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Determination of Young's modulus using optical fiber long-period gratings

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Abstract
Curvature sensitive CO_2-laser induced long-period fiber gratings (LPGs) were employed to measure the Young's moduli of materials. Two techniques, 'bar resonance' and 'through transmission', were used. In the first case, flexural vibrations of bars made of various industrial materials arranged in a cantilever configuration were probed by the LPG. The measured response allowed us to obtain the bar's vertical movement as a function of time, its frequency components and the bar material's Young's modulus. In the second case, the optical response of LPGs was used to determine the propagation velocities of perturbations along a bar, which allowed the straightforward calculation of the Young's modulus. The values obtained show good agreement with the ones reported in the literature. The results obtained in this paper demonstrate the feasibility of using LPGs to dynamically characterize a material's elastic properties. To the best of our knowledge, this is the first demonstration of the use of long-period fiber gratings for dynamically determining Young's modulus values.

Keywords: optical fiber sensor, vibration sensor, Young's modulus, long-period fiber grating

(Some figures may appear in colour only in the online journal)

1. Introduction

Predicting a material's behavior under deformation is a very important task in engineering. Thus, measuring a material's Young's modulus, which is a fundamental parameter allowing us to determine the stiffness of a given material, is essential for the development of engineering projects [1].

Young's modulus can be obtained from static and dynamic measurements. When the Young's modulus is studied from a static point of view, a setup in which a uniaxial stress is applied on the sample is usually employed. By taking into account the slope of the stress–strain curve at the origin, the elastic modulus is then calculated. Dynamic measurements, in turn, are more precise than the static ones because they employ very low strain levels. Moreover, they are nondestructive and repeatable tests [1]. These measurements are generally performed using the 'through transmission' [2] or the 'bar resonance' technique [1, 3].

Although Young's modulus can be dynamically measured by using electrical sensors [1], there are a number of advantages to building up fiber optics-based devices. Among these advantages are the small size of the devices, their ease of fabrication, their high sensitivity and their immunity to electromagnetic interference. Besides, they are very robust and make it possibile to act in harsh environments and can be inserted into the structure to be tested. It is particularly important, for instance, in civil engineering applications, since the fiber can be inserted into walls, beams and floorings [4, 5].

In this paper, we show the feasibility of using CO_2 laser induced long-period gratings (LPGs) for determining the Young's modulus of materials. To do this, two methods were studied. In the first method, we characterized the dynamic response of bars made of different materials put in oscillation. By recovering the bars' flexural oscillations from the LPG time response and by taking the fast Fourier transform (FFT) of it, the movement vibration frequencies could be obtained. Knowledge of these vibration frequencies for the oscillation lengths of different bars allowed us to calculate the materials' Young's moduli. In the second method, the Young's modulus was determined by measuring the propagation velocities of longitudinal and transversal perturbations along a bar also via the observation of the LPG's optical responses.

2. Young's modulus determination

The first setup to be presented for measuring a material's Young's modulus employs the 'bar resonance' method [1, 3]. In this technique, a bar of the material of interest is arranged in a cantilever configuration and put in oscillation. The resulting movement is studied in order to obtain the bar material's Young's modulus value. In contrast to reference [1], where the authors employed a force sensor to register the displacement of the bar as a function of time, here we use a curvature sensitive long-period fiber grating to obtain this measurement.

The Young's modulus of optical fibers was previously measured [6, 7]. However, we emphasize that our interest is in measuring the Young's moduli of materials other than the one that the fiber is made from. Thus, to our knowledge, this is the first paper to report the use of long-period fiber gratings for determining the Young's moduli of materials of interest.

