Interacción entre dos partículas cargadas

Sistema ~~CGS~~ MKS

Cambio de Coulomb ($\nabla\cdot\bar{A}=0$)

$$\nabla\cdot\bar{D}=\rho \qquad \nabla\times\bar{B}=\mu_0\bar{J}+\mu_0\frac{\partial\bar{D}}{\partial t}$$ (ausencia de magnetización)

$$\nabla\cdot\bar{B}=0 \qquad \nabla\times\bar{E}+\frac{\partial\bar{B}}{\partial t}=0$$

En el vacío $\bar{E}=\frac{\bar{D}}{\epsilon_0}$

$$\nabla\cdot\bar{E}=\frac{\rho}{\epsilon_0} \qquad \nabla\times\bar{B}=\bar{J}+\mu_0\epsilon_0\frac{\partial\bar{E}}{\partial t}$$

$$\bar{B}=\nabla\times\bar{A} \longrightarrow \boxed{\nabla\times\nabla\times\bar{A}=\mu_0\bar{J}+\frac{1}{c^2}\frac{\partial\bar{E}}{\partial t}} \quad 1$$

$$\nabla\times\left(\bar{E}+\frac{\partial\bar{A}}{\partial t}\right)=0$$

$$\hookrightarrow \bar{E}+\frac{\partial\bar{A}}{\partial t}=-\nabla\phi$$

$$\boxed{\bar{E}=-\nabla\phi-\frac{\partial\bar{A}}{\partial t}} \quad 2$$

(1-2) $\nabla\times(\nabla\times\bar{a})=\nabla(\nabla\cdot\bar{a})-\nabla^2\bar{a}$

$*$ $$\nabla(\nabla\cdot\bar{A})-\nabla^2\bar{A}=\mu_0\bar{J}-\frac{1}{c^2}\frac{\partial^2\bar{A}}{\partial t^2}-\frac{1}{c^2}\nabla\frac{\partial\phi}{\partial t}$$

$*$ $$-\nabla^2\phi-\nabla\cdot\frac{\partial\bar{A}}{\partial t}=\frac{\rho}{\epsilon_0}$$

$$\hookrightarrow -\nabla^2\phi-\frac{\partial}{\partial t}\nabla\cdot\bar{A}=\rho/\epsilon_0 \text{, como } \nabla\cdot\bar{A}=0$$

$$\boxed{\begin{aligned}&\nabla^2\bar{A}-\frac{1}{c^2}\frac{\partial^2\bar{A}}{\partial t^2}=-\mu_0\bar{J}+\frac{1}{c^2}\nabla\frac{\partial\phi}{\partial t}\\&\nabla^2\phi=-\frac{\rho}{\epsilon_0}\end{aligned}}$$

$$F(\bar{x},t) = \frac{1}{(2\pi)^2}\int d^3k \int d\omega \; F(\bar{k},\omega)\, e^{i(\bar{k}\cdot\bar{x}-\omega t)}$$

$$\rho(\bar{x},t) = ze\,\delta(\bar{x}-\bar{v}t)$$
$$\bar{J} = \bar{v}\,\rho(\bar{x},t)$$

Cambiemos todo al espacio $(\omega,\bar{k})$

$$\rho(\bar{k},\omega) = \frac{1}{(2\pi)^2}\int d^3x \int dt \; ze\,\delta(\bar{x}-\bar{v}t)\, e^{-i(\bar{k}\cdot\bar{x}-\omega t)}$$

$$= \frac{ze}{(2\pi)^2}\int dt \; e^{-i(\bar{k}\cdot\bar{v}-\omega)t}$$

$$\rho(\bar{k},\omega) = \frac{ze}{(2\pi)^2}\cdot 2\pi\,\delta(\omega-\bar{k}\cdot\bar{v})$$

$$\boxed{\rho(\bar{k},\omega) = \frac{ze}{2\pi}\,\delta(\omega-\bar{k}\cdot\bar{v})}$$

$$\boxed{\bar{J}(\bar{k},\omega) = \frac{ze\bar{v}}{2\pi}\,\delta(\omega-\bar{k}\cdot\bar{v})}$$

