

TESTING A BEST-FIT HYDRODYNAMICAL MODEL USING NEW OBSERVABLES

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UNICAMP

EVOLUTION OF A HEAVY-ION COLLISION



Figure: MADAI collaboration, Hannah Petersen and Jonah Bernhard

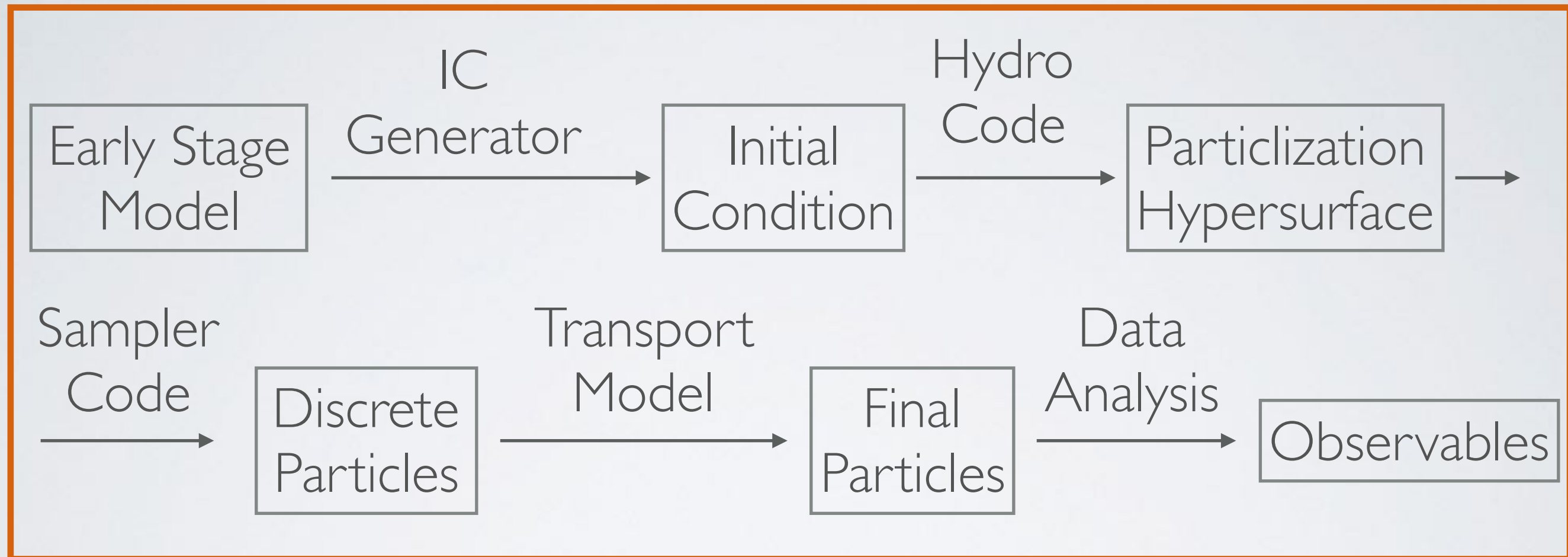
Early
Stage

Hydrodynamic
Stage

Particlization

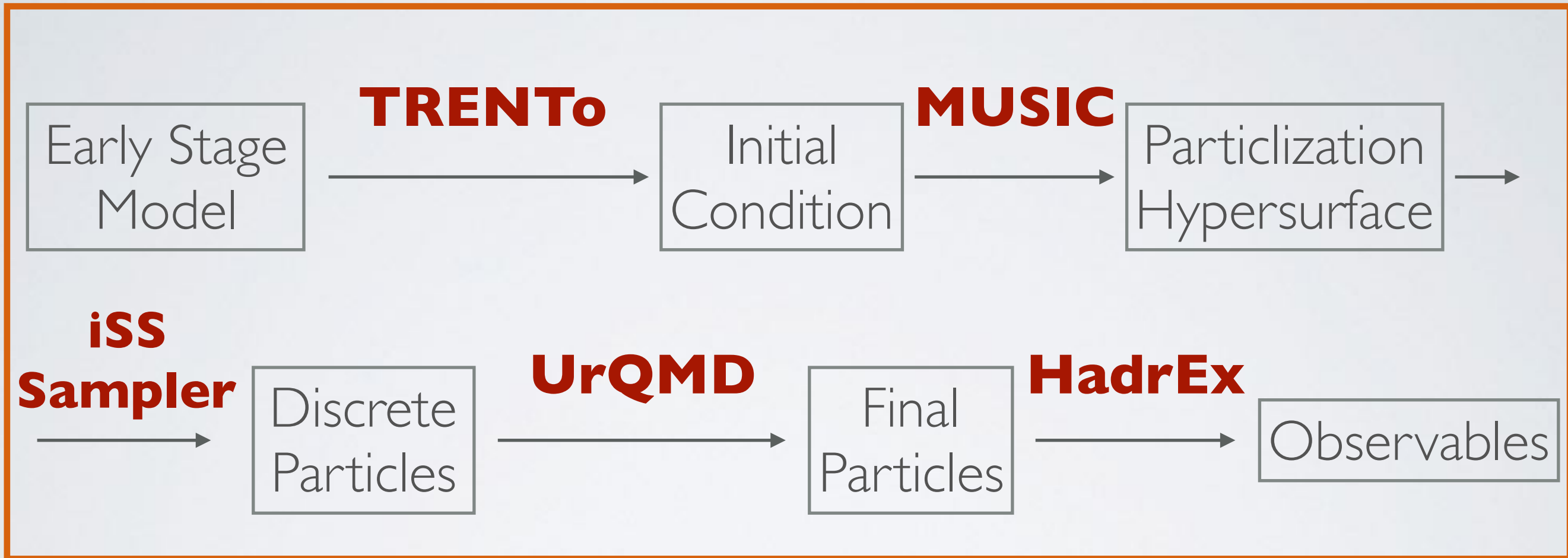
Hadron Gas

SIMULATING A HEAVY-ION COLLISION



The entire chain must be connected in a consistent way and requires state of the art computational resources.

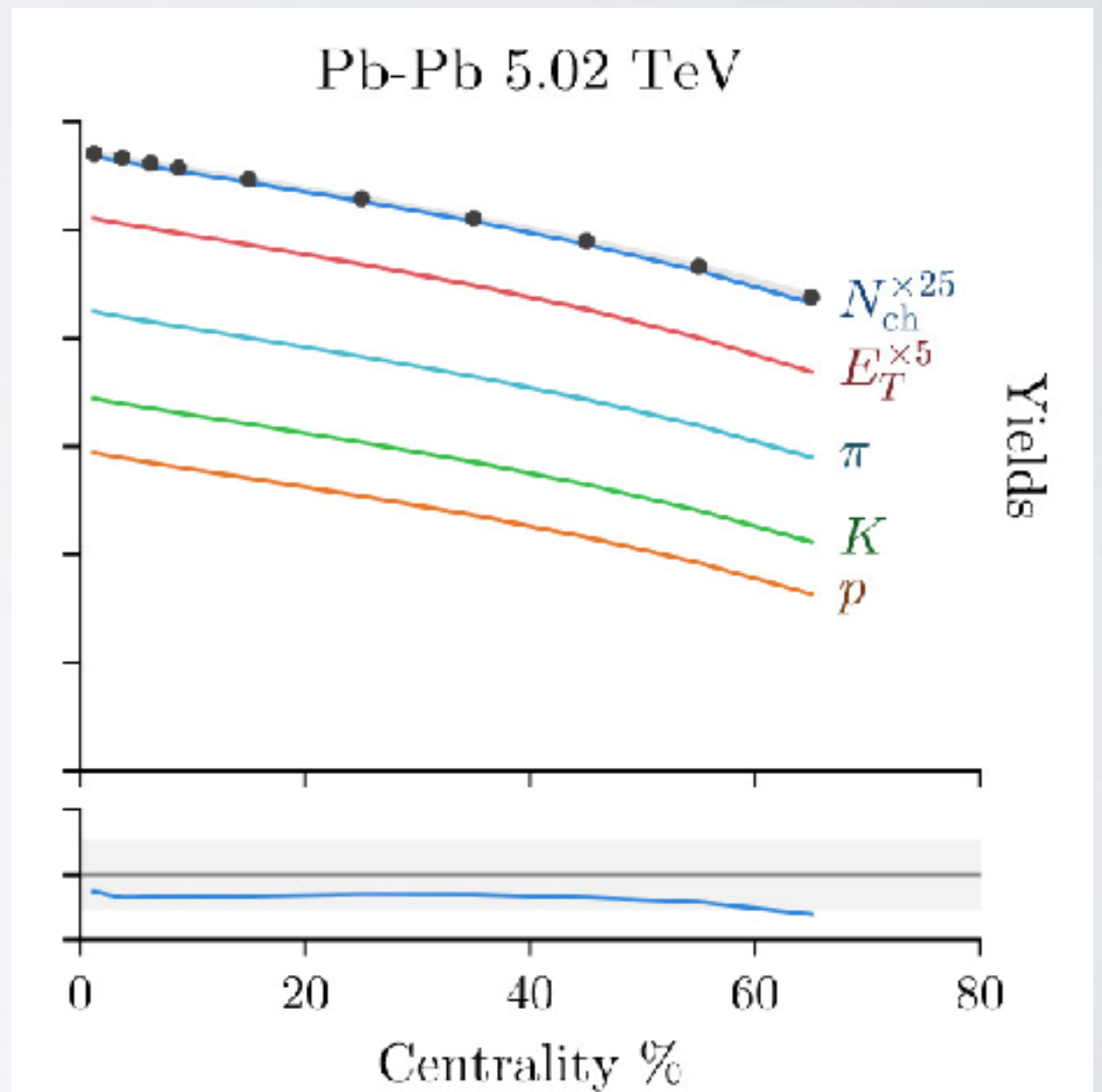
SIMULATING A HEAVY-ION COLLISION



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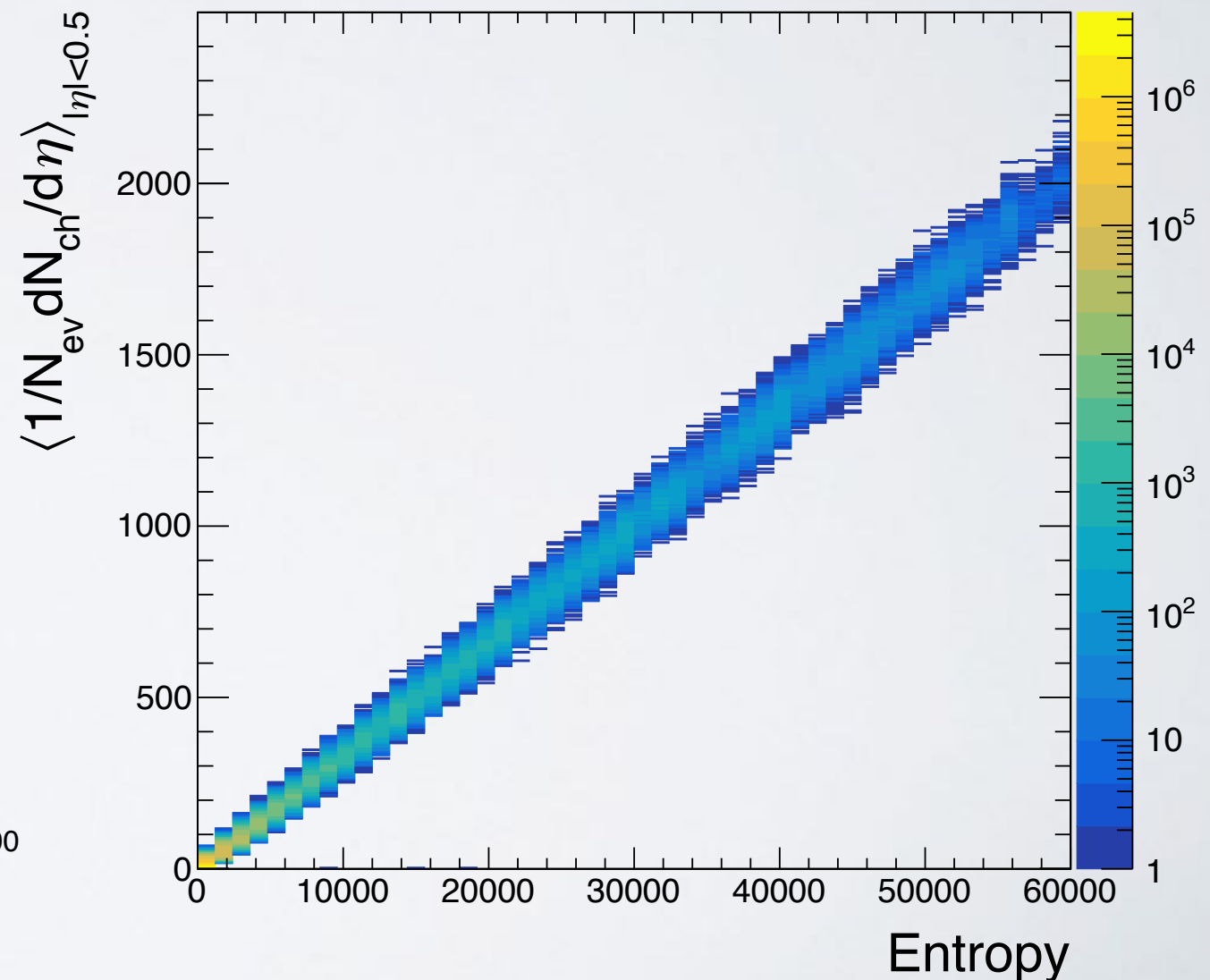
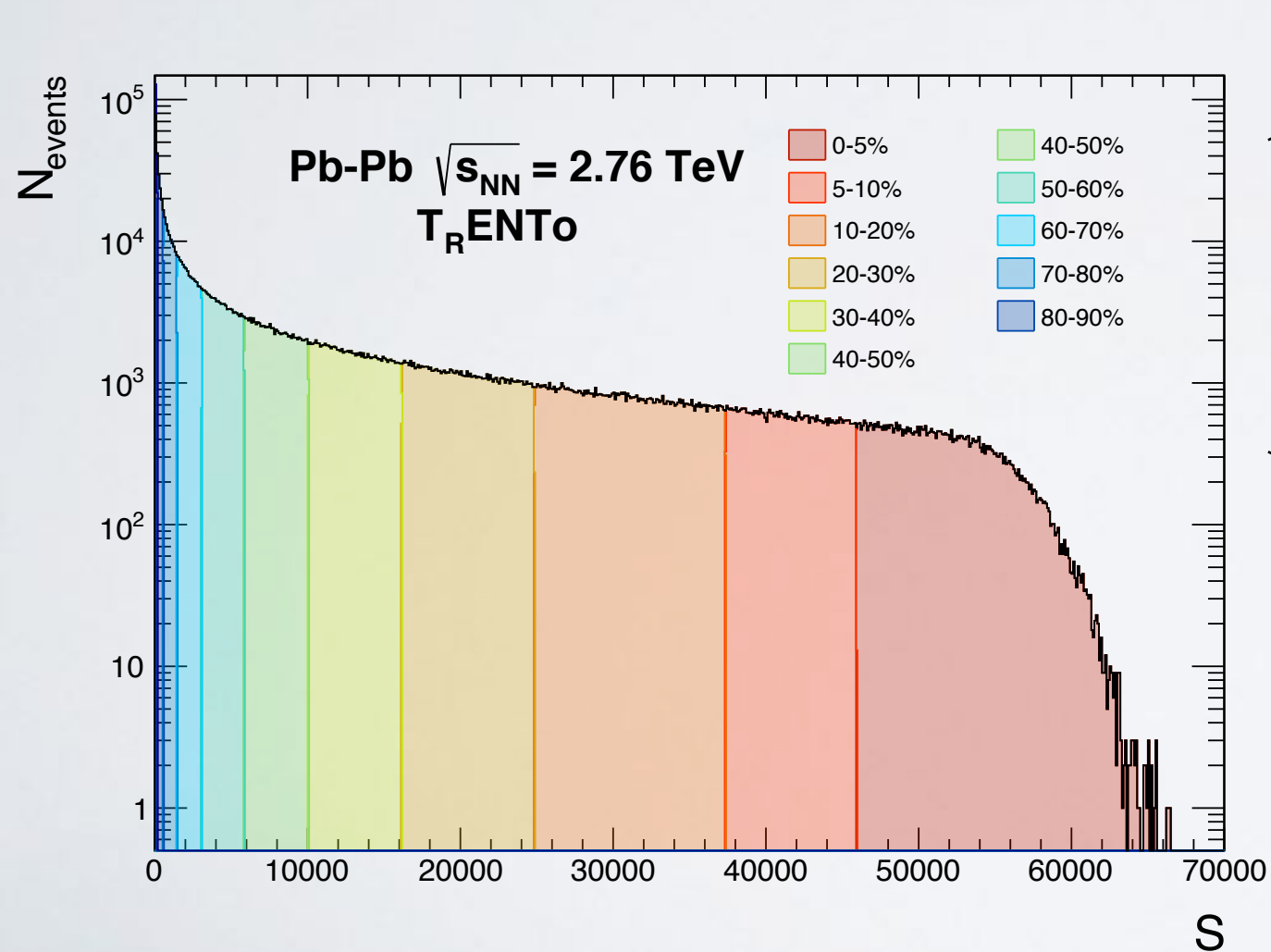
DUKE GROUP BAYESIAN-ANALYSIS

