs when the Lorentz force just balances the force due to the built-in field.

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1. Example: Hall coefficient of intrinsic silicon

Intrinsic silicon has electron and hole concentrations, $n = p = n_i = 1.5 \times 10^{10}$ cm^{-3}, and electron and hole drift mobilities, $\mu_e = 1350$ cm^2 V^{-1} s^{-1}, $\mu_h = 450$ cm^2 V^{-1} s^{-1}. Calculate the Hall coefficient and compare it with a typical metal.

Solution

Given $n = p = n_i = 1.5 \times 10^{10}$ cm^{-3}, $\mu_e = 1350$ cm^2 V^{-1} s^{-1} and $\mu_h = 450$ cm^2 V^{-1} s^{-1} we have

$$b = \mu_e/\mu_h = 1350/450 = 3$$

then,
$$R_H = \frac{(1 \times 10^{16} \text{ m}^{-3}) - (1 \times 10^{16} \text{ m}^{-3})(3)^2}{(1.6 \times 10^{-19} \text{ C})\left[(1 \times 10^{16} \text{ m}^{-3}) + (1 \times 10^{16} \text{ m}^{-3})(3)\right]^2}$$

or
$$R_H = -208 \text{ m}^3 \text{ A}^{-1} \text{ s}^{-1}$$

which is orders of magnitude larger than that for a typical metal. All Hall effect devices use a semiconductor rather than a metal sample.

2. Example: Zero Hall coefficient in a semiconductor

Given the mass action law, $np = n_i^2$, find the electron concentration when the Hall coefficient is zero for a semiconductor. Using $n_i = 1.5 \times 10^{10}$ cm^{-3}, and electron and hole drift mobilities, $\mu_e = 1350$ cm^2 V^{-1} s^{-1} and $\mu_h = 450$ cm^2 V^{-1} s^{-1}, what are n and p in Si for zero R_H?

Solution

Substituting the mass action law $p = n_i^2/n$ into Equation (9) we get

$$R_H = \frac{p - nb^2}{e(p + nb)^2} = \frac{\dfrac{n_i^2}{n} - nb^2}{e\left(\dfrac{n_i^2}{n} + nb\right)^2} = 0$$

that is $\dfrac{n_i^2}{n} - nb^2 = 0$

solving, $n = n_i/b = 0.33n_i = 5 \times 10^9$ cm^{-3}

   Obviously the corresponding hole concentration, $p = bn_i$ or 4.5×10^{10} cm^{-3}.

3. Example: Maximum Hall coefficient in a semiconductor

Given the mass action law, $np = n_i^2$, find n for maximum R_H (negative and positive). Assume that the drift mobilities remain relatively unaffected as n changes (due to doping). Given the electron and hole drift mobilities, $\mu_e = 1350$ cm^2 V^{-1} s^{-1}, $\mu_h = 450$ cm^2 V^{-1} s^{-1} for silicon, determine n for maximum R_H in terms of n_i.

Solution

Substituting the mass action law $p = n_i^2/n$ into Equation (9) we get

$$R_H = \frac{p - nb^2}{e(p + nb)^2} = \frac{\dfrac{n_i^2}{n} - nb^2}{e\left(\dfrac{n_i^2}{n} + nb\right)^2} = \frac{u}{v}$$

where u and v represent the numerator and denominator as a function of n.

Then $\dfrac{dR_H}{dn} = \dfrac{u'v - uv'}{v^2} = 0$

where primes are derivatives with respect to n. This means that $u'v - uv' = 0$, so that,

$$u'v - uv' = \left[-\frac{n_i^2}{n^2} - b^2\right]\left[e\left(\frac{n_i^2}{n} + nb\right)^2\right] - \left[\frac{n_i^2}{n} - nb^2\right]\left[2e\left(\frac{n_i^2}{n} + nb\right)\left(-\frac{n_i^2}{n^2} + b\right)\right] = 0$$

   We can multiply through by n^3 and then combine terms and factor to obtain,

$$b^3 n^4 - [3n_i^2 b(1 + b)]n^2 + n_i^4 = 0$$

or,　　　　　　　　$b^3 x^2 + [-3b(1-b)]x + 1 = 0$

where $x = (n/n_i)^2$. This is a quadratic equation in x. Its solution is,

$$x = \frac{n^2}{n_i^2} = \frac{3b(1+b) \pm \sqrt{9b^2(1+b)^2 - 4b^3}}{2b^3}$$

For Si, $b = 3$, and we have two solutions corresponding to $n/n_i = 1.14$ and $n/n_i = 0.169$, or $p/n_i = 1/0.169 = 5.92$.

4. Example: Hall coefficient of a semiconductor

Given the mass action law, $np = n_i^2$, and the electron and hole drift mobilities, $\mu_e = 1350$ cm^2 V^{-1} s^{-1}, $\mu_h = 450$ cm^2 V^{-1} s^{-1} for silicon, that is $b = 3$, sketch schematically how R_H changes with electron concentration n, given those values of n resulting in $R_H = 0$ and maximum R_H values in the above examples.
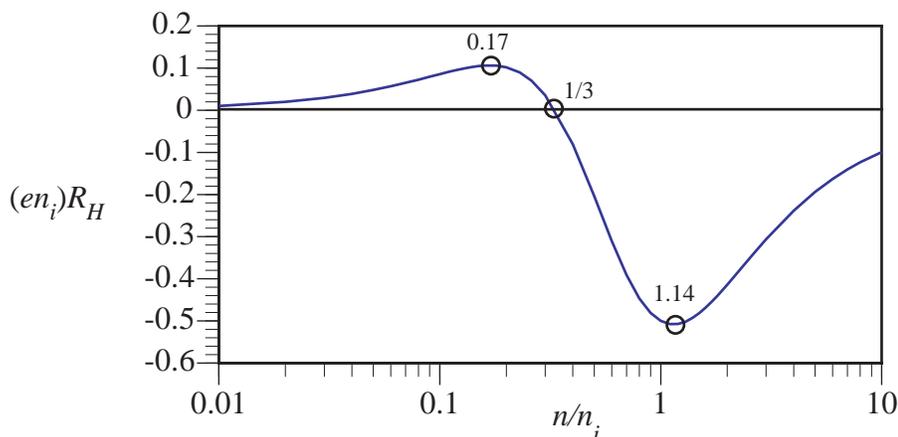
Solution

