Mapping the principal component flow analysis from initial conditions

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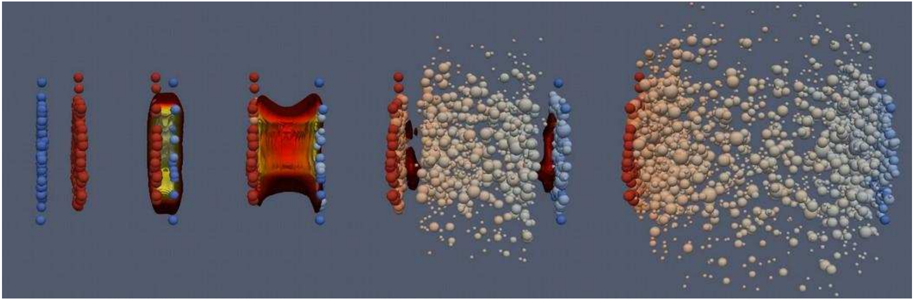
FAPESP

Hydrodynamic response in heavy-ion collisions

- Anisotropic flow is an invaluable probe of the QGP.
- How does it respond to initial geometry?
- What features of the fluctuating initial stages determine the angular correlations between particles?
- Ultimate visualization of two-particle correlations: **PCA**.
- What do the PCA observables tell us?

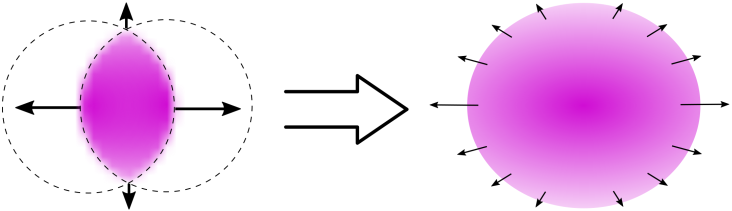
Heavy Ion Collisions



- Early stages, expansion, particlization, hadronic dynamics.
- Here, TRENTO + MUSIC + ISS + UrQMD for $Pb + Pb$ at $\sqrt{s_{NN}} = 5.02$ TeV.

MADAI collaboration, H. Petersen, J. Bernhard.

Anisotropic flow



- Initial density profile results in anisotropic flow.
- Response depends on the QGP properties.
- Characterization by flow harmonics.

Flow harmonics

Angular profile in transverse plane in each event (momentum space):

$$\frac{dN}{p_T\,dp_T\,d\eta\,d\varphi} = \frac{1}{2\,\pi}\frac{dN}{p_T\,dp_T\,d\eta}\sum_{n=-\infty}^{\infty} V_n(p_T,\eta)\,e^{-in\varphi}\,. \tag{1}$$

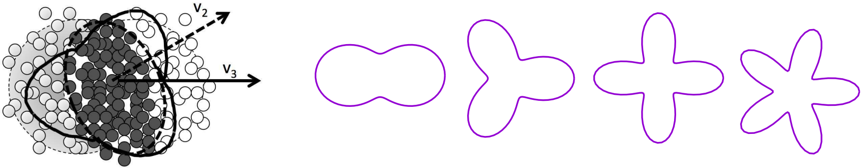


Fig: C. Nonaka, M. Asakawa, PTEP 2012 (2012).

Principal-component analysis (PCA)

- Fluctuations correlate particle pairs in transverse plane:

$$\left\langle \frac{dN_{\text{pairs}}}{d\phi_1\, d\phi_2} \right\rangle = \frac{1}{2\,\pi} \sum_{n=-\infty}^{\infty} \mathcal{V}_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2)\, e^{-in\Delta\varphi}\,. \qquad (2)$$

- Information contained in first eigenvectors and eigenvalues (PCA):

$$\mathcal{V}_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) \simeq \sum_{\alpha=1}^{k} \langle N(\mathbf{p}_1)\rangle \langle N(\mathbf{p}_2)\rangle\, v_n^{(\alpha)}(\mathbf{p}_1) v_n^{(\alpha)}(\mathbf{p}_2) \qquad (3)$$

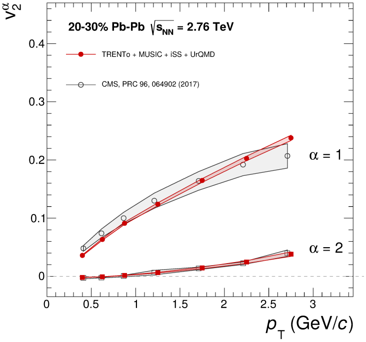
Ordered eigenvalues $\lambda_\alpha = ||\langle N\rangle\, v_n^{(\alpha)}||^2$, $\lambda^\alpha > \lambda^{\alpha+1}$.