- The Duke group originally utilised a framework similar to ours (TRENTo + Free Streaming + VISH2+1 + FRZOUT + UrQMD) and obtained optimal a posteriori TRENTo parameters (arxiv: 1808.02106);
- We utilise these parameters, with a different overall normalisation;

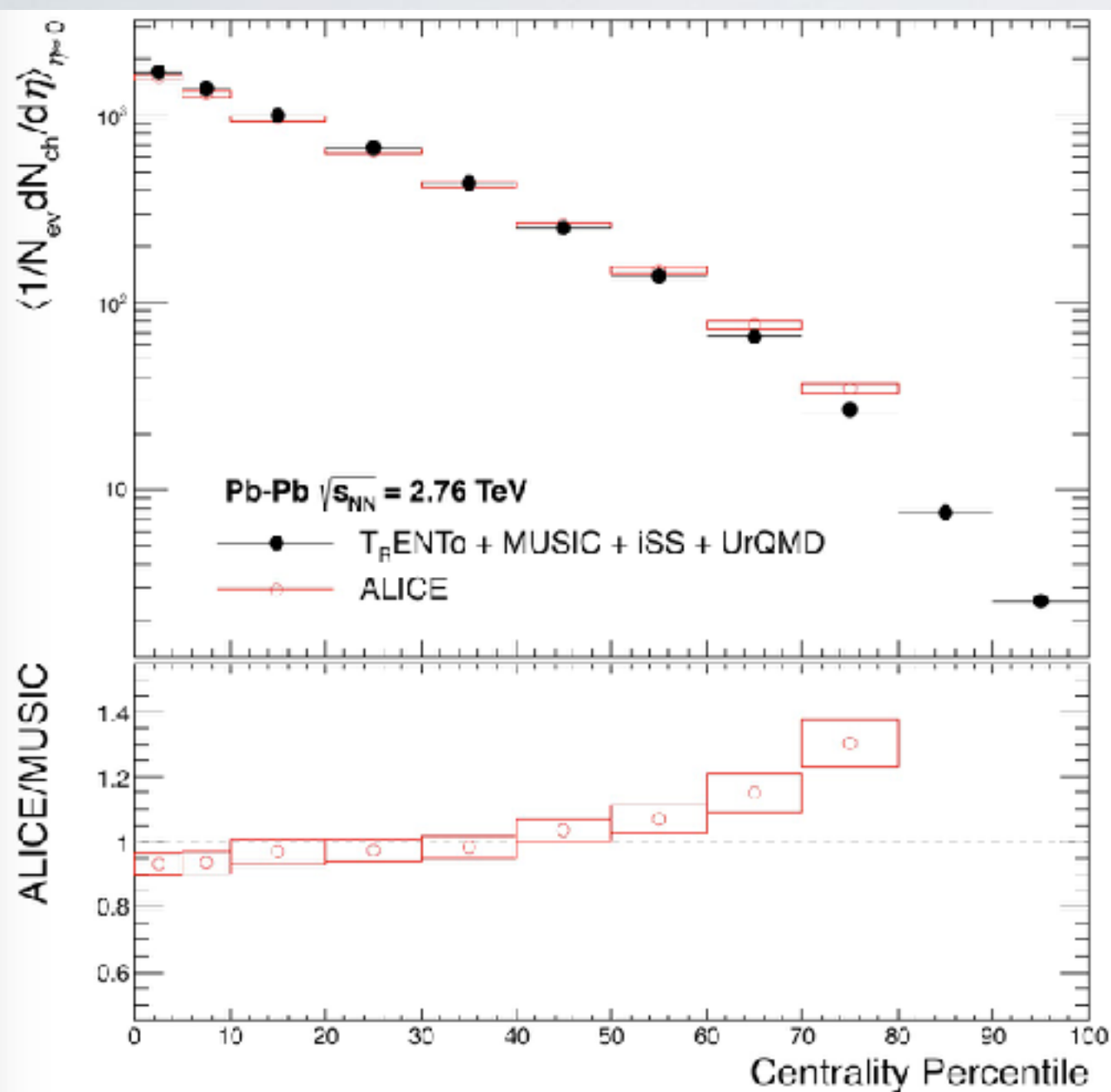


CENTRALITY SELECTION

Events are classified in centrality by comparison to the total entropy distribution from 1 million generated TRENTo events.



RESULTING MULTIPLICITIES



PARTICLE DISTRIBUTION

Single-particle distribution

$$\frac{dN}{d\vec{p}} = \sum_{n=-\infty}^{+\infty} V_n(p) e^{in\varphi} \quad V_n(p) \equiv \frac{1}{2\pi\Delta p_t \Delta\eta} \sum_{j=1}^{M(p)} \exp(in\varphi_j)$$

Pair distribution in the flow picture:

$$\left\langle \frac{dN_{\text{pairs}}}{d\vec{p}_a d\vec{p}_b} \right\rangle = \left\langle \frac{dN}{d\vec{p}_a} \frac{dN}{d\vec{p}_b} \right\rangle + \mathcal{O}(N) = \sum_{n=-\infty}^{n=+\infty} V_{n\Delta}(p_a, p_b) e^{in(\varphi_a - \varphi_b)}$$

THE PAIR CORRELATION MATRIX

Where:

$$V_{n\Delta}(p_a, p_b) \equiv \langle Q_n(p_a) Q_n^*(p_b) \rangle - \frac{\langle M(p_a) \rangle \delta_{p_a, p_b}}{(2\pi \Delta p_t \Delta \eta)^2} - \langle Q_n(p_a) \rangle \langle Q_n^*(p_b) \rangle$$



Covariance matrix



Self-correlations



Azimutal asymmetry

$$V_{n\Delta}(p_a, p_b) = \begin{pmatrix} V_{n\Delta}(p_1, p_1) & V_{n\Delta}(p_1, p_2) & (\dots) \\ V_{n\Delta}(p_2, p_1) & V_{n\Delta}(p_2, p_2) & (\dots) \\ (\dots) & (\dots) & (\dots) \end{pmatrix}_{N_b \times N_b}$$

PRINCIPAL COMPONENT ANALYSIS (PCA)

The PCA approximation consists in doing:

$$V_{n\Delta}(p_a, p_b) \approx \sum_{\alpha=1}^k V_n^{(\alpha)}(p_a) V_n^{(\alpha)*}(p_b)$$

We can also diagonalise the correlation matrix:

$$V_{n\Delta}(p_a, p_b) = \sum_{\alpha} \lambda^{(\alpha)} \psi^{(\alpha)}(p_a) \psi^{(\alpha)*}(p_b)$$

ordered from largest eigenvalue.

Identifying both:

$$V_n^{(\alpha)} \equiv \sqrt{\lambda^{(\alpha)}} \psi^{(\alpha)}$$

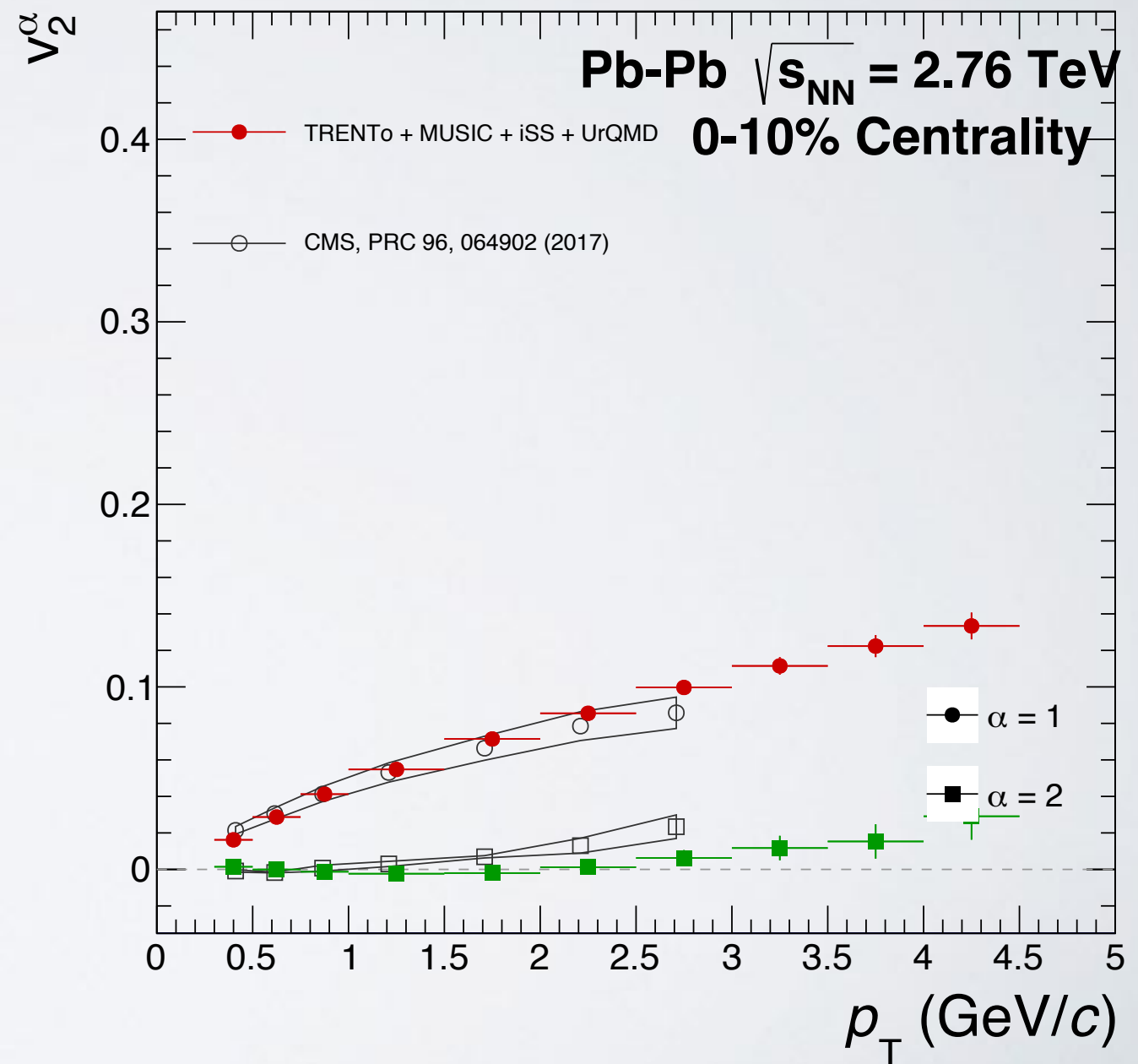
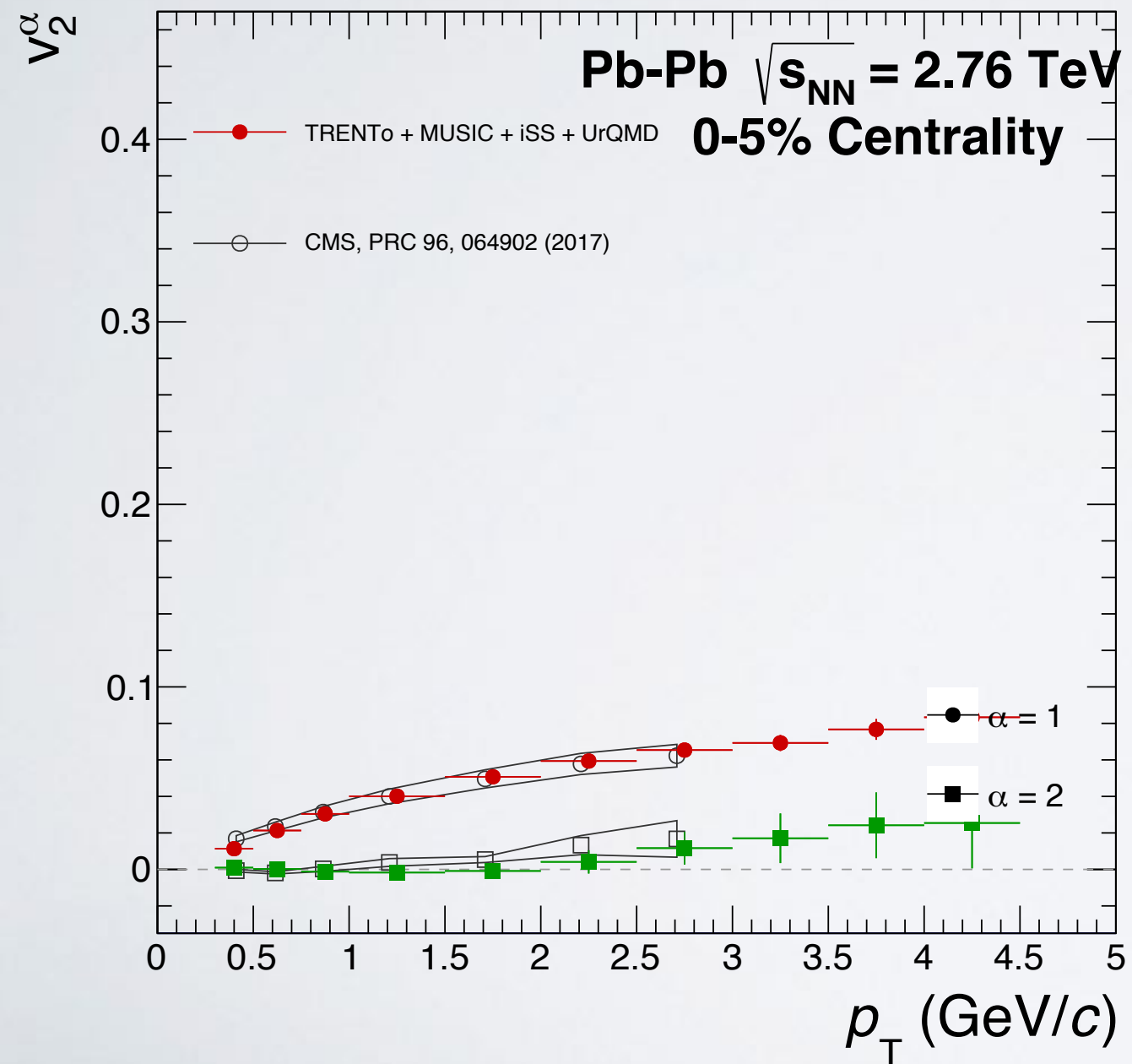
RELATION TO FLOW

For compatibility with the usual flow definition, define:

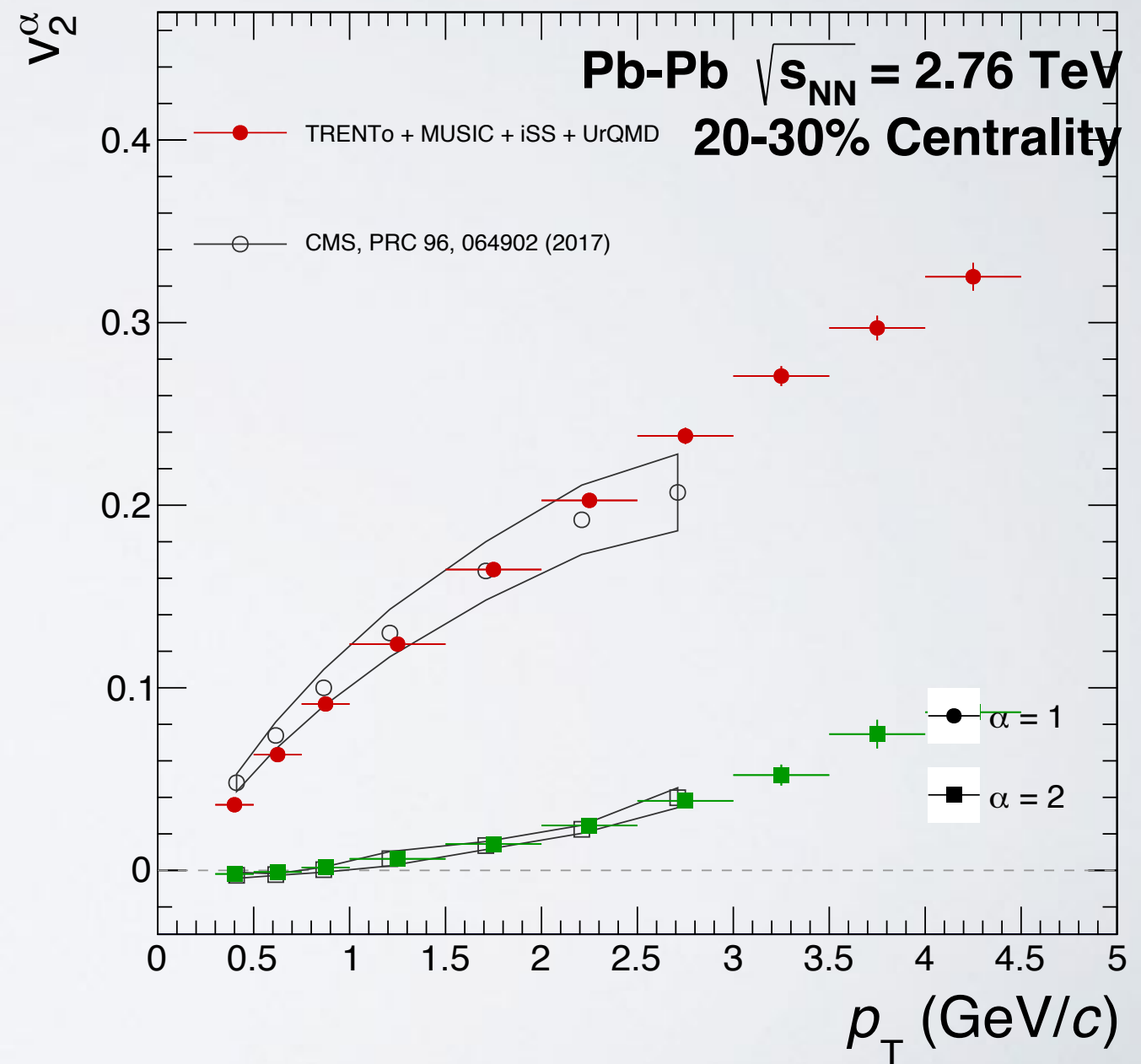
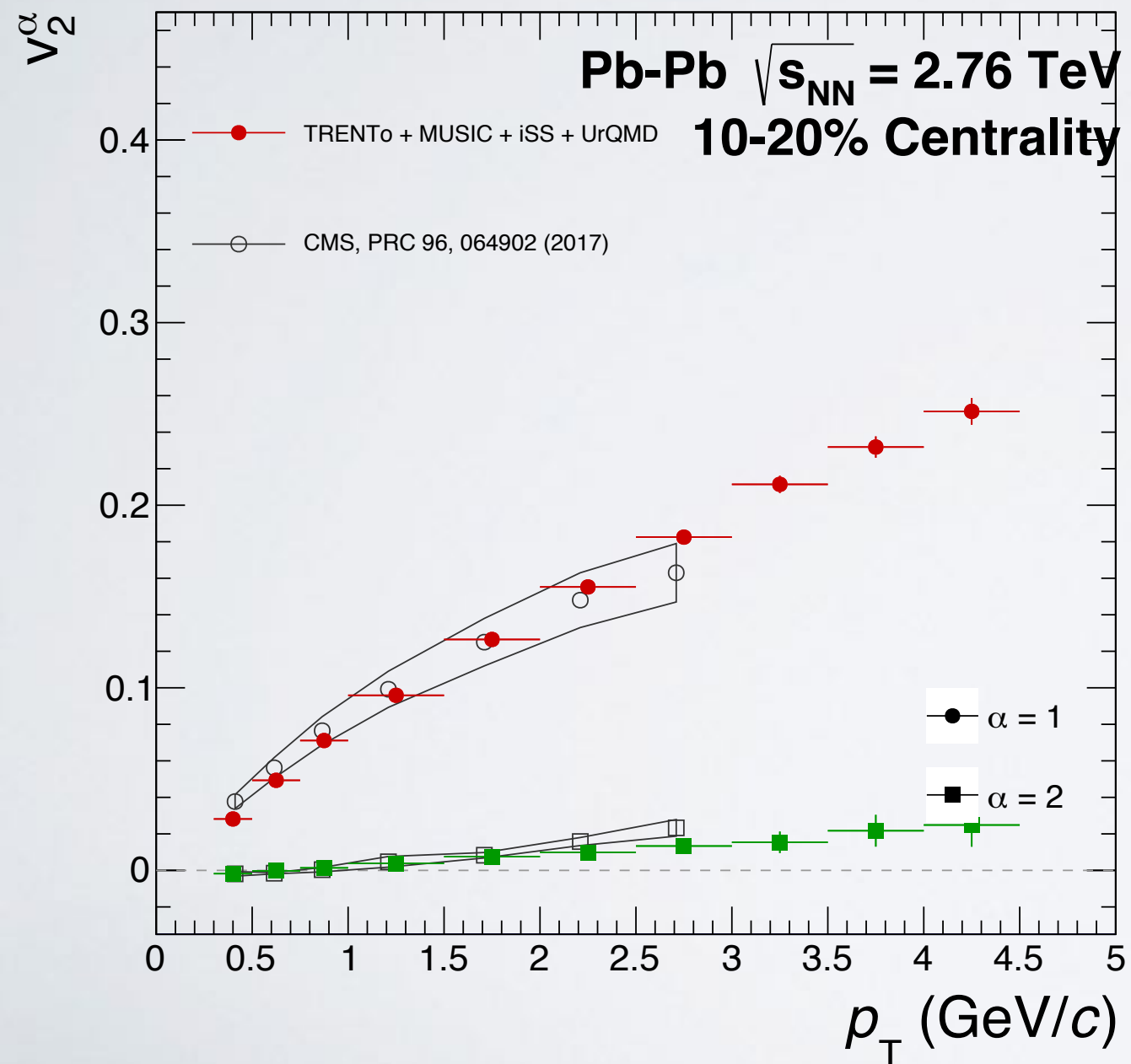
$$v_n^{(\alpha)}(p) \equiv \frac{V_n^{(\alpha)}(p)}{V_0(p)}$$

- Different “n” describe fluctuations of different flow harmonics.
- The leading component ($\alpha = 1$) is equivalent to the usual flow;
- The subleading component measures factorisation breaking at two different bins in phase space;
- More information about the flow by utilising the entire correlation matrix.

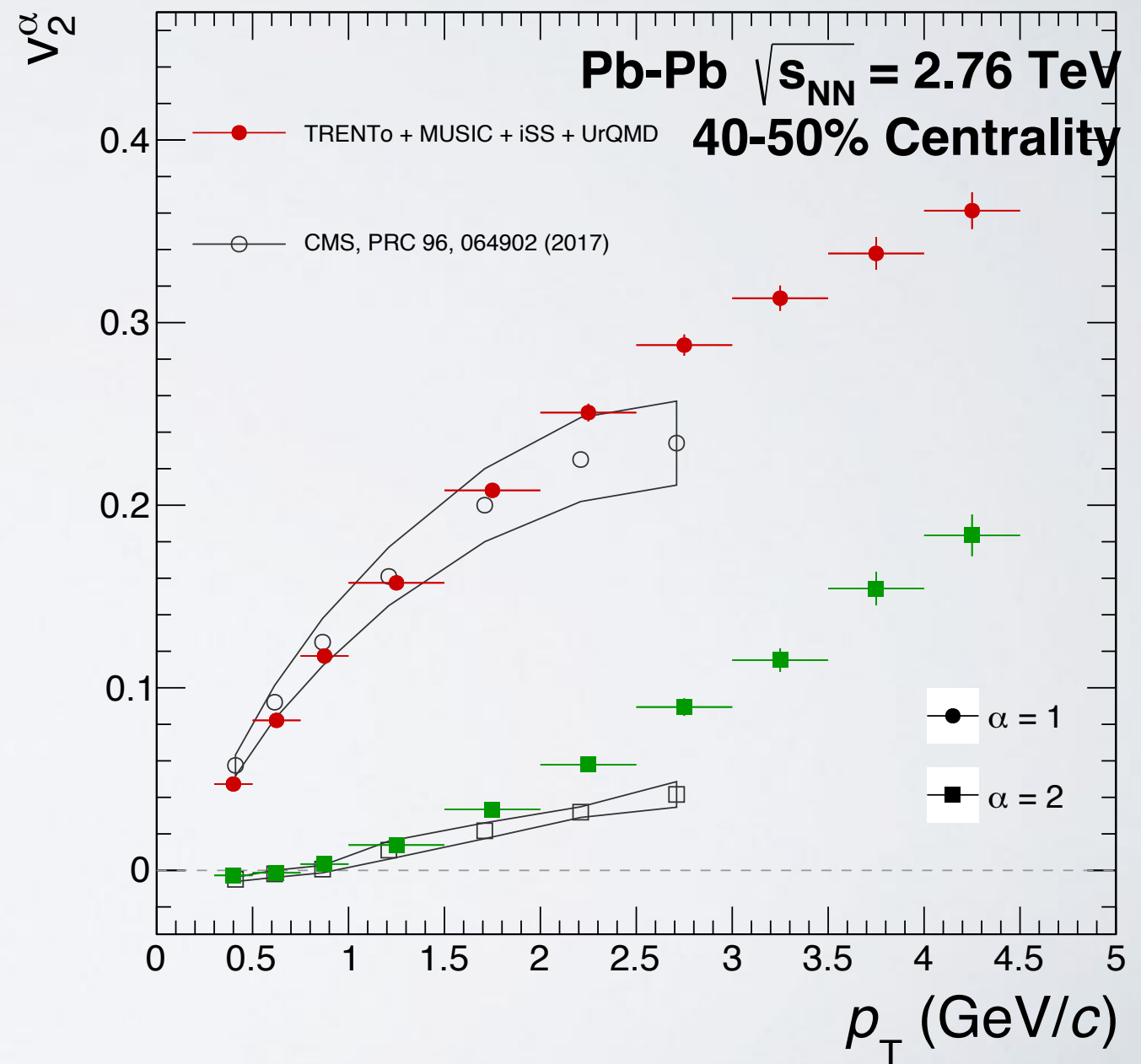
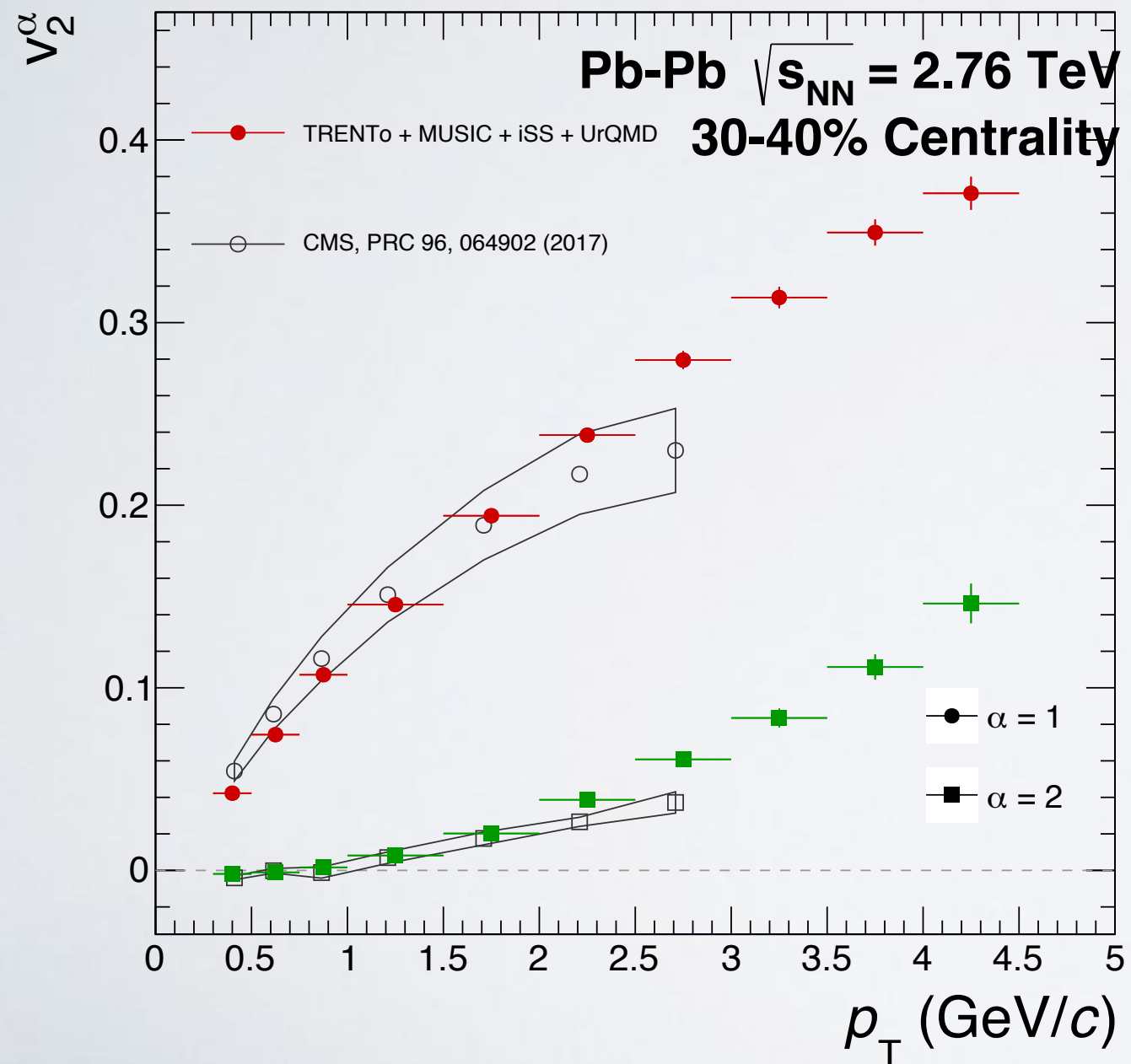
PCA RESULTS 2.76 TEV