Substituting the mass action law $p = n_i^2/n$ into Equation (9) and using a normalized electron concentration $x = n/n_i$, we get,

$$R_H = \frac{p - nb^2}{e(p + nb)^2} = \frac{\dfrac{n_i^2}{n} - nb^2}{e\left(\dfrac{n_i^2}{n} + nb\right)^2} = \frac{\dfrac{1}{x} - xb^2}{en_i\left(\dfrac{1}{x} + xb\right)^2}$$

or　　　　　　　$$y = \frac{R_H}{\left(\dfrac{1}{en_i}\right)} = \frac{\dfrac{1}{x} - xb^2}{\left(\dfrac{1}{x} + xb\right)^2}$$

R_H vs. n obviously follows y vs. x, which is shown in Figure 2 for $b = 3$. It is left as an exercise to show that, when $n \gg n_i$, $R_H = -1/en$ and when $n \ll n_i$, $R_H = +1/ep$.

Normalized Hall coefficient vs. normalized electron concentration. Values 0.17, 1.14 and 0.33 shown are n/n_i values when the magnitude of R_H reaches maxima and zero respectively.

Figure 2

NOTATION

a	acceleration (m s^{-2})
b	ratio of electron to hole drift mobility ($b = \mu_e / \mu_h$)
B_z	applied magnetic field along the z direction, transverse to J_x (T)
e	electronic charge (1.602×10^{-19} C)
E_x	applied electric field along the x-direction, along the direction of current flow, J_x (V m^{-1})
E_y	electric field along the y-direction (or the Hall field), transverse to J_x and B_z (V m^{-1})
F_{ey}	external applied force acting on an electron in the conduction band along y (N)
F_{hy}	external applied force acting on a hole in the valence band along y (N)
F_{net}	net force (N)
J_x	current density along x (A m^{-2})
J_y	current density along y (A m^{-2})
KE	kinetic energy
m_e	mass of electron in free space (9.10939×10^{-31} kg)
n	concentration of electrons (number of electrons per unit volume) in the conduction band (m^{-3})
n_i	intrinsic concentration (m^{-3})
p	concentration of holes in the valence band (m^{-3})
R_H	Hall coefficient (m^3 C^{-1})
v_{ex}	drift velocity of an electron in the x-direction due to an applied external force along x (m s^{-1})
v_{ey}	drift velocity of an electron in the y-direction due to an applied external force along y (m s^{-1})
v_{hx}	drift velocity of a hole in the x-direction due to an applied external force along x (m s^{-1})
v_{hy}	drift velocity of a hole in the y-direction due to an applied external force along y (m s^{-1})
μ_e	drift mobility of electrons in the conduction band (m^2 V^{-1} s^{-1})
μ_h	drift mobility of holes in the valence band (m^2 V^{-1} s^{-1})

USEFUL DEFINITIONS

Hall coefficient (R_H) is a parameter that gauges the magnitude of the Hall effect. If E_y is the electric field set up in the y-direction due to a current density, J_x, along x and a magnetic field, $\mathbf{B}_z$, along z, then $R_H = E_y/J_x\mathbf{B}_z$.

Hall effect is a phenomenon that occurs in a conductor carrying a current when it is placed in a magnetic field perpendicular to the current. The charge carriers in the conductor become deflected by the magnetic field and give rise to an electric field (Hall field) that is perpendicular to both the current and magnetic field. If the current density, J_x, is along x and the magnetic field, B_z, is along z, then the Hall field is either along $+y$ or $-y$ depending on the polarity of the charge carriers in the material.

Drift mobility is the drift velocity per unit applied field. If μ_d is the mobility then the defining equation is $v_d = \mu_d E$ where v_d is the drift velocity and E is the electric field.

Drift velocity is the average velocity, over all the conduction electrons in the conductor, in the direction of an applied electrical force ($\mathbf{F} = -e\mathbf{E}$ for electrons). In the absence of an applied field, all the electrons are moving around randomly and the average velocity, over all the electrons, in any direction is zero. With an applied field, E_x, there is a net velocity per electron, v_{dx}, in the opposite direction to the field where v_{dx} depends on E_x via $v_{dx} = \mu_d E_x$ where μ_d is the drift mobility.

Lorentz force is the force experienced by a moving charge in a magnetic field. When a charge q is moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, then it experiences a force, $\mathbf{F}$, that is proportional to the magnitude of its charge, q, its velocity, $\mathbf{v}$ and the field $\mathbf{B}$ such that $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$.

Mass action law in semiconductor science refers to the law $np = n_i^2$ which is valid under thermal equilibrium conditions and in the absence of external biases and illumination.

Semiconductor is a nonmetallic element (e.g. Si or Ge) that contains both electrons and holes as charge carriers in contrast to an enormous number of electrons only as in metals. A hole is essentially a "half-broken" covalent bond which has a missing electron and therefore behaves effectively as if positively charged. Under the action of an applied field the hole can move by accepting an electron from a neighboring bond thereby passing on the "hole". Electron and hole concentrations in a semiconductor are generally many orders of magnitude less than those in metals, thus leading to much smaller conductivities.