Consider a horizontal bar set in a configuration in which it is fixed at one of its ends. If the bar is vertically deflected from its equilibrium state and then released, the resulting movement is oscillatory and its amplitude decays as a function of time. The formal treatment of this movement, via the Euler–Bernoulli equation, indicates that the general solution has a series form with infinite frequency components, which are given by equation (1), where ρ and Y are, respectively, the material density and Young's modulus; A and L are, respectively, the cross-sectional area and the vibrating length of the bar; I is the second moment of the cross-section, which, for a rectangular bar of width d and depth h, can be calculated as $d\,h^3/12$. Moreover, in equation (1) one can find the parameter λ_n, known as modal eigenvalue, which is determined by the boundary conditions of the problem [1]

$$f_n = \left(\frac{\lambda_n^2}{2\pi}\sqrt{\frac{YI}{\rho A}}\right)\frac{1}{L^2}. \tag{1}$$

As we focus on the determination of the Young's modulus, one can see, from equation (1), that the knowledge of only one frequency component is enough to attain our goal. In this investigation, we determined the first frequency component, f_1, and then found the Young's moduli of the materials of interest.

The second method reported here for obtaining the Young's modulus employs the 'through transmission' technique of acoustic waves [8]. The experiment is carried out by exciting longitudinal and transversal mechanical perturbations on a long bar and then by measuring the propagation velocities of these perturbations. Knowledge of the longitudinal and transversal velocities of the induced mechanical perturbation (v_L and v_T, respectively) allows us to calculate the Young's modulus of the material from equation (2) (where ρ is material density) [8]

$$\mathrm{Y} = \frac{3\rho v_T^2}{v_L^2 - v_T^2}\left(v_L^2 - \frac{4}{3}v_T^2\right). \tag{2}$$

3. Long-period gratings and their curvature sensitivity

Long-period fiber gratings (LPGs) consist of a longitudinal periodic perturbation of the refractive index of an optical fiber which is able to provide coupling between core and cladding modes at certain wavelengths. Equation (3) describes the wavelengths $\lambda^{(m)}$ where the coupling between the referenced modes happen—n_{co} is the effective refractive index of the core mode, $n_{cl}^{(m)}$ is the effective refractive index of the mth order cladding mode and Λ is the period of the refractive index perturbation. Experimentally, the wavelengths at which coupling is attained are seen as dips in the LPG transmission spectrum (one example can be seen in the inset of figure 2) [9]

$$\lambda^{(m)} = (n_{co} - n_{cl}^{(m)})\Lambda. \tag{3}$$

The refractive index modulation for inducing long period gratings can be obtained, for instance, by applying an electrical arc or by pressurizing a corrugated board against the fiber [9]. Also, exposing the fiber to a femtosecond, UV or CO_2 laser is a means to achieve the refractive index longitudinal pattern [10].

In this research, CO_2 laser-induced LPGs imprinted on standard telecom optical fibers were employed. The gratings obtained in this way have the property of being sensitive to the curvature conditions to which the fiber is subjected (the curvature itself and its orientation). This sensitivity arises from the fact that the fabrication method provides a nonuniform refractive index change on the cross-section of the fiber, causing the cladding modes field distribution to be asymmetric. Thus, according to the direction that the fiber is bent, the overlap integral value between the coupled modes can increase or decrease and, therefore, a variation in the coupling coefficient between the modes is observed [11, 12].

This variation on the coupling coefficient can be experimentally realized when observing the depth of the LPG spectral resonances: the deeper the resonance, the higher the coupling coefficient between the modes. As it is associated with the bending conditions, the fiber curvature state can be probed if this feature is characterized.

In order to perform a characterization measurement of the LPG response to curvature variations, a 500 μm pitch and 2.5 mm long LPG was set in a configuration as shown in figure 1. In the experimental setup, a super-luminescent LED (SLED) is used as the light source. An optical spectrum analyzer (OSA) and a photodetector (PDT) coupled to an oscilloscope (OSC) are used for taking measurements. The LPG is glued (using Loctite® Super Bonder) on a bar made of a material of interest, which has one fixed end and the other one is let free. The deflection of the bar (accounted as a vertical displacement Δy of the free bar end), causes the LPG to bend and, thus, its curvature response can be monitored. It's worthwhile emphasizing that, during characterization, the free bar end is deflected by using a micrometric screw.