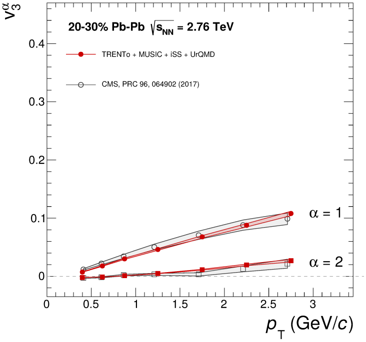
R. S. Bhalerao, J. Y. Ollitrault, S. Pal and D. Teaney, PRL **114** (2015).
A. M. Sirunyan *et al.* [CMS Collaboration], PRC **96** (2017).

Principal-component analysis

In the hydrodynamic picture,

$$\mathcal{V}_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) = \langle N(\mathbf{p}_1)\, V_n(\mathbf{p}_1)\, N(\mathbf{p}_2)\, V_n^*(\mathbf{p}_2)\rangle \,. \tag{4}$$

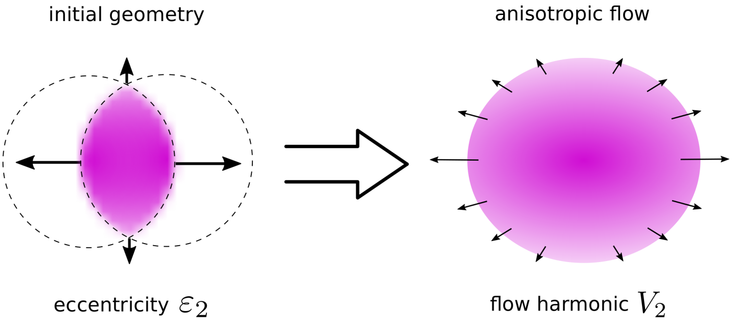




arXiv:1811.05048

Mapping the flow from initial geometry

Larger pressure gradients → stronger flow!



Linear response: $V_n \simeq \kappa_n \, \varepsilon_n$.

Mapping the flow from initial geometry

- Generalization from cumulants of the initial density profile. Example expansion:

$$V_2 \simeq \kappa\, \epsilon_{2,2} + \kappa'\, \epsilon_{2,4} + \kappa''\, |\epsilon_{2,2}|^2 \epsilon_{2,2} + \dots$$

- Dimensionless eccentricities from cumulants:

$$\epsilon_{n,m} := W_{n,m}/(W_{0,2})^{m/2}\,. \tag{5}$$

- Cumulant-generating function from density profile $\rho(\vec{x})$:

$$W(\vec{k}) = \log\left(\int d^2x\, \rho(\vec{x})\, e^{i\,\vec{k}\cdot\vec{x}}\right) = \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} W_{n,m}\, k^m\, e^{-i\,n\,\phi_{\vec{k}}}\,. \tag{6}$$

F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, PRC **85** (2012).
D. Teaney and L. Yan, PRC **83** (2011).

Hydrodynamic response

- Flow harmonics V_n can be predicted by $\epsilon_{n,m}$.
- The κ's are found by minimizing

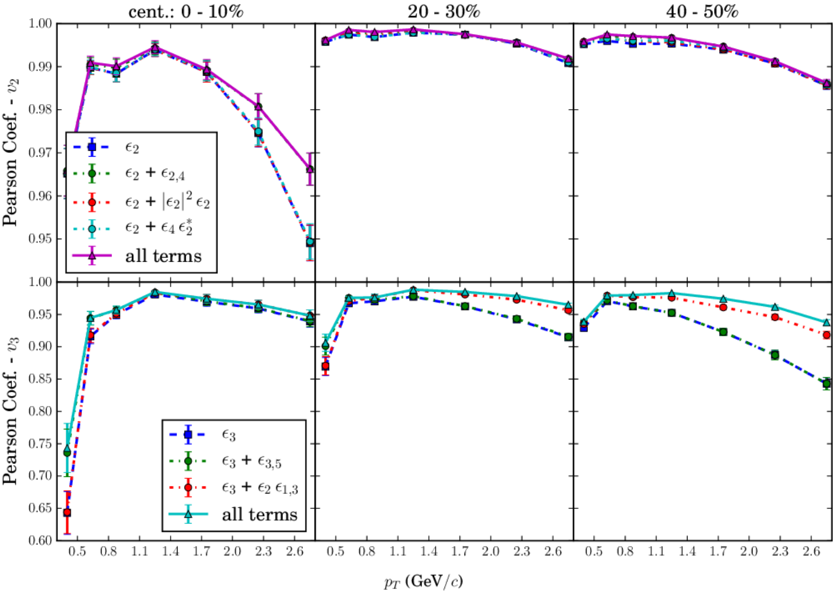
$$\langle|\delta_{\text{residual}}|\rangle^2 := \langle|V_n^{\text{est.}} - V_n|^2\rangle\,. \tag{7}$$