Ahora

$$\nabla^2\phi(\bar{x},t) = -\frac{\rho(\bar{x},t)}{\epsilon_0}$$

$$\nabla^2 \frac{1}{\cancel{(2\pi)^2}}\int d^3k \int d\omega\; \phi(\bar{k},\omega)\, e^{i(\bar{k}\cdot\bar{x}-\omega t)} = -\frac{1}{\epsilon_0}\frac{1}{\cancel{(2\pi)^2}}\int d^3k \int d\omega\; \frac{ze}{2\pi}\,\delta(\omega-\bar{k}\cdot\bar{v})\, e^{i(\bar{k}\cdot\bar{x}-\omega t)}$$

$$-k^2\,\phi(\bar{k},\omega) = -ze\,\delta(\omega-\bar{k}\cdot\bar{v})/\epsilon_0\cdot 2\pi$$

$$\boxed{\phi(\bar{k},\omega) = +\frac{ze}{2\pi\, k^2\epsilon_0}\,\delta(\omega-\bar{k}\cdot\bar{v})}$$

Calculemos $\bar{A}(\bar{k},\omega)$

$$\nabla^2\bar{A} - \frac{1}{c^2}\frac{\partial^2\bar{A}}{\partial t^2} = -\mu_0\bar{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\nabla\phi$$

$$\bar{A}(\bar{x},t) = \frac{1}{(2\pi)^2}\int d^3k\int d\omega\ \bar{A}(\bar{k},\omega)\ e^{i(\bar{k}\cdot\bar{x}-\omega t)}$$

$-i\omega \cdot i\bar{k}$

$$-k^2 A(\bar{k},\omega) + \frac{\omega^2}{c^2}A(\bar{k},\omega) = -\mu_0\frac{\bar{v}ze}{2\pi}\delta(\omega-\bar{k}\cdot\bar{v}) + \frac{\omega\bar{k}}{c^2}\phi(\bar{k},\omega)$$

$$A(\bar{k},\omega)\left[k^2-\frac{\omega^2}{c^2}\right] = +\frac{\mu_0 ze\bar{v}}{2\pi}\delta(\omega-\bar{k}\cdot\bar{v}) - \frac{\omega\bar{k}}{c^2}\phi(\bar{k},\omega)$$

pero $\phi(\bar{k},\omega) = +\frac{ze}{2\pi k^2\varepsilon_0}\delta(\omega-\bar{k}\cdot\bar{v})$

$$A(\bar{k},\omega)\left[k^2-\omega^2/c^2\right] = \mu_0\varepsilon_0 k^2\bar{v}\,\phi(\bar{k},\omega) - \frac{\omega\bar{k}}{c^2}\phi(\bar{k},\omega)$$

$$A(\bar{k},\omega)\left[c^2k^2-\omega^2\right] = \phi(\bar{k},\omega)\left[k^2\bar{v} - \omega\bar{k}\right]$$

$$\boxed{A(\bar{k},\omega) = \phi(\bar{k},\omega)\ \frac{k^2\bar{v}-\omega\bar{k}}{c^2k^2-\omega^2}}$$

Determinemos ahora $\phi(\bar{x},t)$ y $\bar{A}(\bar{x},t)$

$$\phi(\bar{x},t) = \frac{1}{(2\pi)^2}\int d^3k\int d\omega\ \frac{ze}{k^2\cdot 2\pi\varepsilon_0}\delta(\omega-\bar{k}\cdot\bar{v})\ e^{i(\bar{k}\cdot\bar{x}-\omega t)}$$

$$= \frac{ze}{(2\pi)^2}\int d^3k\ \frac{e^{i(\bar{k}\cdot\bar{x}-\bar{k}\cdot\bar{v}t)}}{2\pi\varepsilon_0 k^2}$$

$\bar{v}t \to \bar{r}$
$\bar{x} \to \bar{r}'$

$$= \frac{ze}{\varepsilon_0(2\pi)^3}\int d^3k\ e^{i\bar{k}\cdot(\bar{r}'-\bar{r})}$$

$$\boxed{\phi(\bar{r},t) = \frac{ze}{(2\pi)^3\varepsilon_0}\int\frac{d^3k}{k^2}\ e^{i\bar{k}\cdot(\bar{r}'-\bar{r})}}$$ OK