Figure 2 shows how the optical response of the LPG, whose transmission spectrum is presented in the inset, varies according to the bar end vertical displacement Δy. The situation in which the bar is straight is defined as $\Delta y = 0$. Positive

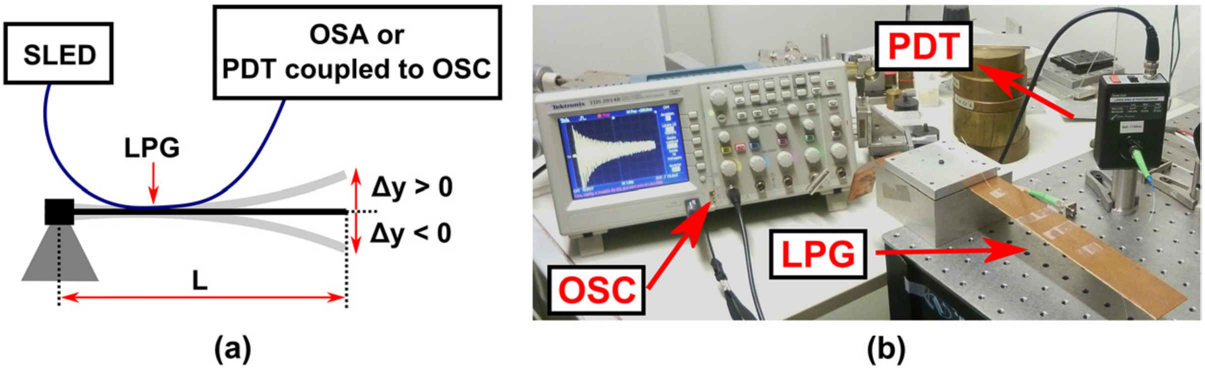

Figure 1. (a) Scheme and (b) picture of the experimental setup for 'bar resonance' method. SLED: super-luminescent LED; OSA: optical spectrum analyzer; PDT: photodetector; OSC: oscilloscope; Δy: vertical displacement; L: vibrating bar length.

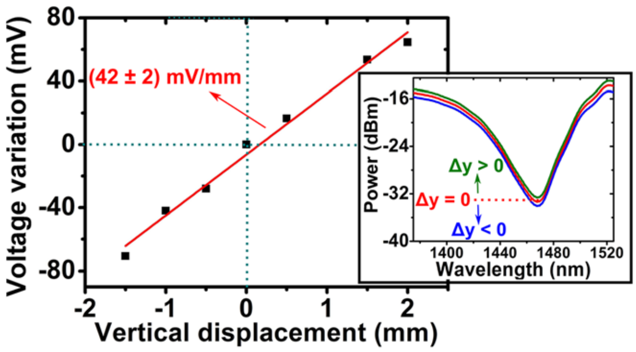

Figure 2. System calibration. The vertical displacement of the bar implies a depth variation of the LPG resonance. It can be accounted for as a voltage variation in the photodetector. Inset shows the spectral characteristics of the LPG resonance.

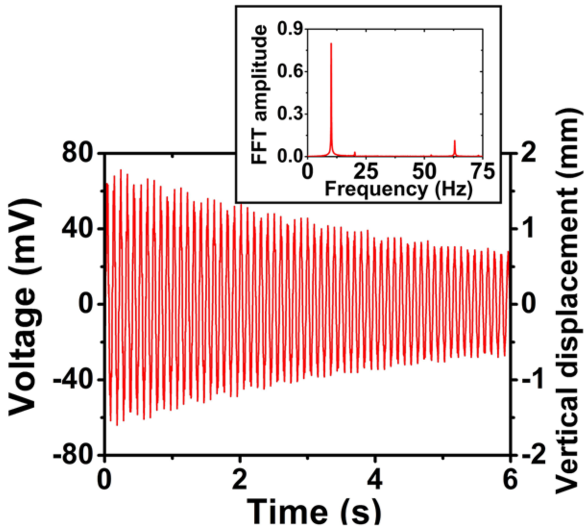

Figure 3. Photodetector (left axis) time response and its conversion to vertical displacement (right axis). Inset shows the fast Fourier transform (FFT) amplitude as a function of frequency.

variations in Δy indicate upward displacements and negative variations downward displacements.