- The quality of the predictor can be measured by the Pearson coefficient

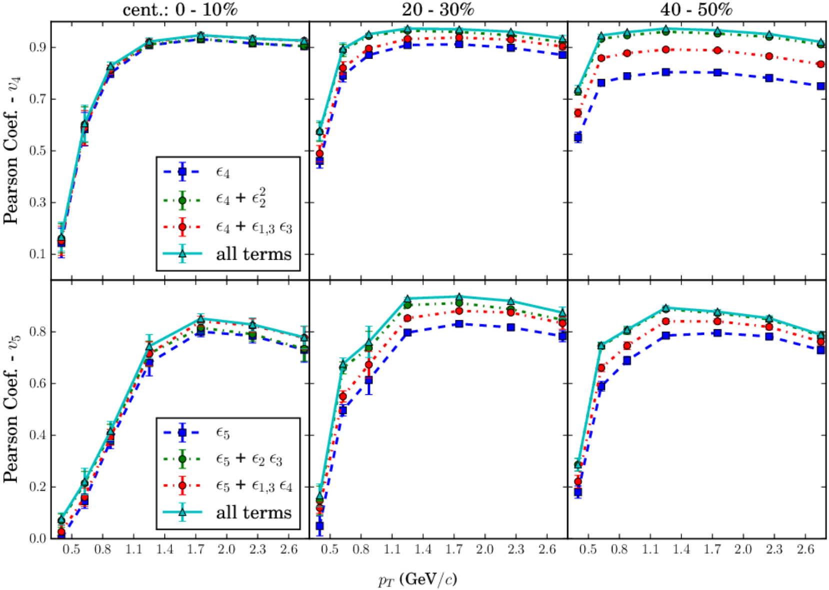
$$\frac{\langle V_n^* \, V_n^{\text{est.}}\rangle}{\sqrt{\langle|V_n|^2\rangle \, \langle|V_n^{\text{est.}}|^2\rangle}} \leq 1\,. \tag{8}$$

- To predict differential flow, $V_n(p_T)$, find $\kappa(p_T)$.

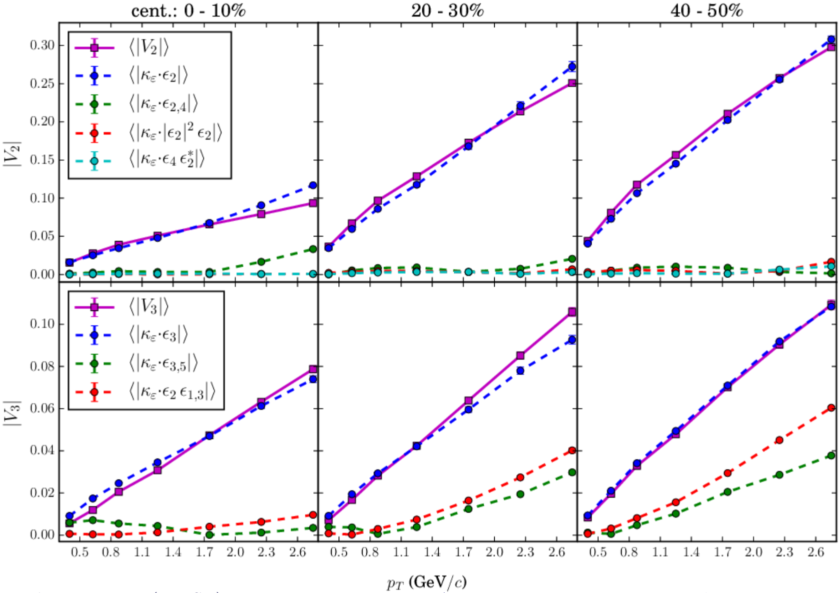
Estimator performance – V_2 and V_3



Estimator performance– V_4 and V_5



Importance of each term– V_2 and V_3



Remarks

- Excellent predictor for V_2!
- In $\epsilon_{n,m}$: $n \sim$ transformation under rotations; $m \sim$ distinct length scales.
- More terms are required at high centrality and p_T.
- Better predictions for $p_T \gtrsim 1$ GeV/c.
- Do predictions cover the details of flow fluctuations?
- Check via principal-component analysis.

Principal-component analysis (PCA)

- In the hydrodynamic picture,

$$\mathcal{V}_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) = \langle N(\mathbf{p}_1)\, N(\mathbf{p}_2)\, V_n(\mathbf{p}_1)\, V_n^*(\mathbf{p}_2)\rangle\,. \tag{9}$$

- Only $\langle N(\mathbf{p}_1)\rangle\langle N(\mathbf{p}_2)\rangle$ is compensated by definition of $v_n^{(\alpha)}$:

$$\mathcal{V}_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) \simeq \sum_{\alpha=1}^{k} \langle N(\mathbf{p}_1)\rangle\langle N(\mathbf{p}_2)\rangle\, v_n^{(\alpha)}(\mathbf{p}_1) v_n^{(\alpha)}(\mathbf{p}_2) \tag{10}$$

- Multiplicity fluctuations not predicted by the “mapping”.