CALCULEMOS AHORA $\bar{A}(\bar{x},t)$

$$\bar{A}(\bar{x},t) = \frac{1}{(2\pi)^2}\int d^3k \int d\omega \; \bar{A}(\bar{k},\omega)\, e^{i(\bar{k}\cdot\bar{x}-\omega t)}$$

$$= \frac{1}{(2\pi)^2}\int d^3k \int d\omega \; \frac{k^2\bar{v}-\omega\bar{k}}{c^2k^2-\omega^2}\,\frac{ze}{2\pi\varepsilon_0 k^2}\,\delta(\omega-\bar{k}\bar{v})\, e^{i(\bar{k}\cdot\bar{x}-\omega t)}$$

$$= \frac{ze}{(2\pi)^3\varepsilon_0}\int \frac{d^3k}{k^2}\;\frac{k^2\bar{v}-(\bar{k}\bar{v})\bar{k}}{c^2k^2-(\bar{k}\cdot\bar{v})^2}\; e^{i(\bar{k}\cdot\bar{x}-\bar{k}\cdot\bar{v}t)} \quad ; \; \bar{x}\to\bar{r}', \; \bar{v}t\to\bar{r}$$

$$\boxed{\bar{A}(\bar{r},\bar{r}') = \frac{ze}{(2\pi)^3\varepsilon_0}\int \frac{d^3k}{k^2}\;\frac{\bar{v}-(\hat{k}\cdot\bar{v})\hat{k}}{c^2-(\hat{k}\cdot\bar{v})^2}\; e^{i\bar{k}\cdot(\bar{r}'-\bar{r})}}$$

en Gauss $\frac{1}{4\pi\varepsilon_0}=1 \quad \therefore \quad \frac{1}{(2\pi)^3}\frac{1}{\varepsilon_0} = \frac{1}{2\pi^2\cdot 4\pi\varepsilon_0}$

$$\therefore \quad \boxed{\begin{aligned} \phi(\bar{r},\bar{r}') &= \frac{ze}{2\pi^2}\int \frac{d^3k\, e^{i\bar{k}\cdot(\bar{r}'-\bar{r})}}{k^2} \\ \bar{A}(\bar{r},\bar{r}') &= \frac{ze}{2\pi^2}\int \frac{d^3k}{k^2}\;\frac{\bar{\beta}-(\hat{k}\cdot\bar{\beta})\hat{k}}{c\,(1-(\hat{k}\cdot\bar{\beta})^2)}\; e^{i\bar{k}\cdot(\bar{r}'-\bar{r})} \end{aligned}}$$

$$k^2-k^2(\hat{k}\cdot\bar{\beta})^2 = k^2-(\bar{k}\cdot\bar{v}/c)^2 = k^2-\left(\frac{\hbar\bar{k}\cdot\bar{v}}{\hbar c}\right)^2 = k^2-(E/\hbar c)^2$$

$$\boxed{\bar{A}(\bar{r},\bar{r}') = \frac{ze}{2\pi^2}\int \frac{d^3k}{c}\;\frac{\bar{\beta}-(\hat{k}\cdot\bar{\beta})\hat{k}}{k^2-(E/\hbar c)^2}\; e^{i\bar{k}\cdot(\bar{r}'-\bar{r})}}$$

E - ENERGÍA TRANSFERIDA EN LA COLISIÓN

Cálculo de la amplitud de transición

$$a_{if} = \langle \psi_f(\bar r)\varphi_f(\bar r')| -e\phi - e\bar v_e\cdot\bar A\,|\varphi_i(\bar r')\psi_i(\bar r)\rangle$$
$$= \int d^3r\int d^3r'\ \psi_f^*(\bar r)\,\varphi_f^*(\bar r')(-e\phi - e\bar v_e\cdot\bar A)\,\varphi_i(\bar r')\psi_i(\bar r)$$