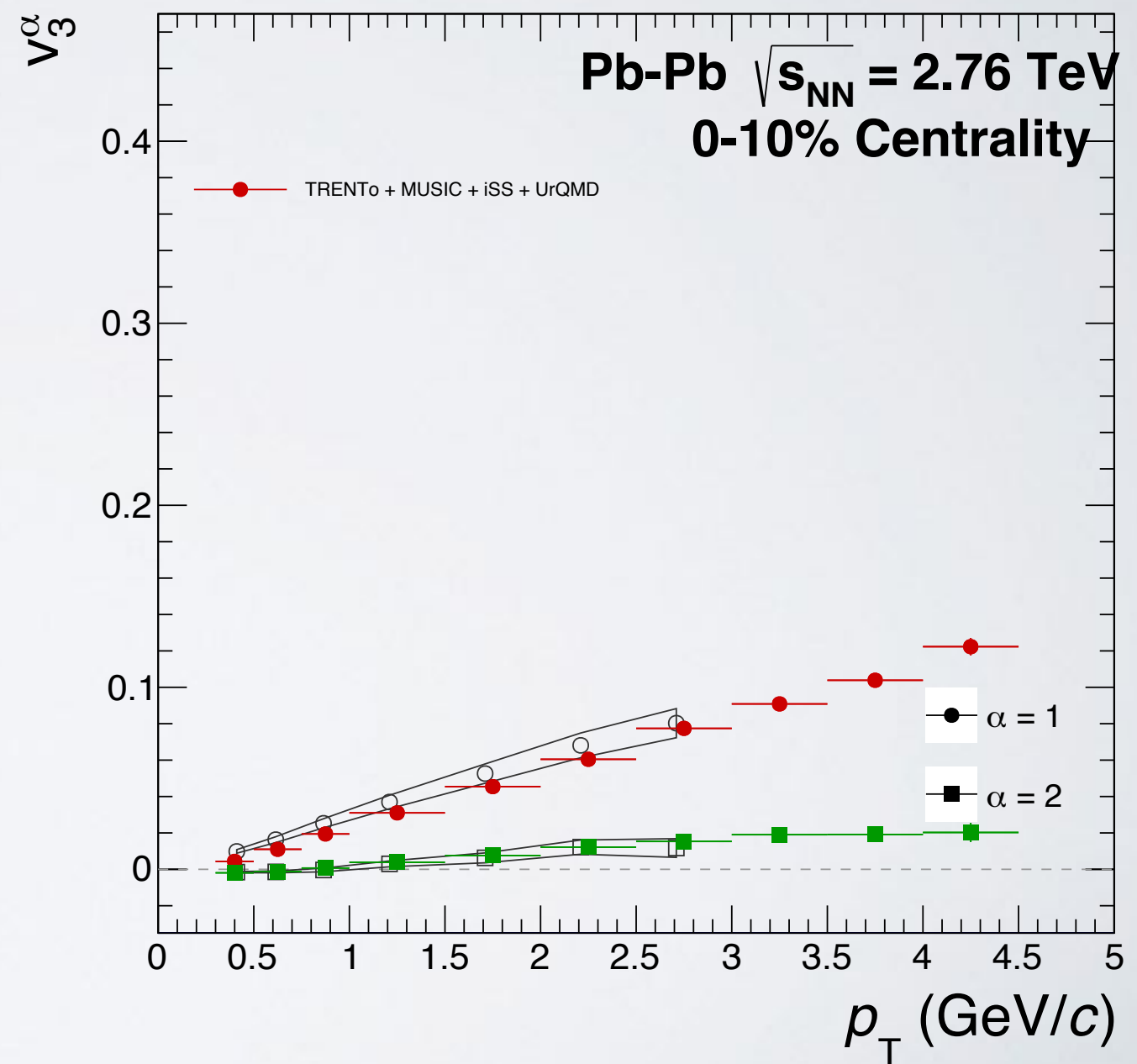
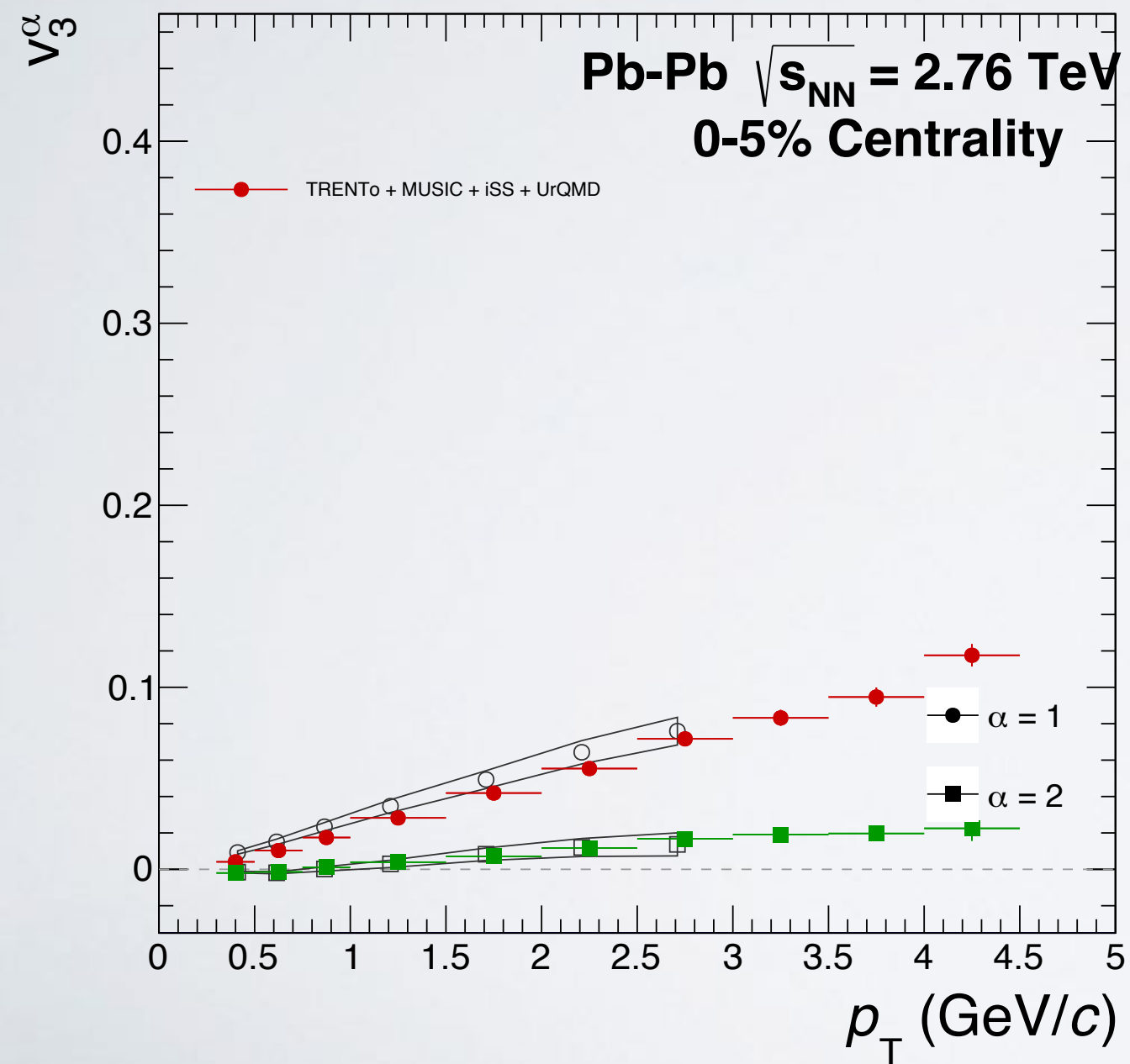
PCA RESULTS 2.76 TEV



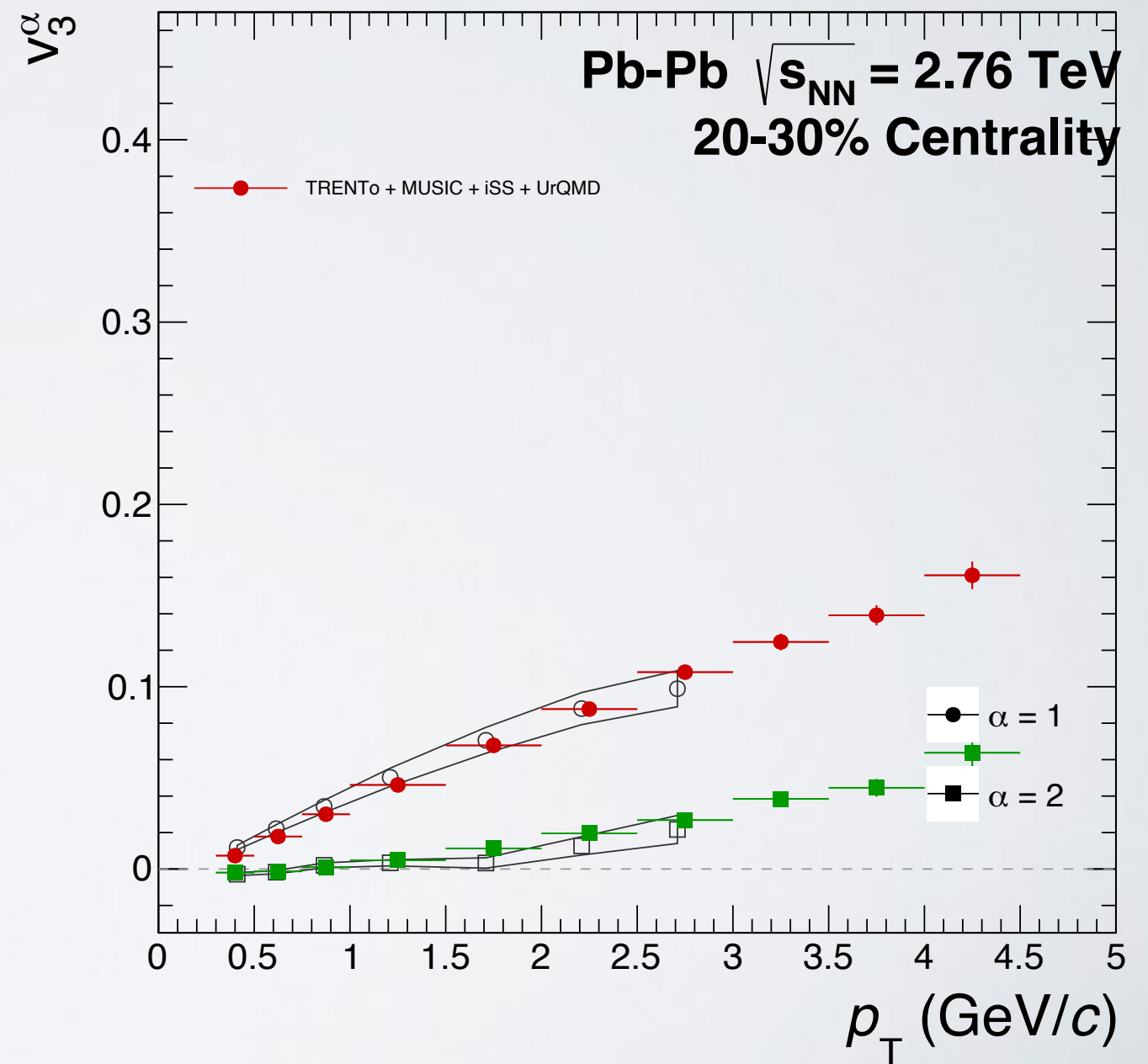
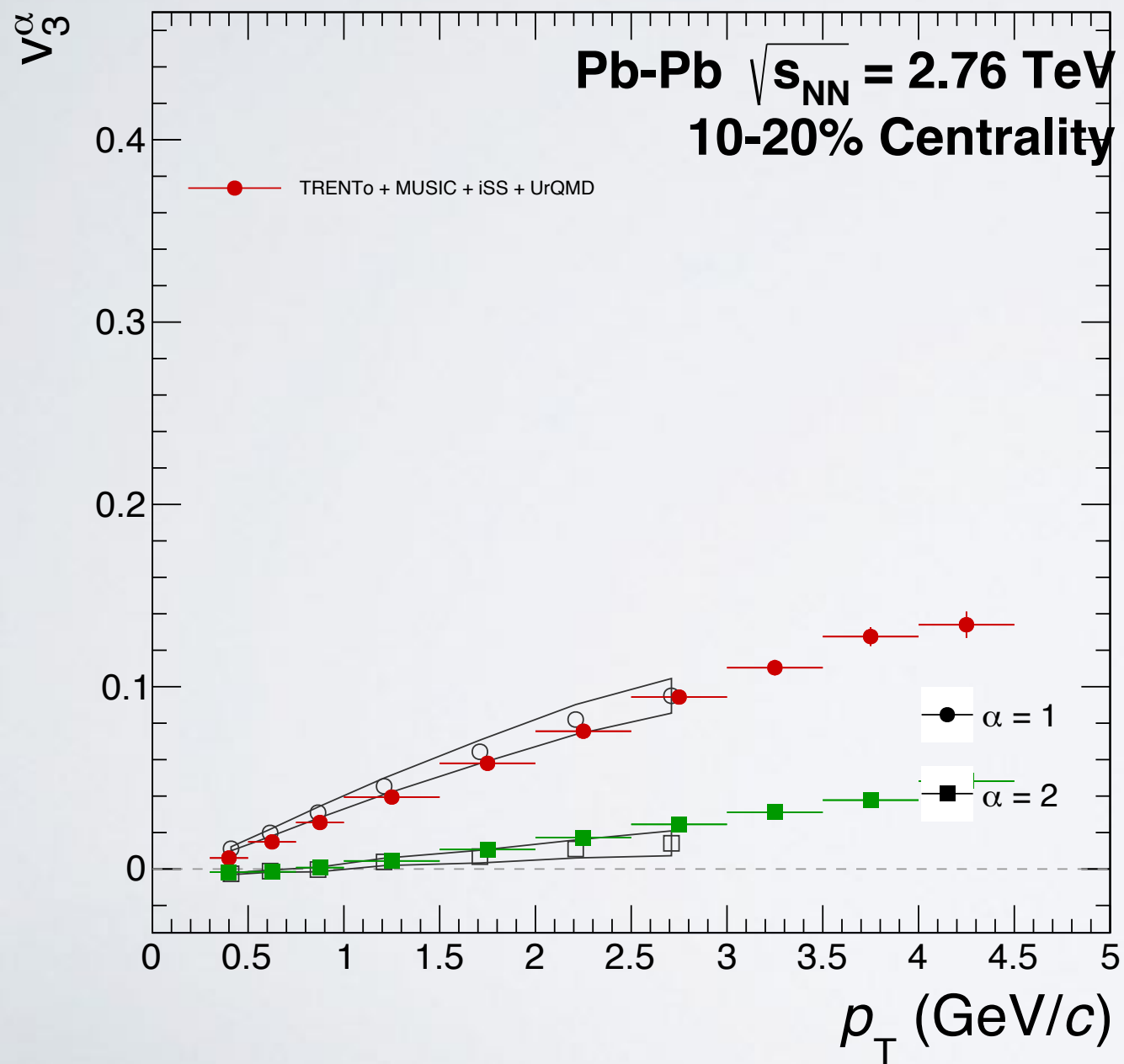
PCA RESULTS 2.76 TEV