- Change of definition...

Normalized principal-component analysis

- Write angular distribution of pairs as:

$$\left\langle \frac{1}{N_{\text{pairs}}} \frac{dN_{\text{pairs}}}{d\phi_1\, d\phi_2} \right\rangle = \frac{1}{2\,\pi} \sum_{n=-\infty}^{\infty} V_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2)\, e^{-in\Delta\varphi}\,. \qquad (11)$$

- In the hydrodynamic picture,

$$V_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) = \langle V_n(\mathbf{p}_1)\, V_n^*(\mathbf{p}_2)\rangle\,. \qquad (12)$$

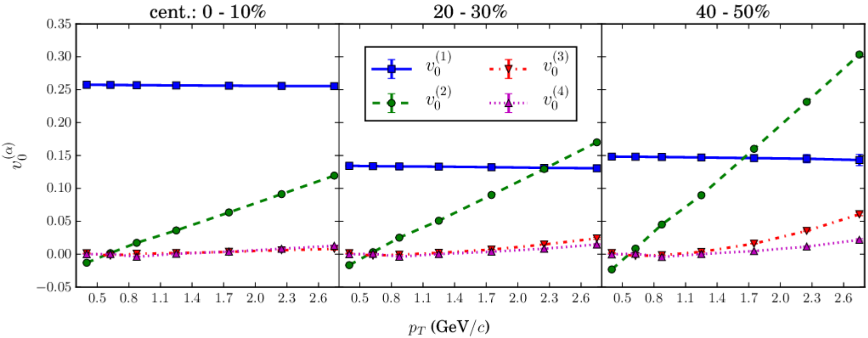
- Information contained in first eigenvectors and eigenvalues (PCA):

$$V_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) \simeq \sum_{\alpha=1}^{k} v_n^{(\alpha)}(\mathbf{p}_1) v_n^{(\alpha)}(\mathbf{p}_2) \qquad (13)$$

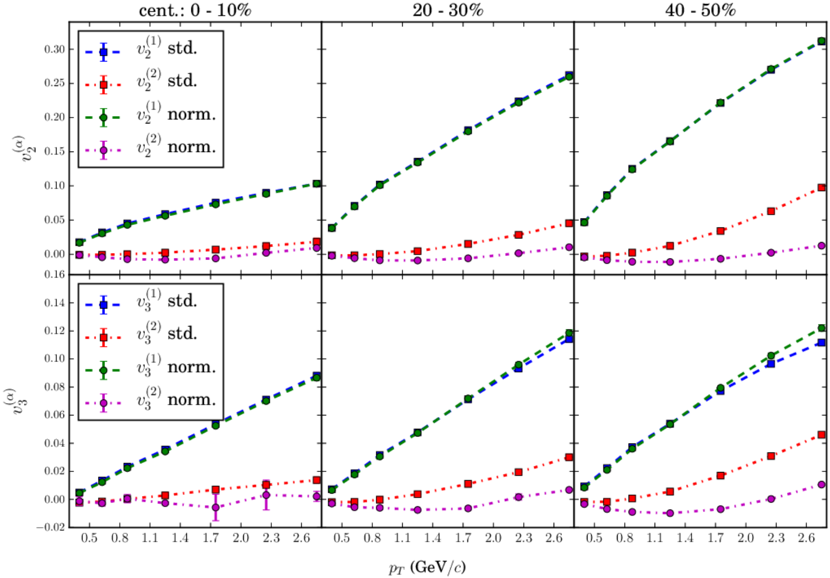
Ordered eigenvalues $\lambda_\alpha = ||v_n^{(\alpha)}||^2$, $\lambda^\alpha > \lambda^{\alpha+1}$.