The vertical axis in the figure 2 plot follows the same logic. The voltage measured in the photodetector is assumed to be zero when the bar is straight. If there is an increment in the transmitted light power, the voltage variation is positive; if, in contrast, there is a light power decrement, the voltage variation is negative.

By observing the figure 2 inset, one can see that when Δy is positive, the LPG resonance is shallower than when Δy is negative. Ergo, when detecting light using the photodetector, as its voltage response is proportional to the overall optical power from the broadband light source, negative voltage variations are related to negative Δy values and positive voltage variations identify positive Δy values. It happens due to the fact that, for a shallower resonance, a higher overall light power level sensitizes the photodetector and, for a deeper one, a lower light power level is detected.

The dependence of voltage on bar vertical displacement is seen to be linear. The angular coefficient of the fitted line, (42 ± 2) mV mm^{-1}, acts as a calibration factor for recovering the vertical displacement of the bar from optical data. It's worth emphasizing, however, that the linear behavior is observed since the measurements were performed in a small curvature range. For macrobending conditions, a nonlinear behavior is expected [13].

4. 'Bar resonance' method results

The 'bar resonance' method allows us to obtain the Young's modulus of a material by probing the flexural movements of bars put in oscillation in a cantilever setup. In order to probe the oscillatory movement of the bar, we again use the configuration shown in figure 1 with the photodetector as the measurement system.

Initially, the bar is deflected ($\Delta y < 2$ mm) and then released. The subsequent movement around the equilibrium position causes the LPG to bend upwards and downwards. The signal measured in the oscilloscope takes a sinusoidal form whose amplitude decays as a function of time (figure 3).

The left axis of the plot in figure 3 shows the photodetector signal measured by the oscilloscope when a 21 cm long steel bar is put in oscillation—the LPG was glued at a distance $L = (13.00 \pm 0.05)$ cm from the fixed end of the bar. Dividing this data by the calibration constant, (42 ± 2) mV mm^{-1}, one can calculate the vertical displacement Δy of the bar—right axis in figure 3. Furthermore, it is worth observing that the movement acceleration can be obtained by simply taking the second derivative of the signal shown in figure 3. Experimental

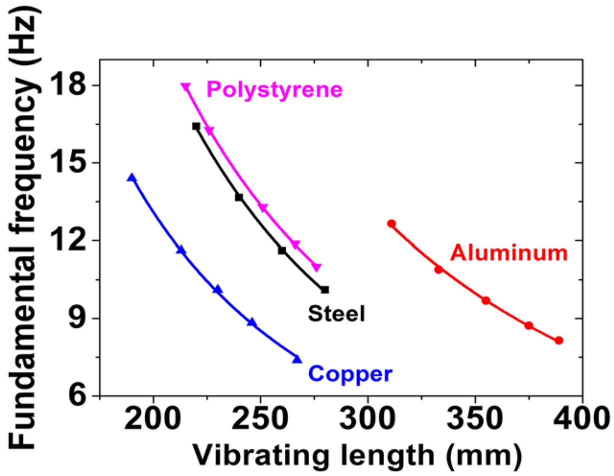

Figure 4. Fundamental frequency versus vibrating bar length plot for tested materials. Dots represent the measured data and solid lines the fitted curves.

Table 1. Characteristics of the bars employed in the experiments.