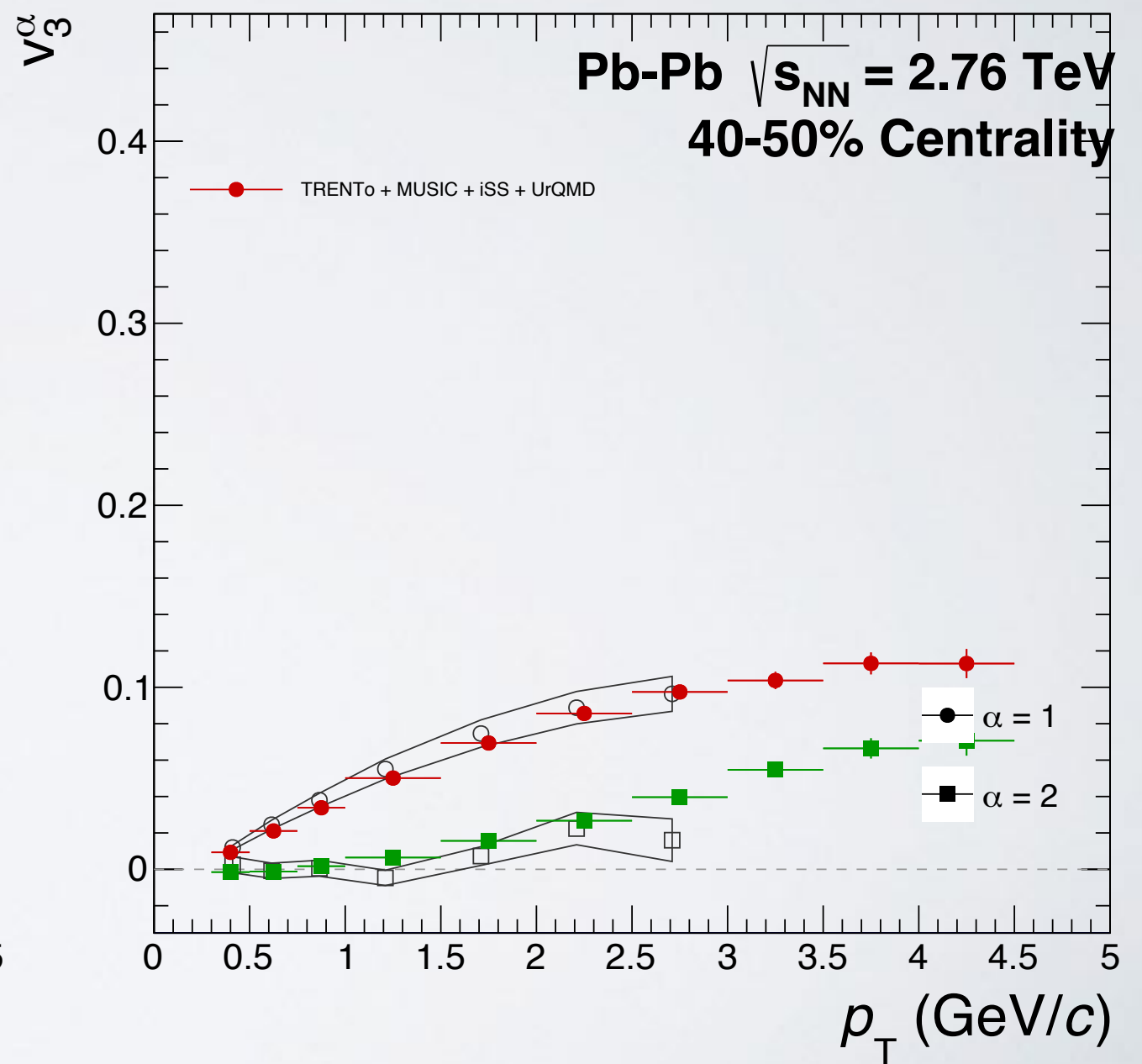
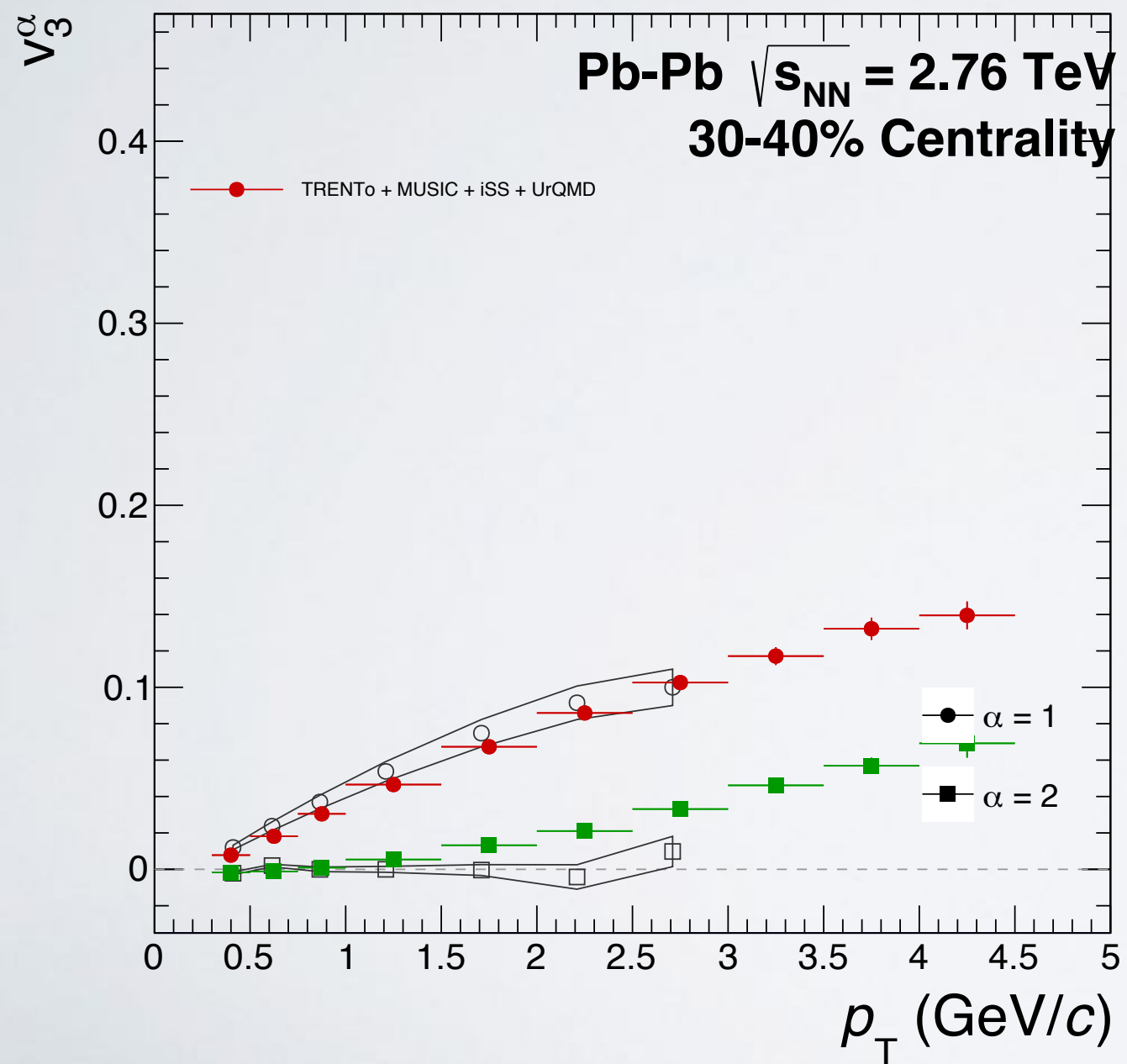
PCA RESULTS 2.76 TEV