Multiplicity fluctuations

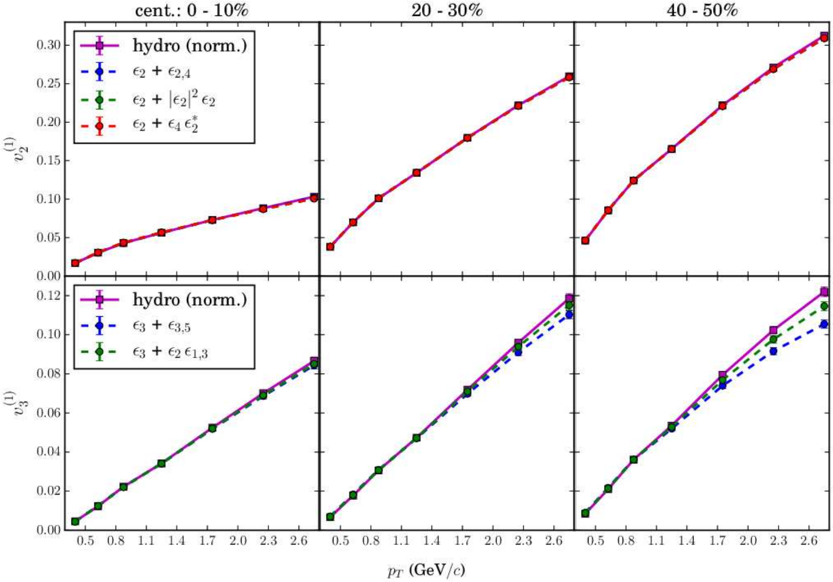
Non-trivial multiplicity fluctuations via $\mathcal{V}_{0\Delta} := \langle \Delta N(\mathbf{p}_1)\, \Delta N(\mathbf{p}_2) \rangle$:



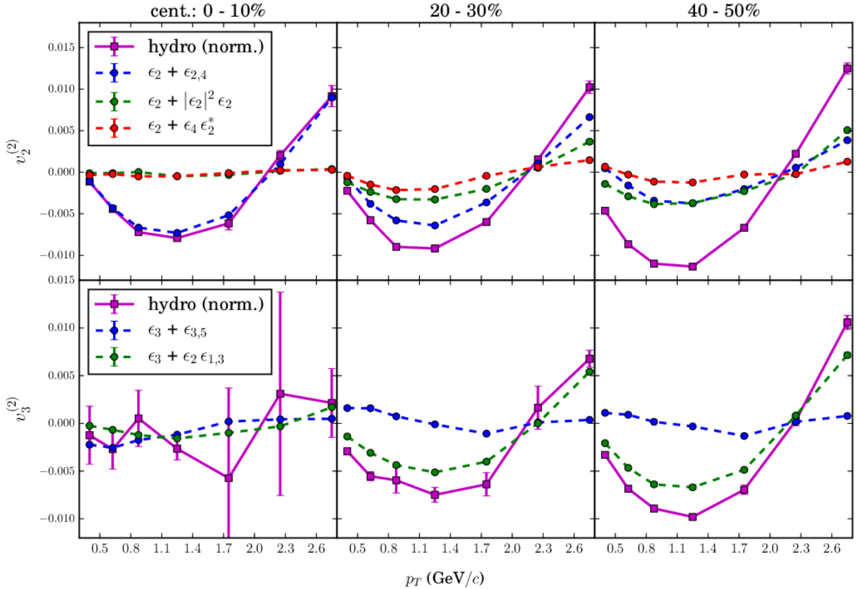
Standard *vs.* Normalized PCA



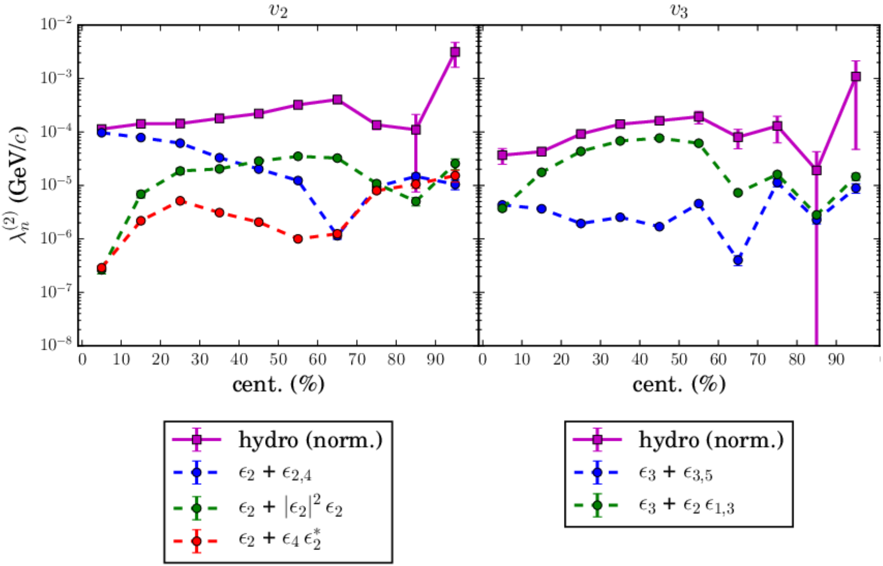
Predictor performance – 1st PCA mode



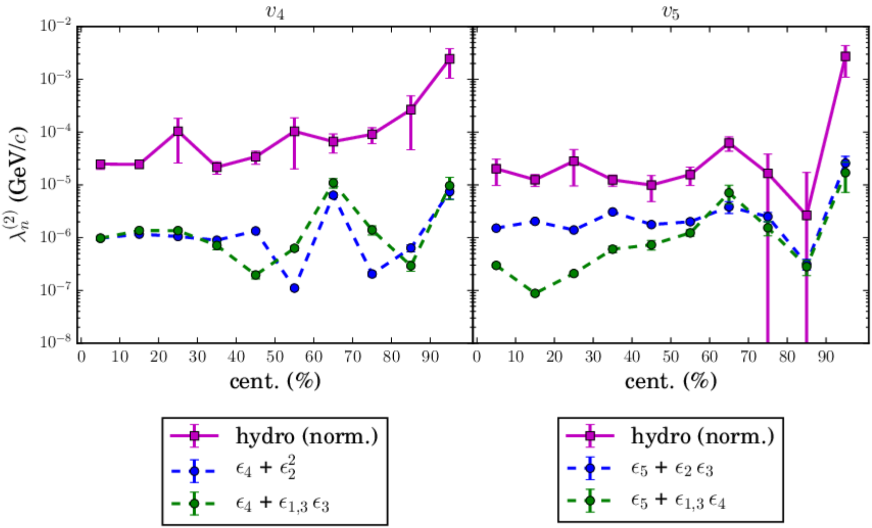
Predictor performance – 2$^{\text{nd}}$ PCA mode



Predictor performance – 2^{nd} PCA eigenvalue



Predictor performance – 2nd PCA eigenvalue



Remarks

- Nontrivial multiplicity fluctuations!
- Subleading mode connected to subleading *terms*.
- Subleading PCA mode is much more sensitive to extra terms!
- Only V_2 at central collisions is reproduced. More terms?
- More peripheral collisions $\rightarrow$ nonlinear terms missing?

Final remarks

- The subleading PCA *eigenvalue* is a very sensitive observable.
- Identification of the most important features of the fluctuating geometry.
- Improvement with a larger number of terms? On the way!

Acknowledgements

Thank you.

Cumulant examples

$$W_{0,2} = \frac{i^2}{2!}\frac{1}{2}\,\left[\{r^2\} - \{r\,e^{-i\,\phi}\}\,\{r\,e^{-i\,\phi}\}\right]\,, \tag{14}$$

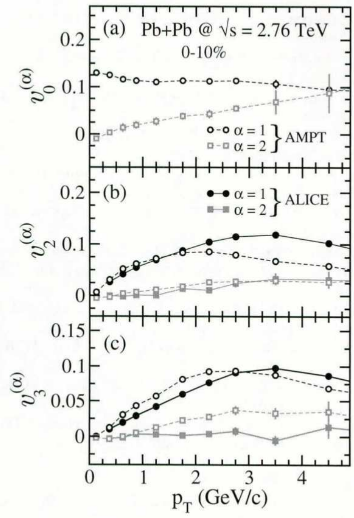
$$W_{2,2} = \frac{i^2}{2!}\frac{1}{4}\,\left[\{r^2\,e^{2\,i\,\phi}\} - \{r\,e^{i\,\phi}\}^2\right]\,, \tag{15}$$

$$W_{1,3} = \frac{i^3}{3!}\frac{3}{8}\,\left[\{r^3\,e^{i\,\phi}\} - \{r^2\,e^{2\,i\,\phi}\}\,\{r\,e^{-i\,\phi}\}\right]\,, \tag{16}$$

where

$$\{\ldots\} := \frac{\int d^2x\,\rho(\vec{x})\,(\ldots)}{\int d^2x\,\rho(\vec{x})} \tag{17}$$