Material	Density, ρ (kg m^{-2})	Cross-section area, A (mm^2)	Cross-section second moment, I (mm^4)
Steel	7935 [14]	24.774 ± 0.003	1.9827 ± 0.0006
Aluminum	2700 [15]	73.204 ± 0.005	13.543 ± 0.003
Copper	8960 [15]	34.197 ± 0.004	2.5181 ± 0.0008
Polystyrene	1072 [16]	112.775 ± 0.003	99.265 ± 0.009

Table 2. Measured and literature Young's moduli values.

Material	Y_{measured} (GPa) (this paper)	$Y_{\text{literature}}$ (GPa)
Steel	200 ± 3	190–213 [17, 18]
Aluminum	70.9 ± 0.8	70–72 [1]
Copper	108 ± 2	110–120 [1]
Polystyrene	3.03 ± 0.06	2–4 [19]

results show that the movement acceleration reaches values in the order of 20 m s^{-2}.

By taking the Fourier transform of the measured signal, one can identify the frequency components of the oscillating bar movement. The inset in figure 3 shows the amplitude of the fast Fourier transform (FFT), calculated from figure 3 data, as a function of the frequency. The fundamental frequency, identified as the highest peak in the figure 3 inset plot, is found to be (10.10 ± 0.02) Hz.

Figure 4 shows the fundamental frequency data measured for different bar vibration lengths and for different bar materials (steel, aluminum, copper and polystyrene). This plot allowed us to obtain each of the tested materials' Young's modulus by fitting the experimental data using equation (1). It is worth observing that the variation in the bar vibrating length is done by simply changing the point where the bar is fixed.

As can be seen in equation (1), if one wants to obtain the Young's modulus from fitting a fundamental frequency versus vibrating length plot, it is necessary to know the material density, bar cross-sectional area and its second moment values. These values are shown in table 1. Also, one needs to know the modal eigenvalue for the fundamental mode, λ_1. This value, calculated to be 1.875 for the studied configuration, is obtained from an eigenvalue equation that arises when considering the problem boundary conditions in the Euler–Bernoulli equation (nonzero vertical displacement and zero initial velocity). The graphical method for obtaining this value is explained in [1].

Table 2, in turn, shows the measured Young's moduli values and the expected ones from the literature. From data shown in table 2, one can analyze that the Young's moduli values obtained herein show a good resemblance to the ones found in the literature. It makes the optical method proposed in this paper a good alternative for performing a fast, efficient and nondestructive measurement of a material's Young's modulus.

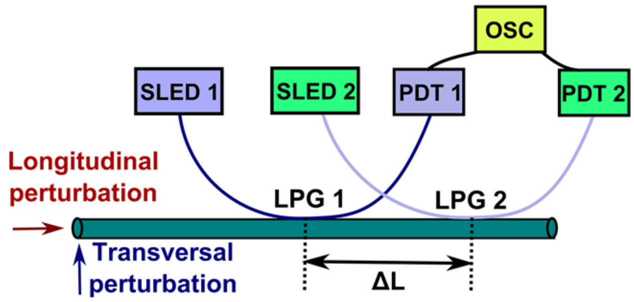

Figure 5. Scheme of the experimental setup for 'through transmission' technique. SLED: super-luminescent LED; PDT: photodetector; OSC: oscilloscope; ΔL: distance between LPGs.

5. 'Through transmission' technique results

The 'through transmission' method for determining a material's Young's modulus is based on the measurement of the propagation velocities of longitudinal and transversal acoustic perturbations along a bar of the material of interest. Here, we used the setup shown in figure 5, where two LPGs were glued on the tested bar (made of aluminum) at a distance ΔL apart from each other.

When longitudinal and transversal acoustic perturbations are induced on a bar end, they propagate along the bar with different velocities, v_L and v_T, respectively. These perturbations can be induced on the bar by simply hitting it longitudinally or transversally, as indicated in figure 5.

The propagation of the acoustic waves along the bar causes the LPGs' optical responses to alter when the pulses reach the position where they are glued. As the LPGs are a distance $\Delta L = (2.55 \pm 0.01)$ m apart from each other, the pulse reaches the second LPG position after the time interval Δt that it reached the first LPG one. By taking into account this time interval and the distance between the LPGs, one can calculate the pulse velocity.