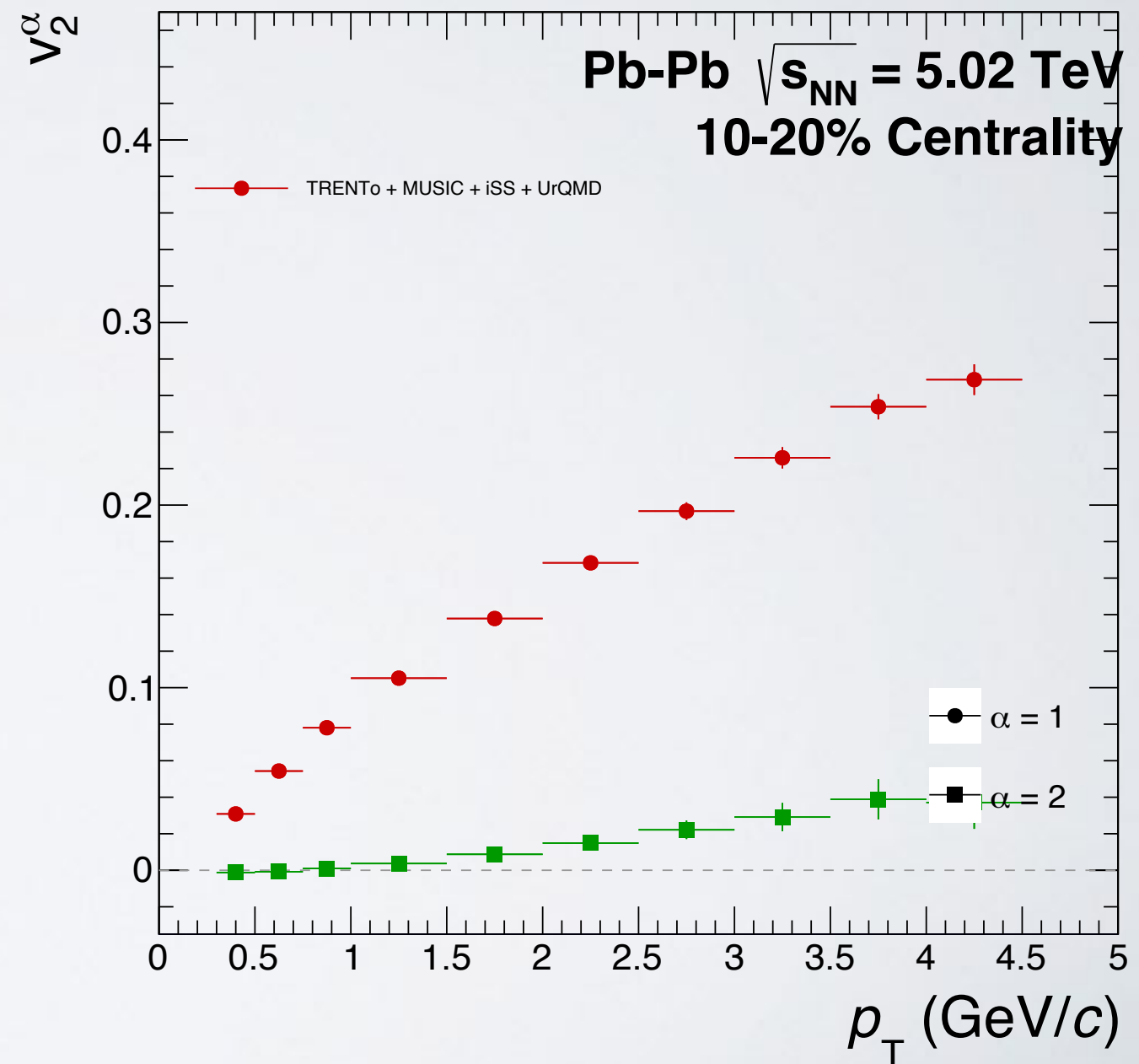
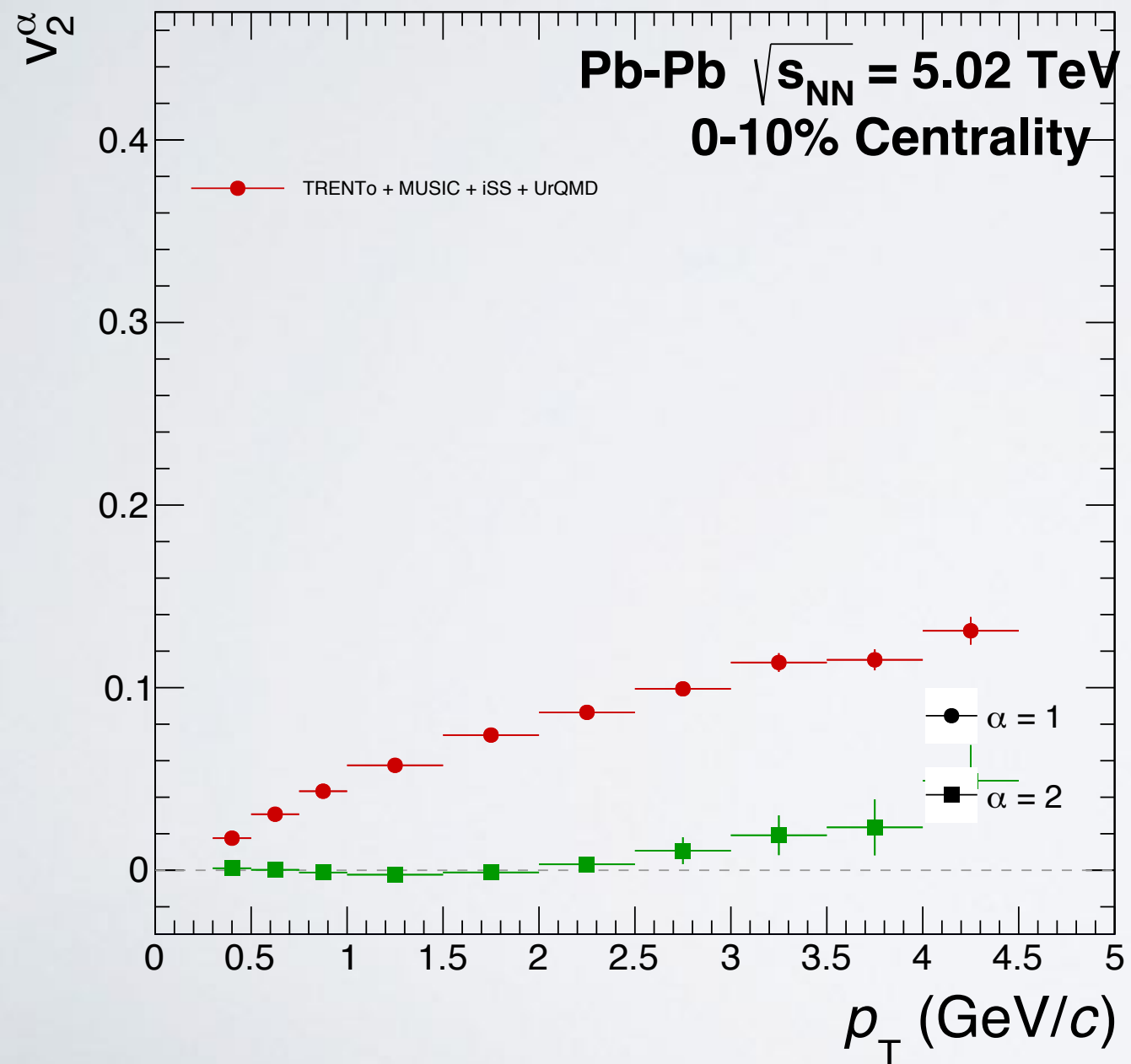
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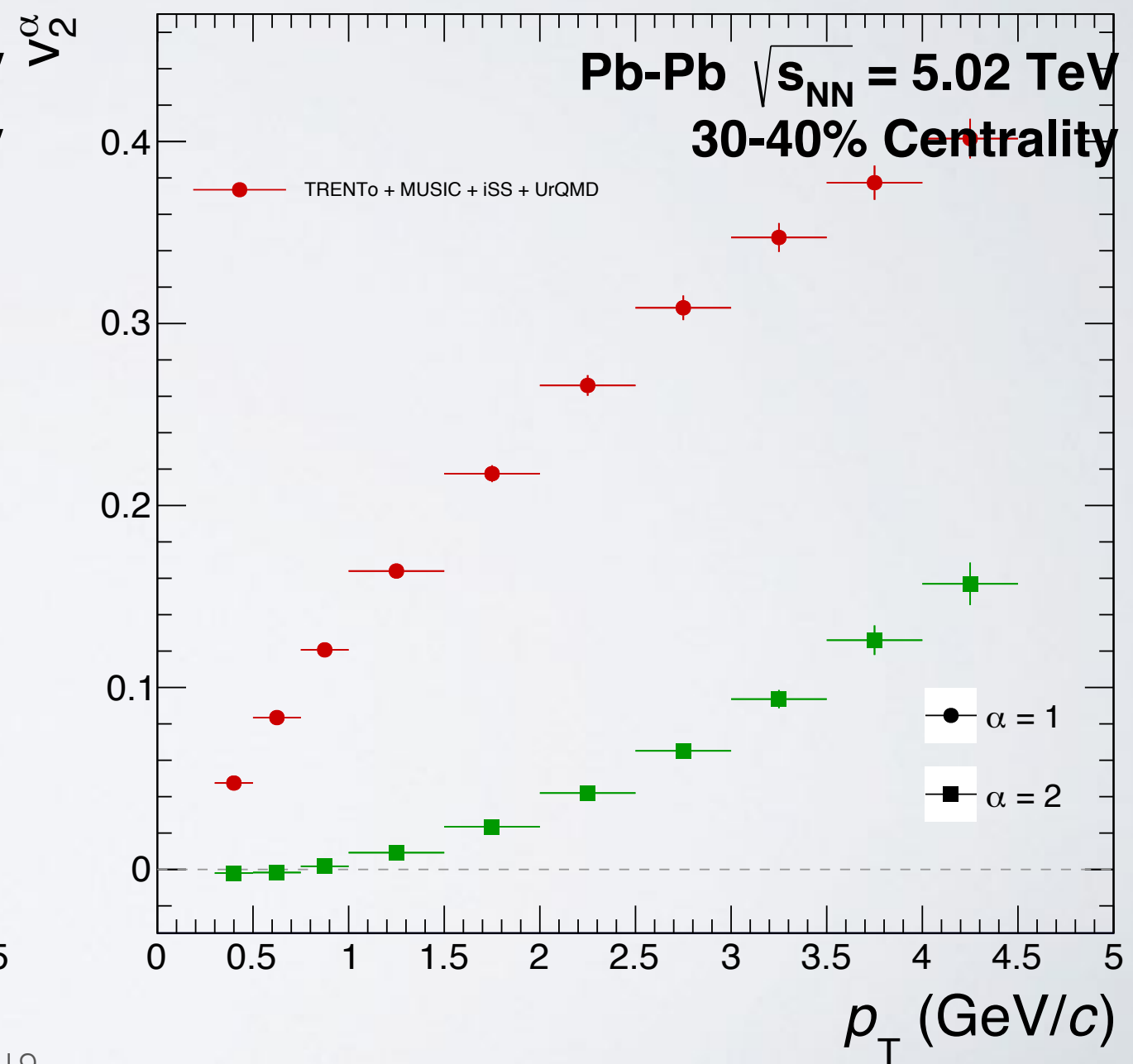
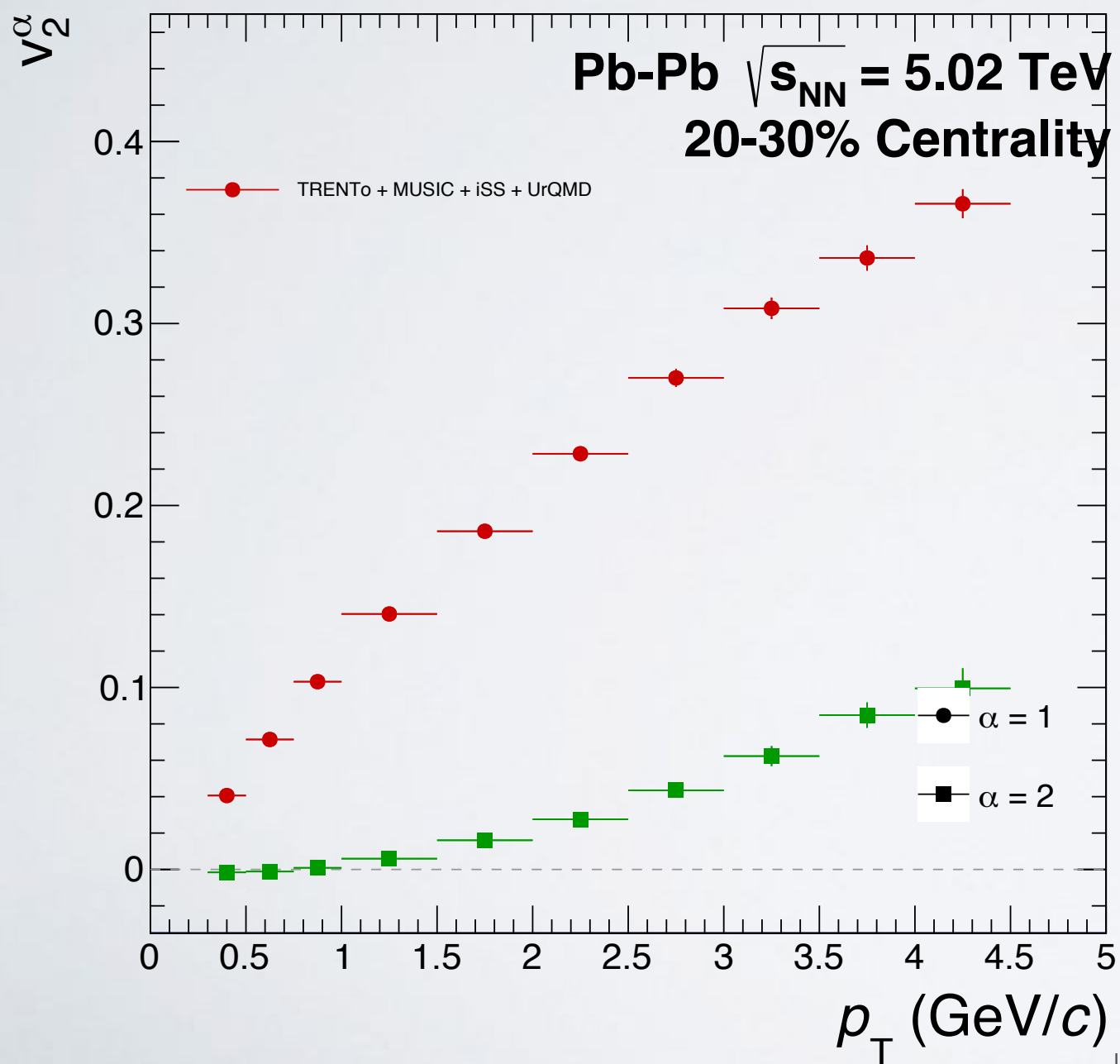
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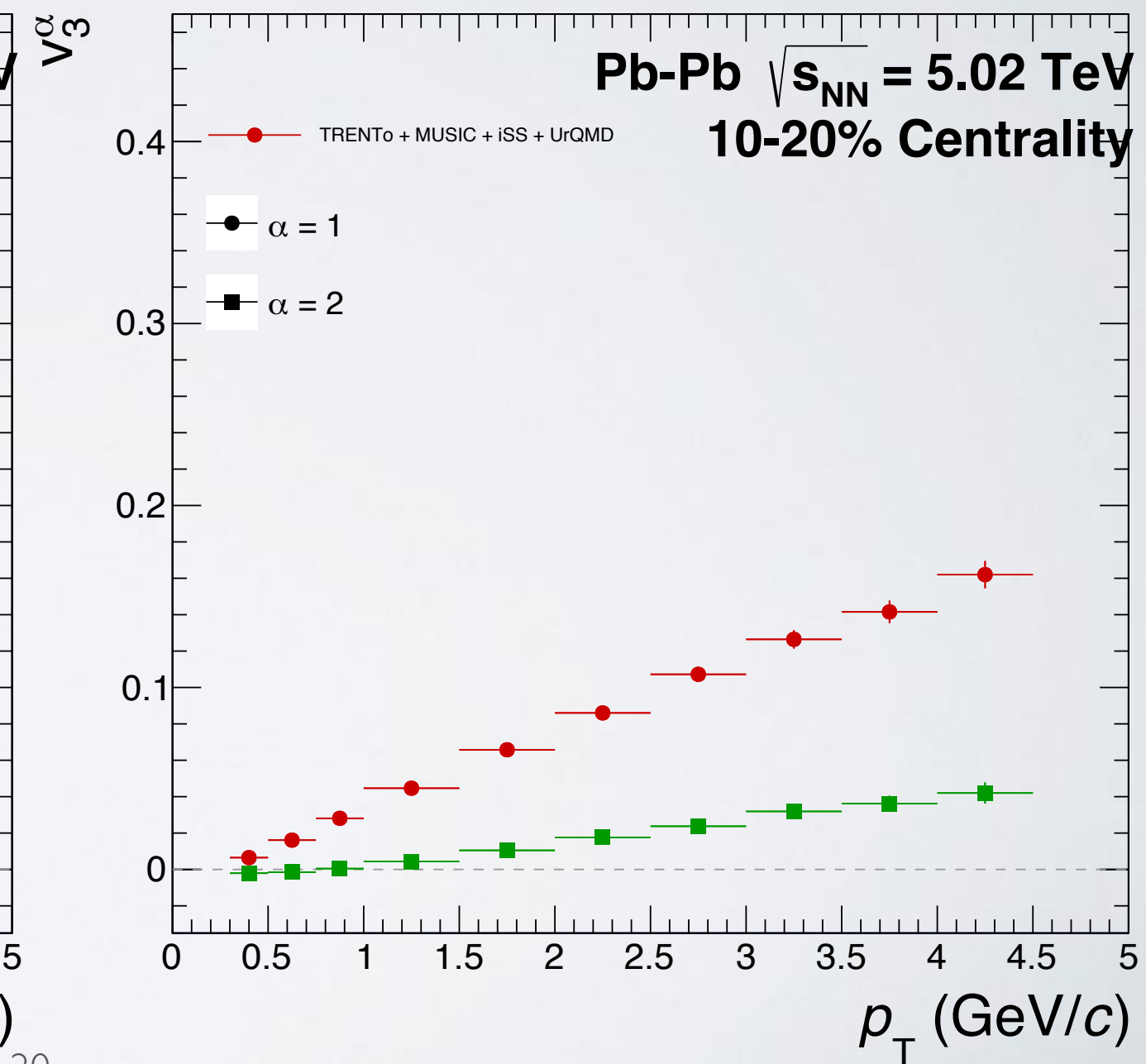
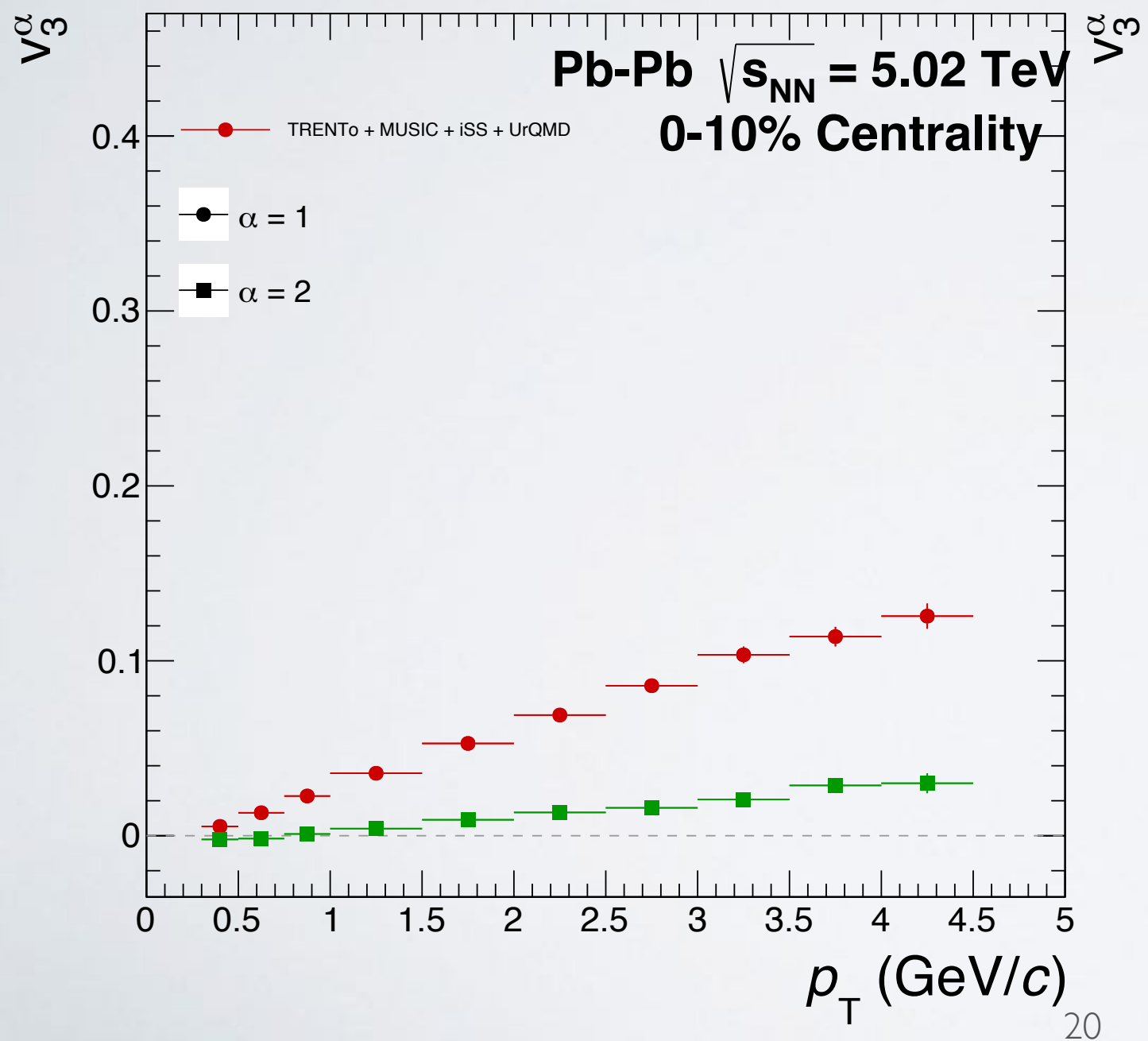
PCA RESULTS 5.02 TeV