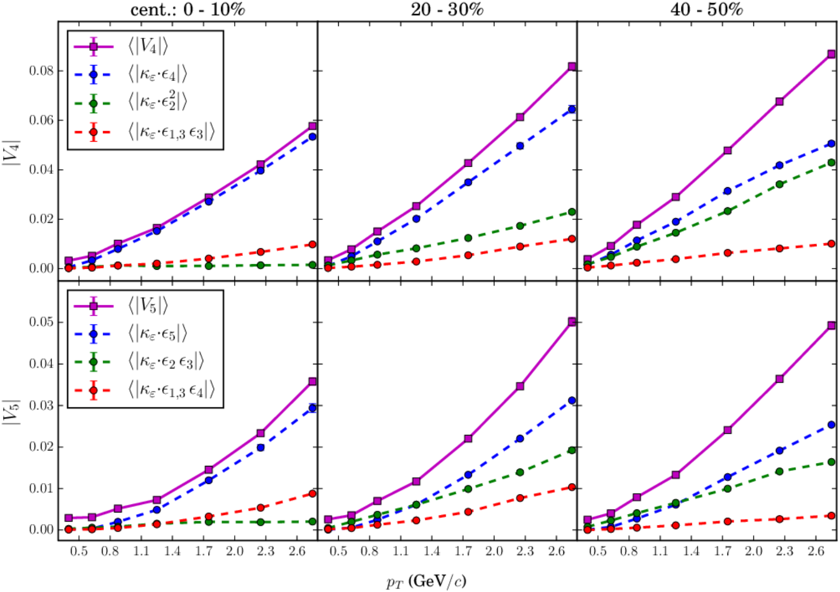
Original PCA from AMPT



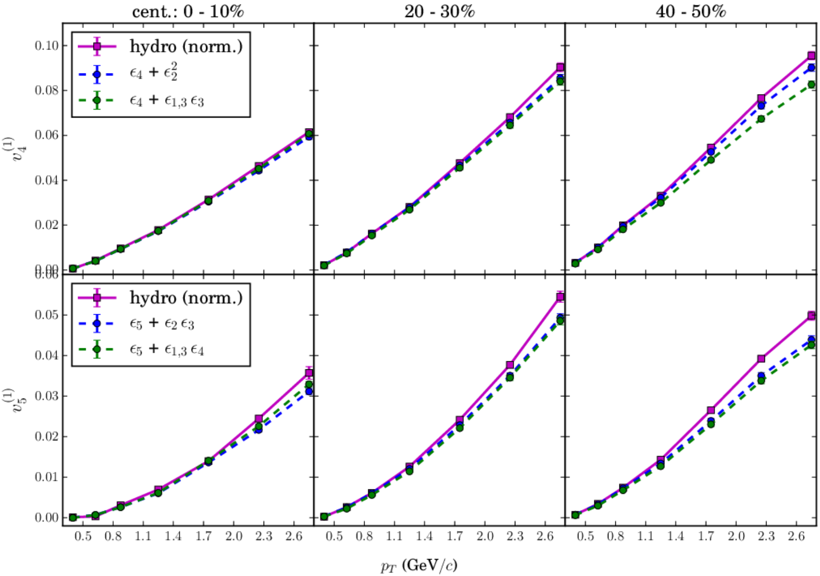
General hydrodynamic response

$$V_n^{\mathrm{est}}(p_T, y) = \sum_{m=\max[n,2]}^{m_{\max}} \kappa_{n,m}(p_T, y)\, \epsilon_{n,m} + \\ + \sum_{p=2}^{p_{\max}} \sum_{\substack{\{n_i, m_i\} \\ \sum n_i = n \\ m < m_{\max}}} \kappa^{(p)}_{\{n_i, m_i\}}(p_T, y) \prod_{i=1}^{p} \epsilon_{n_i, m_i}\, , \tag{18}$$

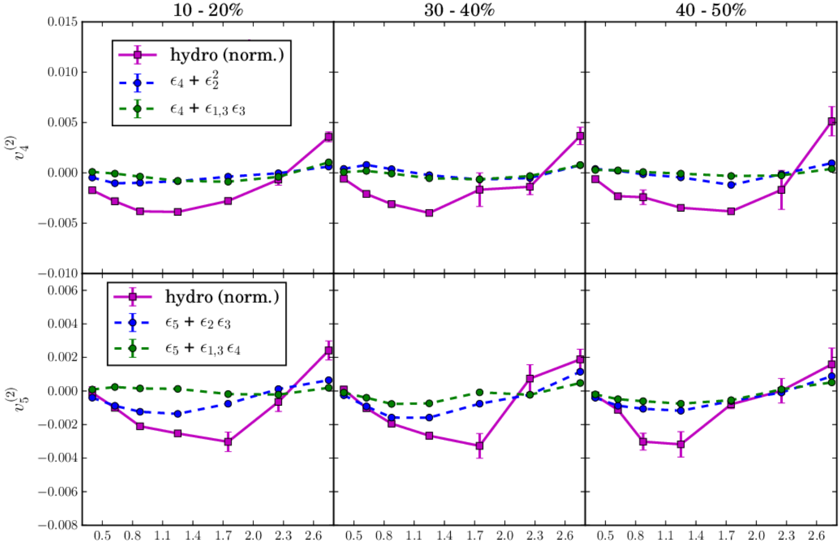
Importance of each term– V_4 and V_5



Predictor performance for V_4 and V_5 – 1$^{\text{st}}$ PCA mode



Predictor performance for V_4 and V_5 – 2$^{\text{nd}}$ PCA mode



Bin-independent orthonormality condition

Naive diagonalization of $\mathcal{V}^{ab}_{n\Delta}$ with conventional algorithms will result in

$$\sum_a \psi_n^{(\alpha)a}\,\psi_n^{(\beta)a} = \delta_{\alpha\beta}\,, \tag{19}$$

not compatible with

$$\int dp_T\,\psi_n^{(\alpha)}(p_T)\,\psi^{(\beta)}(p_T) = \delta_{\alpha\beta}\,. \tag{20}$$