Figure 6 shows how the time interval Δt is measured. The optical responses of the LPGs are measured as a function of time by using two photodetectors connected to an oscilloscope (figure 5). The voltage measured while there is no perturbation on the bar is assumed to be zero. When the perturbation reaches the LPG's position, a voltage variation is observed.

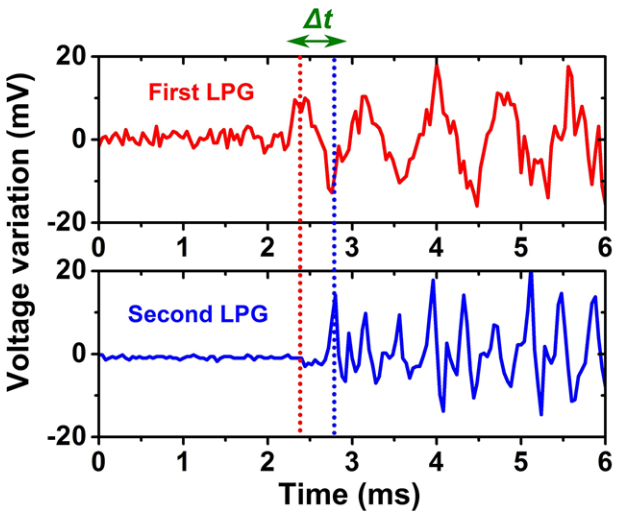

Figure 6. Voltage variation as a function of time for a situation in which a longitudinal perturbation was induced on the bar. Δt: time interval the perturbation took to cover the distance between the LPGs.

To account for the time interval Δt that the perturbation pulse took to cover the distance ΔL between the gratings, the first peak in the voltage variation versus time plot was considered. A threshold was defined for a local maximum to be considered a peak that had arisen due to pulse passage. One considered a peak a local maximum with greater amplitude than three times the standard deviation of the voltage variation points between zero and 2 ms (which is the voltage fluctuation level without any external perturbation).

The measurement of the time interval Δt was performed ten times for longitudinal and transversal perturbations separately. For longitudinal perturbations, Δt was found to be (4.7 ± 0.5) ms and, for transversal perturbations, it was found to be (8 ± 1) ms. By dividing the distance between the gratings by the time interval just presented, longitudinal and transversal perturbation velocities were calculated to be (5400 ± 500) m s^{-1} and (3200 ± 400) m s^{-1} respectively.

The knowledge of these velocities allowed us to obtain the Young's modulus of the bar material (aluminum) via equation (2). The attained value was (70 ± 10) GPa when using 2700 kg m^{-3} as the material density.

Concerning the frequency response of the system, one can say that we were able to measure signals that ranged from a few hertz up to 5 kHz. A specific study, however, is still needed to explore the frequency limits of the sensor.

The Young's modulus value obtained by the 'through transmission' method, (70 ± 10) GPa, is comparable to the ones obtained using 'bar resonance' technique and found in the literature, (70.9 ± 0.8) GPa and 70–72 GPa, respectively (see table 2), although it is less accurate. We believe that this lack of accuracy arises from the difficulty in defining the moment in time when the perturbations sensitize the LPGs. The measurement of the time interval between the moments when the perturbations reach the first and second LPGs provides a value in the order of milliseconds and an error in the same order of magnitude. Thus, we believe that if the measurement could be performed using a longer bar and greater spacing between the LPGs, the percentage error could be reduced. Furthermore, our future research will include a more thorough study of how to attach the LPGs to the bars more efficiently. We believe that improvements to the gluing process will contribute to the optimization of the optical response.

6. Conclusions

This paper has reported the application of fiber long-period gratings to vibration monitoring and Young's modulus determination. To the best of our knowledge, this is the first article to deal with the determination of a material's Young's modulus using long-period fiber gratings.