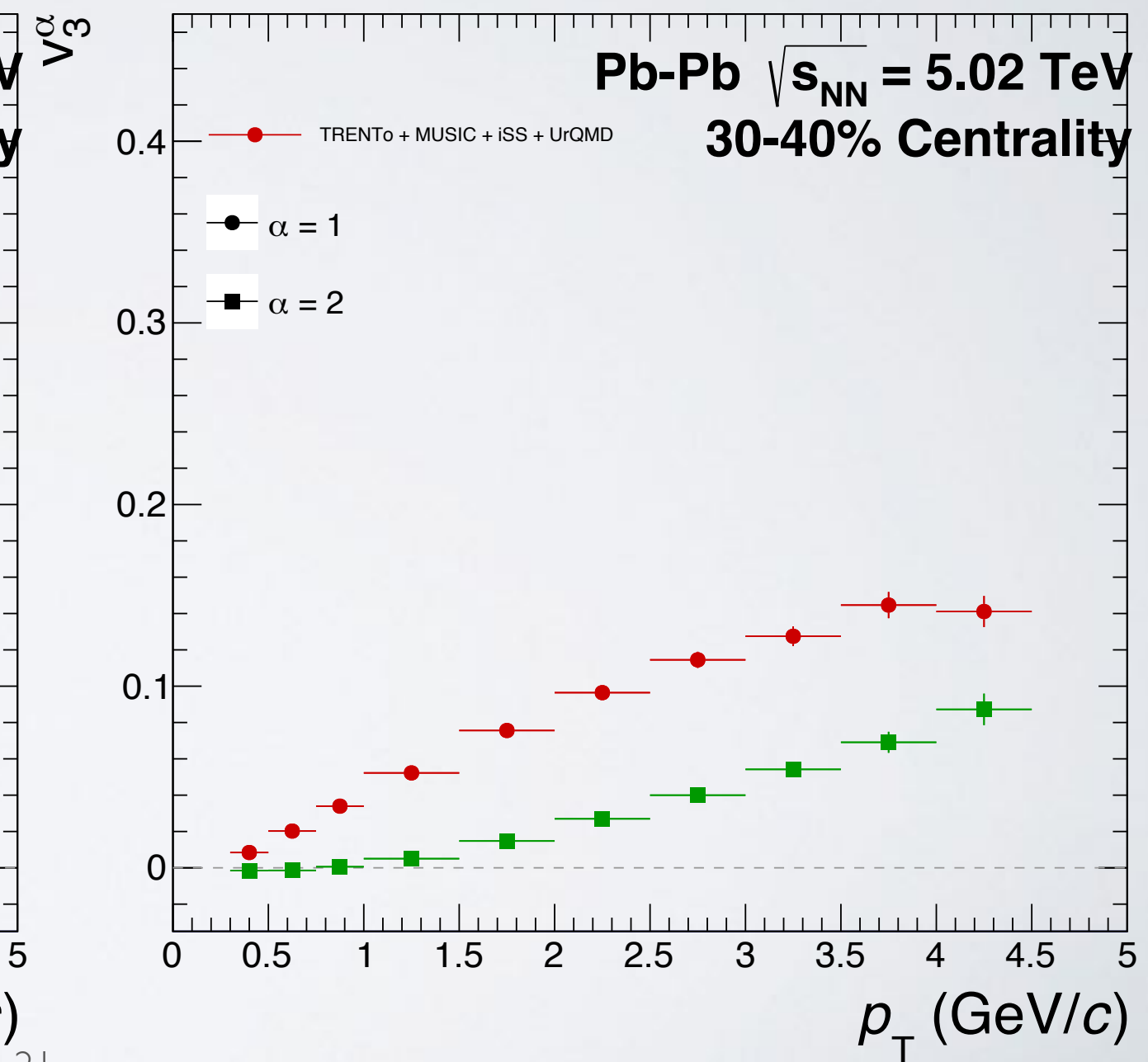
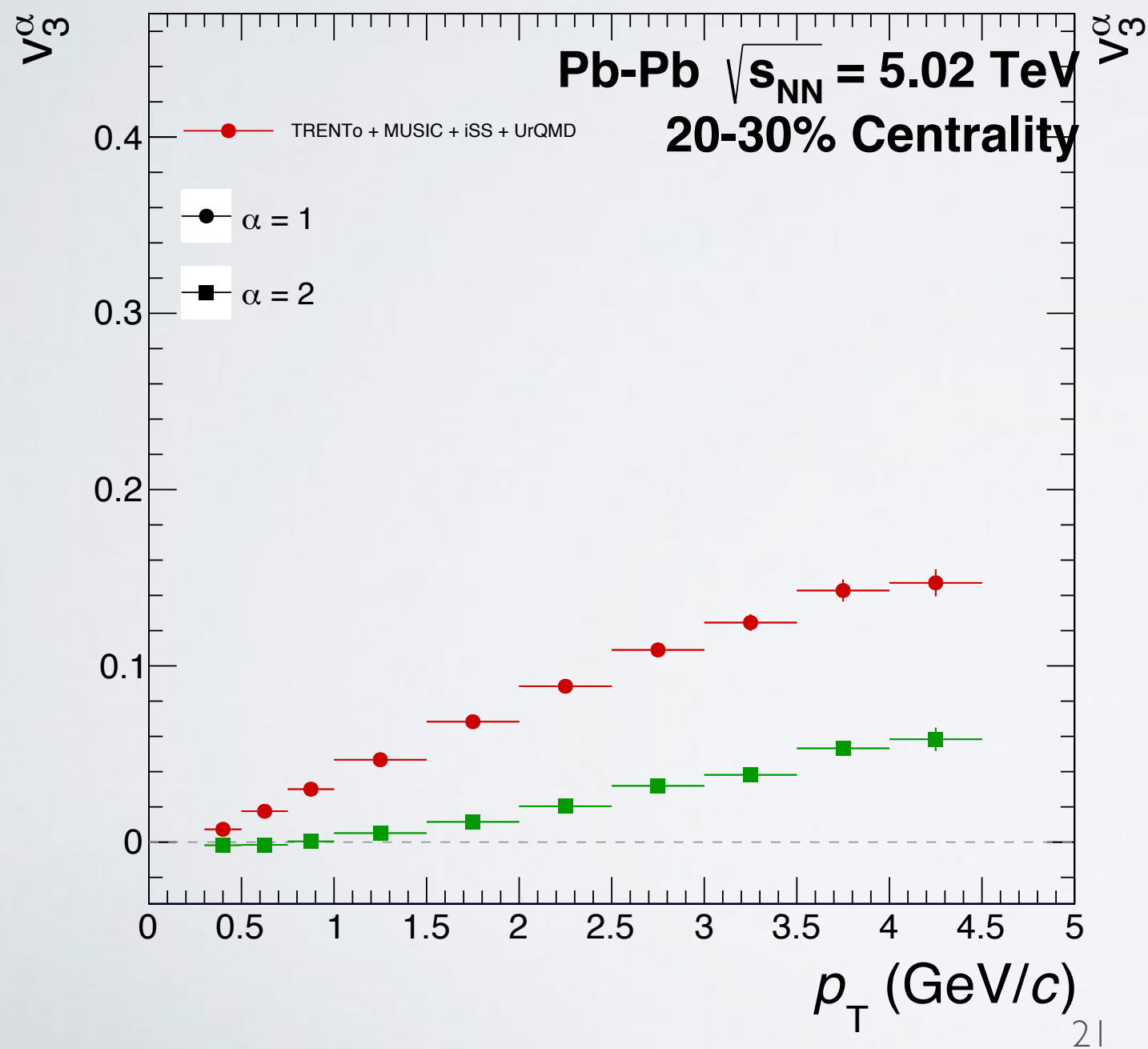
PCA RESULTS 5.02 TeV



PCA RESULTS 5.02 TeV



PCA RESULTS 5.02 TeV



CONCLUSIONS AND OUTLOOK

- Good concordance between our PCA analysis and CMS data for central events and low p_T ;
- Ongoing analysis of further observables;
- Extension to RHIC BES-II energies;
- The framework will be made public.

BACKUP

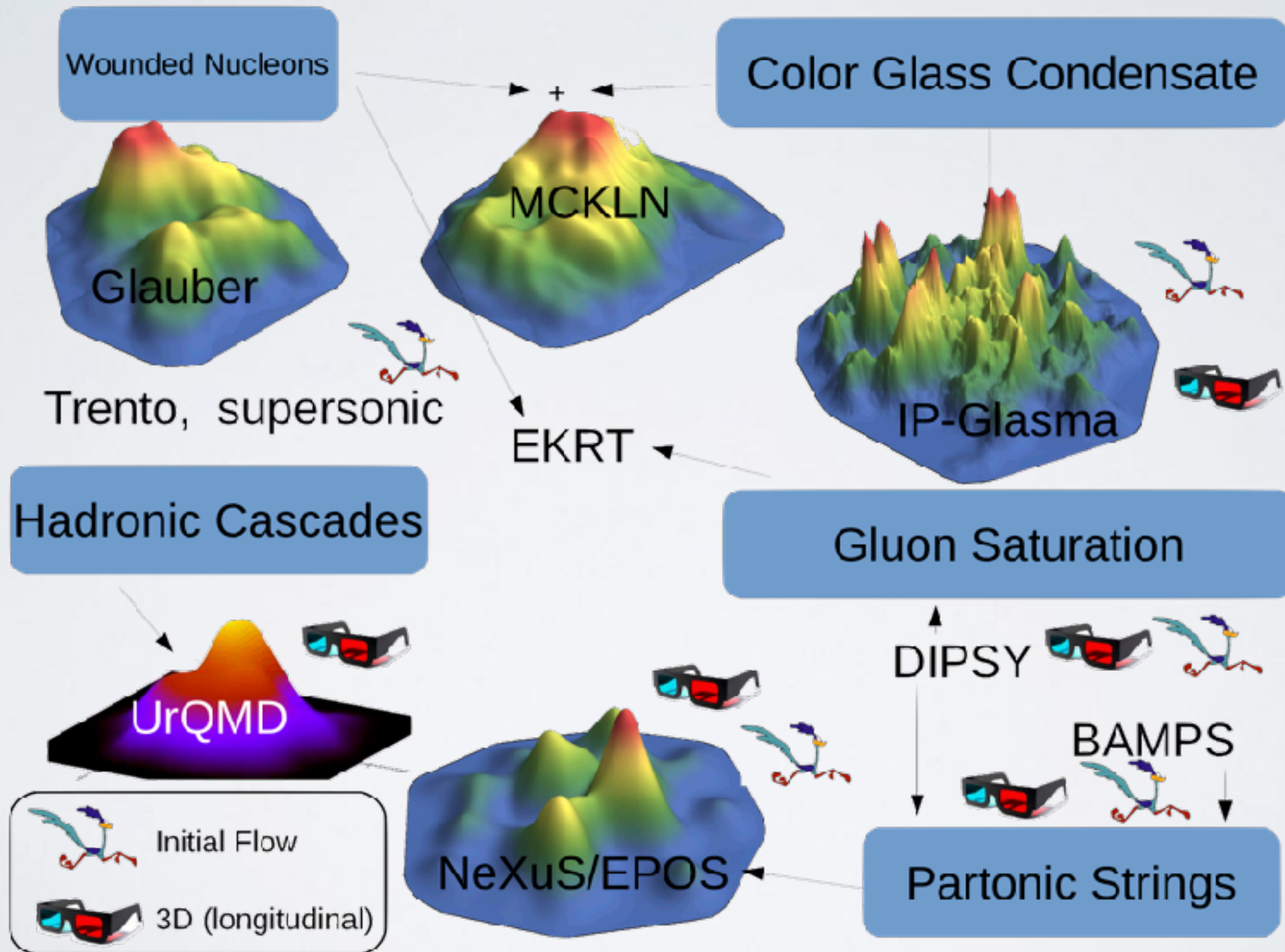
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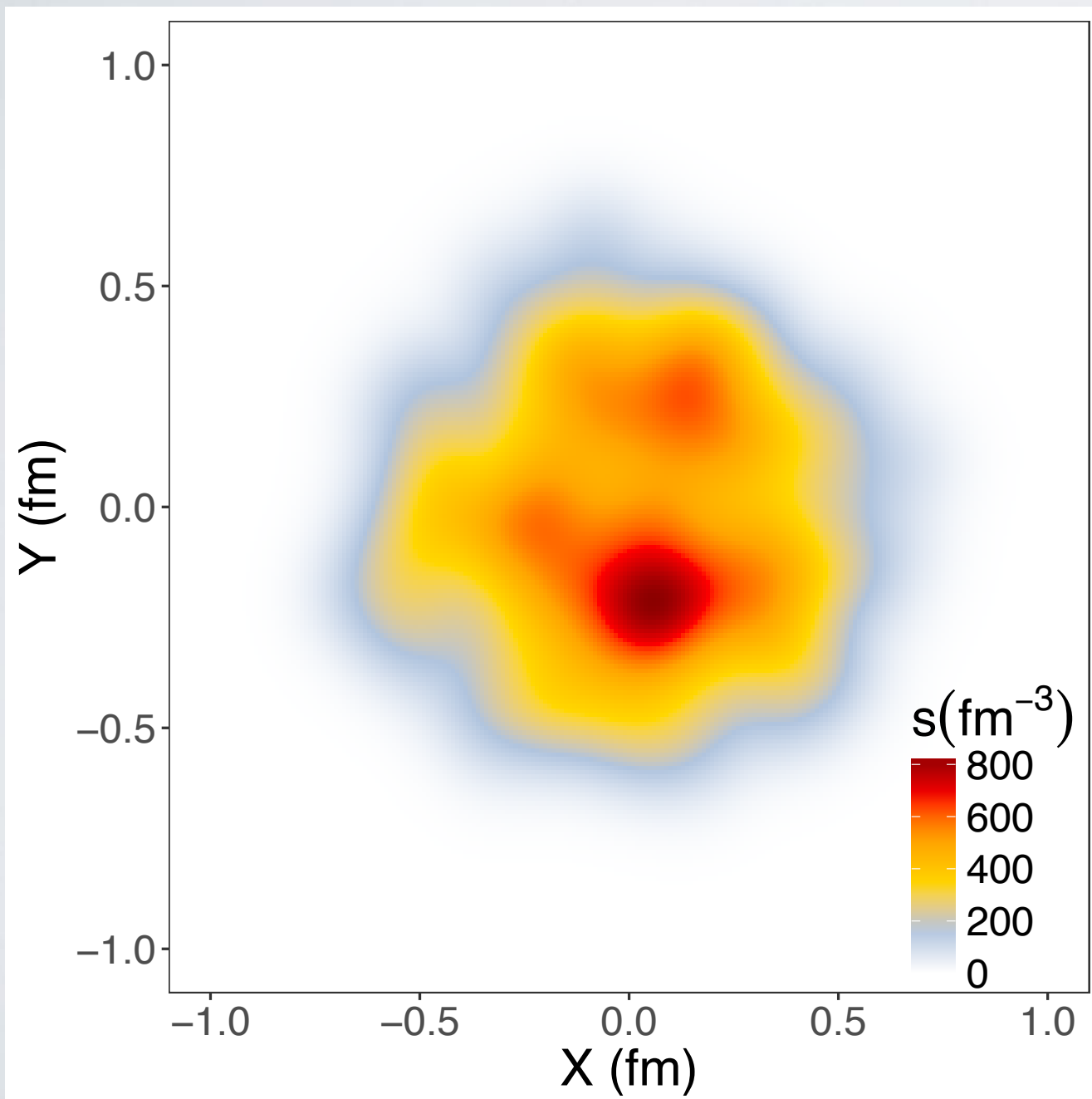
Figure: MADAI collaboration, Hannah Petersen and Jonah Bernhard

TRENT_o + MUSIC + iSS Sampler + UrQMD Gas
+ HadrEx Data Analysis suite

INITIAL CONDITIONS



INITIAL CONDITIONS WITH TRENTO



- T_{REN}To is a parametric wounded nucleon IC model;

$$T_{A,B}(x, y) = \int dz \rho_{A,B}^{\text{part}}(x, y, z)$$

$$\frac{dS}{dy} \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

HYDRODYNAMICS

Ideal Hydro

$$T_{\text{ideal}}^{\mu\nu} = (\epsilon + \mathcal{P})u^\mu u^\nu - \mathcal{P}g^{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0 \quad + \quad \partial_\mu j_i^\mu = 0$$

Energy and momentum
conservation

Other conserved
quantities
(B, S, Q)

HYDRODYNAMICS

Second-order viscous Hydro (Israel-Stewart type)

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \pi^{\mu\nu} - (g^{\mu\nu} - u^\mu u^\nu)\Pi$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu j_i^\mu = 0$$

$$\begin{aligned} \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = & 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\alpha^{\langle\mu} \omega^{\nu\rangle\alpha} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \varphi_7 \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha} \\ & - \tau_{\pi\pi} \pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} + \varphi_6 \Pi \pi^{\mu\nu} \end{aligned}$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi} \Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu} \pi_{\mu\nu}$$

See Denicol et al., PRC90:024912 (2014)

HYDRODYNAMICS

Second-order viscous Hydro (Israel-Stewart type)