We instead diagonalize

$$\sqrt{\Delta p_T^a \Delta p_T^b}\,\mathcal{V}^{ab}_{n\Delta} = \sum_i \lambda^{(\alpha)}\,\tilde{\psi}_n^{(\alpha)a}\,\tilde{\psi}_n^{(\alpha)b} \tag{21}$$

$$\sum_a \tilde{\psi}_n^{(\alpha)a}\,\tilde{\psi}_n^{(\beta)a} = \delta_{\alpha\beta}\,. \tag{22}$$

Replacing $\tilde{\psi}^{(\alpha)a} \to \sqrt{\Delta p_T^a}\psi^{(\alpha)a}$:

$$\mathcal{V}^{ab}_{n\Delta} = \sum_\alpha \lambda^{(\alpha)}\,\psi_n^{(\alpha)a}\,\psi_n^{(\alpha)b}\,, \qquad \sum_a \Delta p_T^a\,\psi_n^{(\alpha)a}\,\psi_n^{(\beta)a} = \delta_{\alpha\beta}\,. \tag{23}$$

Generalized eccentricities

Anisotropy and orientation of the initial density profile $\rho(\vec{x})$ given by

$$W(\vec{k}) := \log\left(\int d^2x\, \rho(\vec{x})\, e^{i\,\vec{k}\cdot\vec{x}}\right) := \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} W_{n,m}\, k^m\, e^{-i\,n\,\phi_{\vec{k}}}\,. \quad (24)$$

Dimensionless eccentricities from cumulants:

$$\epsilon_{n,m} := W_{n,m}/(W_{0,2})^{m/2}\,. \quad (25)$$

- $\epsilon_{n,m} \to e^{-in\delta}\,\epsilon_{n,m}$ under $\phi_{\vec{k}} \to \phi_{\vec{k}} + \delta$.
- $\epsilon_{n\neq 1,m}$ is invariant under $\vec{x} \to \vec{x} + \vec{R}$.

Taylor expansion

Assuming harmonics are given by $\rho(\vec{x})$:

$$V_n = V_n[\rho(\vec{x})] = V_n[W(\vec{k})] = V_n[\{\epsilon_{n,m}\}]\,. \tag{26}$$

Linear response:

$$V_n^{a\,\mathrm{est.}} \simeq \sum_{m=n}^{m_{max}} \kappa^a_{n,m}\,\epsilon_{n,m} + \delta_{\mathrm{residual}}\,. \tag{27}$$

Nonlinear response:

$$\begin{aligned} V_n^{a\,\mathrm{est.}} \simeq \sum_{m=n}^{m_{max}} \kappa^a_{n,m}\,\epsilon_{n,m} + \sum_{l=1}^{m_{max}} \sum_{m=l}^{m_{max}} \sum_{m'=|m-l|}^{m_{max}} \kappa^a_{l,m,m'}\,\epsilon_{l,m}\,\epsilon_{n-l,m'} + \\ + \delta_{\mathrm{residual}}\,. \end{aligned} \tag{28}$$

Factorization breaking

Notice that, if $\mathcal{V}_{n\,\Delta}$ has only one non-vanishing eigenvalue,

$$\mathcal{V}^{ab}_{n\,\Delta} = \sqrt{\lambda_n}\,\psi_n(p^a_T)\,\sqrt{\lambda_n}\,\psi_n(p^b_T)^*\,, \tag{29}$$

$$\left\langle \frac{dN}{d\vec{p}_1}\,\frac{dN}{d\vec{p}_2} \right\rangle = \left\langle \frac{dN}{d\vec{p}_1} \right\rangle \left\langle \frac{dN}{d\vec{p}_2} \right\rangle\,, \tag{30}$$

and the subleading modes are connected to factorization breaking.

Anisotropic flow′

We use the flow vector

$$Q_n^a = \sum_{i=1}^{N^a} e^{i\,n\,\varphi_i^{(a)}}\,. \tag{31}$$

While $\langle Q_{n\neq 0}\rangle_{\text{isotropic}} = 0$, angular correlations contained in

$$Q_{n\,\Delta}^{ab} = \langle Q_n^{a*} Q_n^b\rangle - \delta^{ab}\,\langle Q_0^a\rangle \neq 0\,. \tag{32}$$

If we write

$$\frac{dN}{p_T\,dp_T\,d\eta\,d\varphi} = \frac{1}{2\,\pi}\frac{dN}{p_T\,dp_T\,d\eta}\sum_{n=-\infty}^{\infty} V_n(p_T,\eta)\,e^{-in\varphi}\,, \tag{33}$$

then $V_n^a \simeq Q_n^a/N^a$ and $v_n^a = |V_n^a|$.