Steel, aluminum, copper and polystyrene samples were tested and the experimental results for Young's modulus values, obtained by 'bar resonance' and 'through transmission' techniques, are in good agreement with the ones reported in the literature. Besides, the 'bar resonance' measurement method reported herein allows us to obtain the acceleration of the studied movement—which was in the order of 20 m s^{-2}.

Therefore, the presented results indicate that long-period fiber gratings can be straightforwardly employed in the dynamic characterization of a material's elastic properties.

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References

[1] Digilov R M and Abramovich H 2013 Flexural vibration test of a beam elastically restrained at one end: a new approach for Young's modulus determination *Adv. Mater. Sci. Eng.* **2013** 1–6
[2] Birks A S and Green R E 1991 *Nondestructive Testing Handbook* (*Ultrasonic Testing* vol 7) (Columbus, OH: American Society for Nondestructive Testing)
[3] Radovic M, Lara-Curzio E and Riester L 2004 Comparison of different experimental techniques for determination of elastic properties of solids *Mater. Sci. Eng.* A **368** 1–2
[4] Li H, Li D and Song G 2004 Recent applications of fiber optic sensors to health monitoring in civil engineering *Eng. Struct.* **26** 1647–57
[5] Lee B 2003 Review of the present status of optical fiber sensors *Opt. Fiber Technol.* **9** 57–79
[6] Pigeon F, Pelissier S, Mure-Ravaud A, Gagnaire H and Veillas C 1992 Optical fibre Young's modulus measurement using an optical method *Electron. Lett.* **28** 11
[7] Antunes P, Lima H, Monteiro J and André P S 2008 Elastic constant measurement for standard and photosensitive single mode optical fibres *Microw. Opt. Technol. Lett.* **50** 9
[8] Lago S, Brignolo S, Cuccaro R, Musacchio C, Giuliano Albo P A and Tarizzo P 2014 Application of acoustic methods for a non-destructive evaluation of the elastic properties of several typologies of materials *Appl. Acoust.* **75** 10–6
[9] Osório J H, Mosquera L, Gouveia C J, Biazoli C R, Hayashi J G, Jorge P A S and Cordeiro C M B 2013 High

sensitivity LPG Mach-Zehnder sensor for real time fuel conformity analysis *Meas. Sci. Technol.* **24** 015102

[10] James S W and Tatam R P 2003 Optical fibre long-period grating sensors: characteristics and application *Meas. Sci. Technol.* **14** R49–61

[11] Wang Y 2010 Review of long period fiber gratings written by CO_2 laser *J. Appl. Phys.* **108** 081101

[12] Tan K M, Chan C C, Tjin S C and Dong X Y 2006 Embedded long-period fiber grating bending sensor *Sensors Actuators* A **125** 267–72

[13] He Z, Zhu Y and Du H 2007 Effect of macro-bending on resonant wavelength and intensity of long-period gratings in photonic crystal fiber *Opt. Express* **15** 4

[14] Weast R C and Astle M J 1979 *Handbook of Chemistry and Physics* 59th edn (Boca Raton, FL: CRC Press)

[15] Ruben S 1985 *Handbook of the Elements* (La Salle, IL: Open Court Publishing Company)

[16] Lavrentyev A I and Rokhlin S I 1997 Determination of elastic moduli, density, attenuation, and thickness of a layer using ultrasonic spectroscopy at two angles *J. Acoust. Soc. Am.* **102** 6

[17] Wolfden A and Schwanz W R 1995 An evaluation of three methods to measure the dynamic elastic modulus of steel *J. Test. Eval.* **23** 176–9

[18] Ledbetter H 1993 Dynamic versus static Young's moduli: a case study *Mater. Sci. Eng.* A **165** L9–10

[19] Lubarsky G V, Davidson M R and Bradley R H 2004 Elastic modulus, oxidation depth and adhesion force of surface modified polystyrene studied by AFM and XPS *Surf. Sci.* **558** 135–44