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \pi^{\mu\nu} - (g^{\mu\nu} - u^\mu u^\nu)\Pi$$

$$\partial_\mu T^{\mu\nu} = 0 \quad + \quad \partial_\mu j_i^\mu = 0$$

+ complicated equations of motion for $\pi^{\mu\nu}, \Pi$

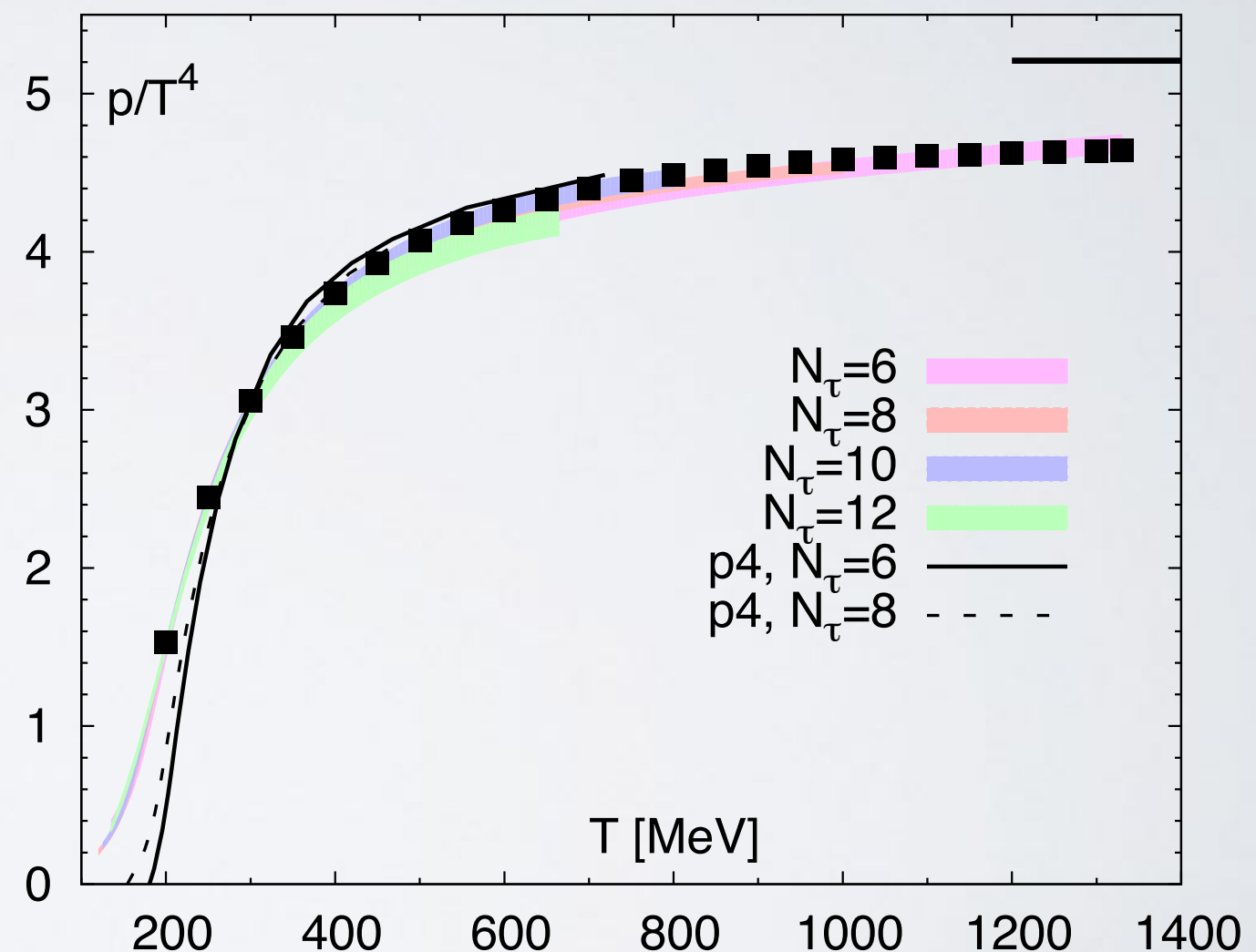
Total of 14 non-linear coupled PDEs, with 13 transport coefficients:

$$\eta(T, \mu), \zeta(T, \mu), \tau_\pi(T, \mu), \delta_{\pi\pi}(T, \mu), \dots$$

See Denicol et al., PRC90:024912 (2014)

EQUATION OF STATE

- External input necessary to close the evolution system of equations;
- Calculated from first principles (e.g. Lattice QCD);
- Details of EOS can influence the extraction of transport coefficients.



**A. Bazavov et al.,
Phys. Rev. D 95, 054504 (2017)**

MUSIC

- Eulerian 3D+1 relativistic second-order viscous hydrodynamics code for event by event (EBE) HIC simulations;
- Evolution is solved through the Kurganov-Tadmor method; (J.Comp.Phys 160, 214 - 2000)
- Code is written in C++ and supports parallelization;
- Code is publicly available (physics.mcgill.ca/music/)

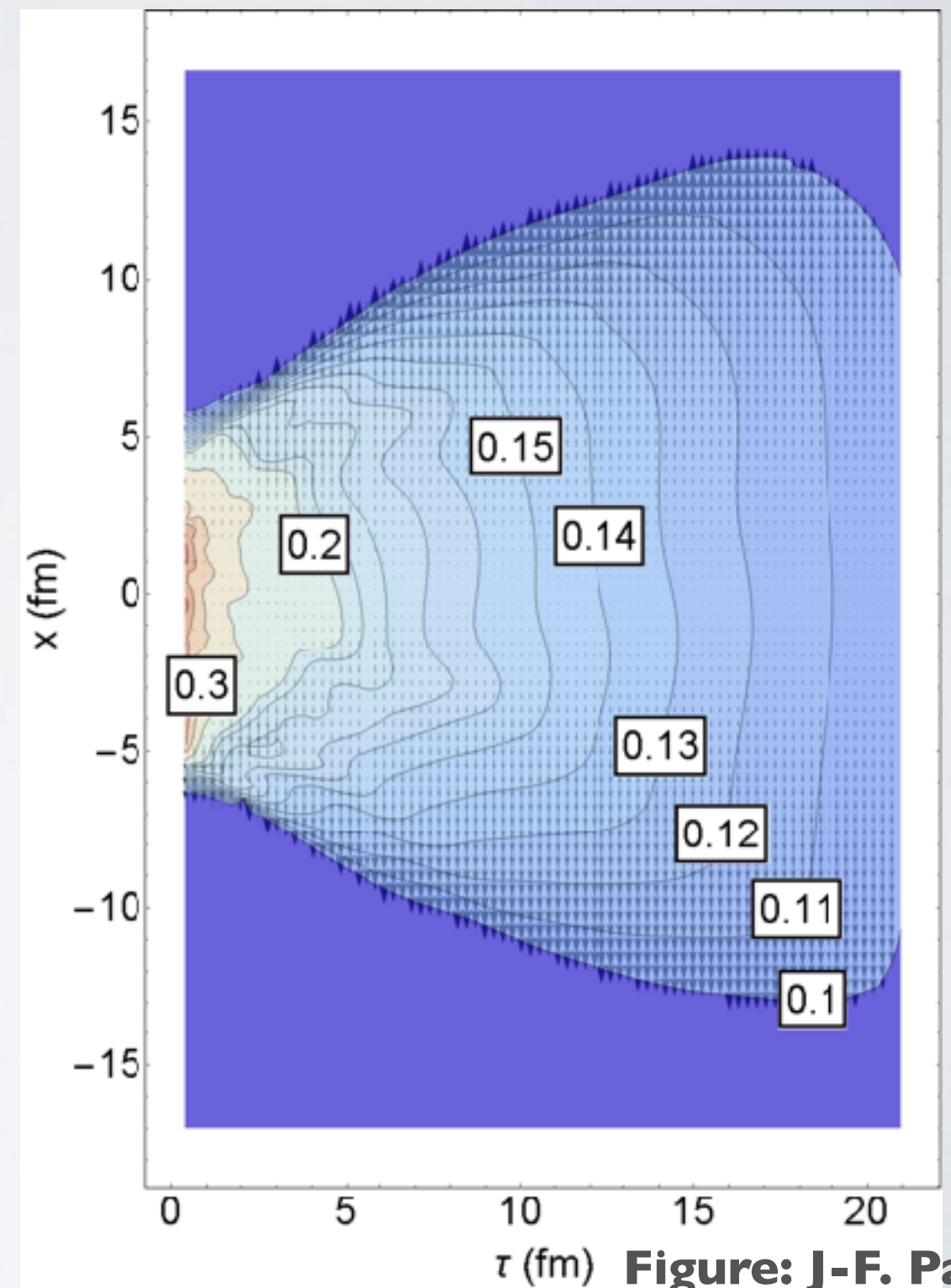
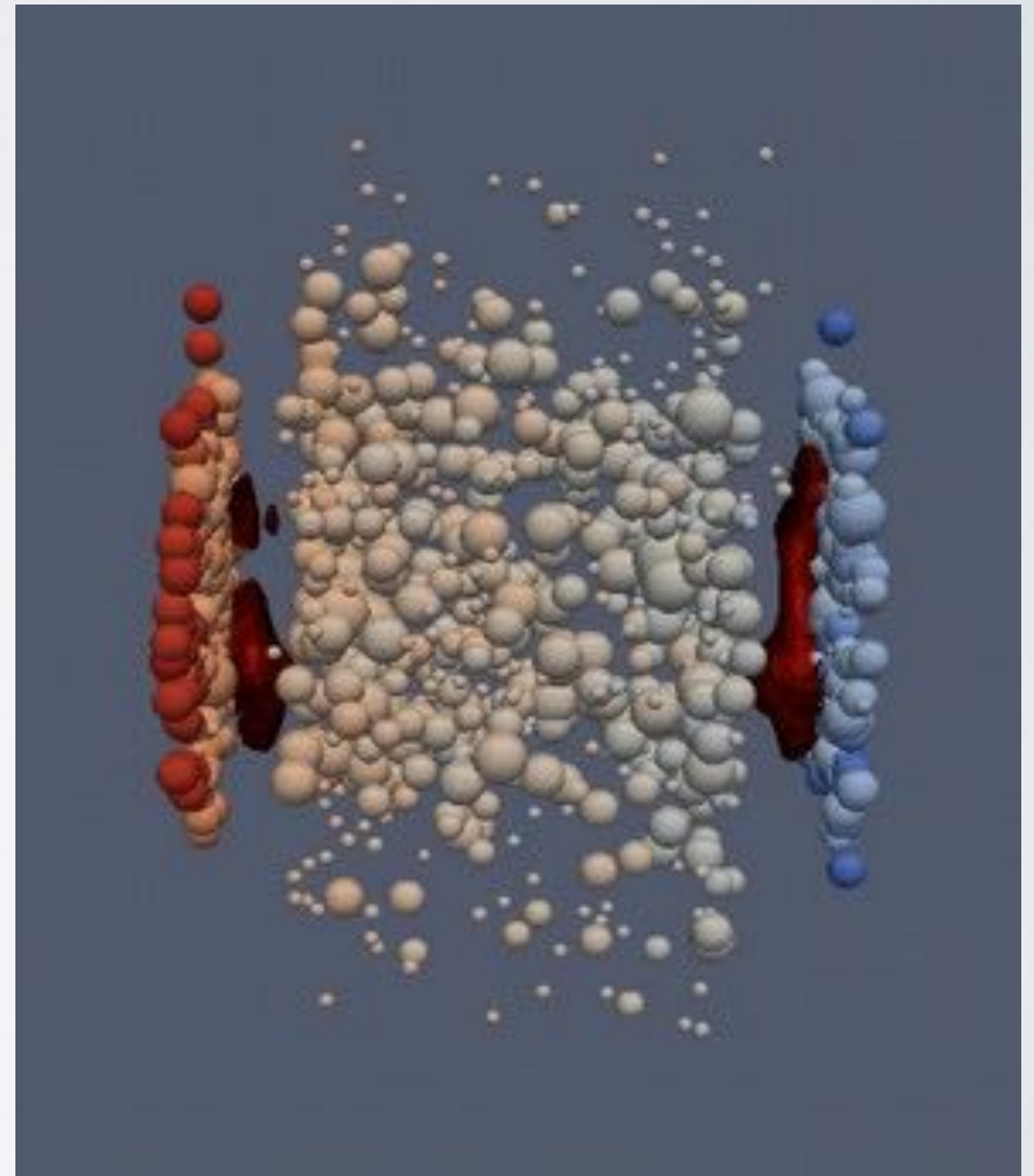


Figure: J-F. Paquet

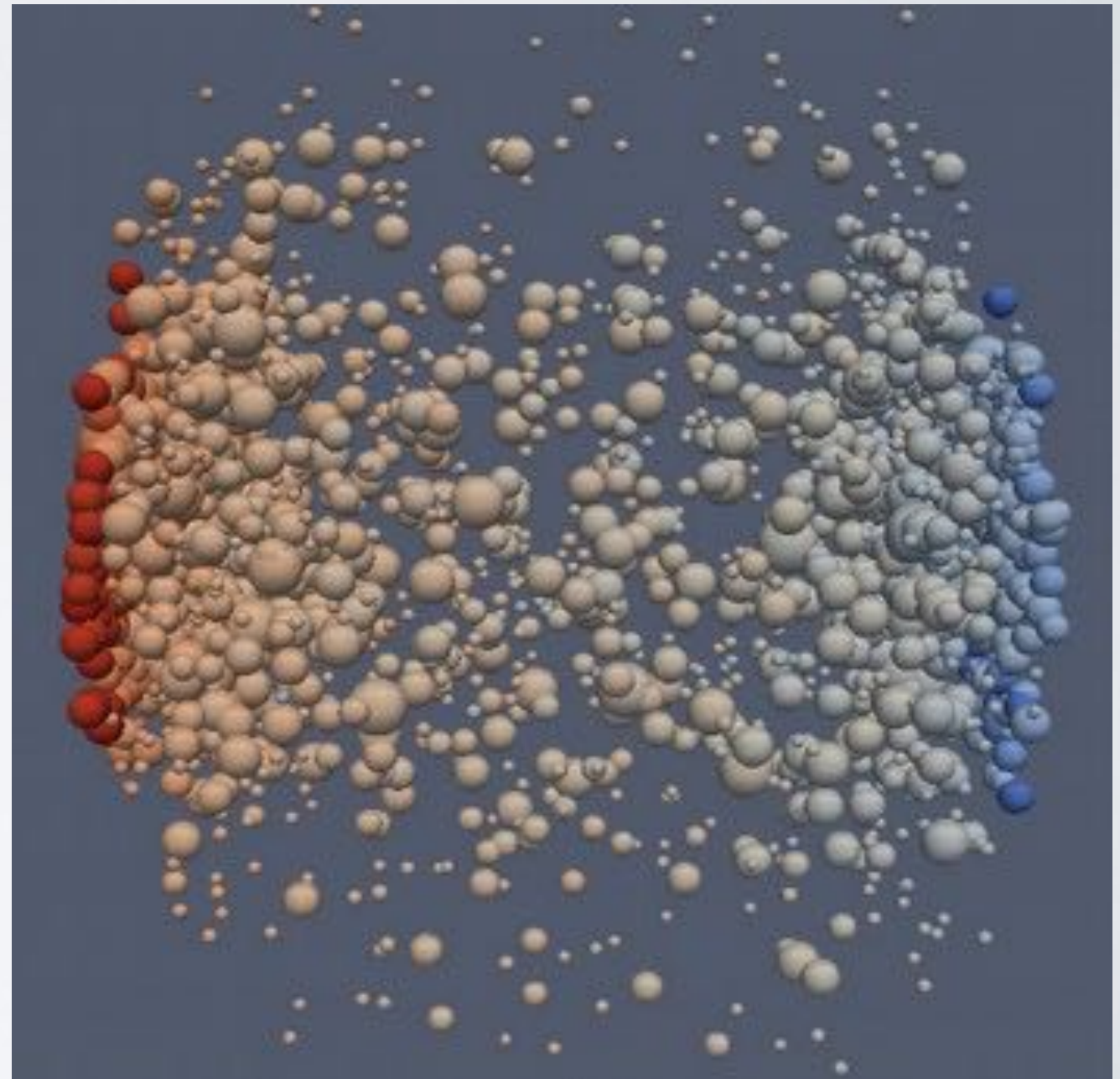
PARTICLIZATION

- The fluid description is matched to a kinetic one, by sampling discrete particles;
- Sampling is carried over the freeze-out hypersurface obtained at the end of hydro;
- iSS code is a publicly available sampler (github.com/chunshen1987/iSS)



URQMD

- Most widely used microscopic transport model that simulates dynamics of the hadronic system;
- Simulates binary collisions, resonance formation and decays;
- Code is publicly available with restrictions (urqmd.org)



ANALYSIS: HADREX CLASS

- ROOT-compliant C++ class for storing HIC events produced with different generator codes;
- Compatible with the OSCAR initiative file format for HIC (<https://karman.physics.purdue.edu/OSCAR/>) ;
- Allows for the usage of standard ROOT macros for calculation of observables.