$= \bar v_{\text{electrón atómico}}$

Veamos el término electrostático (instantáneo) (coulombiano)

$$\int d^3r\int d^3r'\ \psi_f^*(\bar r)\varphi_f^*(\bar r')e\phi\,\varphi_i(\bar r')\psi_i(\bar r)\ ;\quad \psi_f(\bar r)= e^{i\bar k_f\cdot\bar r};\ \psi_i(\bar r) = e^{i\bar k_i\cdot\bar r}$$

$$=\int d^3r\int d^3r'\ \frac{Z_p e^2}{2\pi^2}\int\frac{d^3k}{k^2}\ e^{i\bar k\cdot(\bar r'-\bar r)}\,e^{i(\bar k_i-\bar k_f)\cdot\bar r}\,\varphi_f^*(\bar r')\,\varphi_i(\bar r')$$

$$=\frac{Z_p e^2}{2\pi^2}\int d^3r'\int\frac{d^3k}{k^2}\underbrace{\int e^{i(\bar k_i-\bar k_f-\bar k)\cdot\bar r}\,d^3r}_{(2\pi)^3\delta(\bar q/\hbar-\bar k)}\ e^{i\bar k\cdot\bar r'}\,\varphi_f^*(\bar r')\varphi_i(\bar r')$$

$$= 4\pi Z_p e^2\int d^3r'\int\frac{d^3k}{k^2}\varphi_f^*\varphi_i\,\delta(\bar q/\hbar-\bar k)\,e^{i\bar k\cdot\bar r'} = Z_p e^2\hbar^3\int\frac{e^{i\bar q\cdot\bar r'/\hbar}}{q^2}\,d^3r'\,\varphi_f^*\varphi_i$$

$$= 4\pi Z_p e^2\hbar^2\int\frac{e^{i\bar q\cdot\bar r'/\hbar}}{q^2}\,d^3r'\,\varphi_f^*\varphi_i$$

$$= 4\pi Z_p e^2\hbar^2\int\varphi_f^*(\bar r')\varphi_i(\bar r')\ \frac{e^{i\bar q\cdot\bar r'/\hbar}}{q^2}\,d^3r'$$

$$= \frac{4\pi Z_p e^2\hbar^2}{q^2}\int\varphi_f^*(\bar r')\varphi_i(\bar r')\ e^{i\bar q\cdot\bar r'/\hbar}\,d^3r'$$

xc

herzda

Término del potencial magnético (transversa)

$$\int d\tau \int d^3r' \; \bar{\alpha}\cdot\bar{A} \; \psi_f^*(\bar{r})\psi_i(\bar{r}) \, \varphi_f^*(\bar{r}')\varphi_i(\bar{r}')$$

$$= \frac{Z_p e^2}{2\pi^2}\int \frac{d^3k \; \bar{\alpha}\cdot\bar{\beta}_\tau}{k^2-(E/\hbar c)^2}\int d^3r' \underbrace{\int d^3r \; e^{i(\bar{k}_i-\bar{k}_f-\bar{k})\cdot\bar{r}}}_{(2\pi)^3\delta(\bar{q}/\hbar-\bar{k})} e^{i\bar{k}\cdot\bar{r}'} \varphi_f^*(\bar{r}')\varphi_i(\bar{r}') \quad ; \bar{\alpha}=\frac{\bar{v}_e}{c}$$

$$= 4\pi Z_p e^2 \iint \frac{d^3k \; e^{i\bar{k}\cdot\bar{r}'}}{k^2-(E/\hbar c)^2}\delta(\bar{q}/\hbar-\bar{k}) \; \bar{\alpha}\cdot\bar{\beta}_\tau \; \varphi_f^*(\bar{r}')\varphi_i(\bar{r}') \, d^3r'$$

$$= 4\pi Z_p e^2 \int \hbar^2 \frac{e^{i\bar{q}\cdot\bar{r}'/\hbar}}{q^2-(E/c)^2} \bar{\alpha}\cdot\bar{\beta}_\tau \; \varphi_f^*(\bar{r}')\varphi_i(\bar{r}') \, d^3r'$$

$$= 4\pi Z_p e^2 \hbar^2 \int \frac{e^{i\bar{q}\cdot\bar{r}'/\hbar}}{q^2-(E/c)^2} \bar{\alpha}\cdot\bar{\beta}_\tau \; \varphi_f^*(\bar{r}')\varphi_i(\bar{r}') \, d^3r'$$

$$\bar{\beta}_\tau = \bar{\beta} - (\bar{q}\cdot\bar{\beta})\bar{q}/\hbar^2$$

$$= 4\pi Z_p e^2 \hbar^2 \bar{\beta}_\tau \cdot \int \frac{\bar{\alpha}\, e^{i\bar{q}\cdot\bar{r}'/\hbar}}{q^2-(E/c)^2} \varphi_f^*(\bar{r}')\varphi_i(\bar{r}') \, d^3r'$$

$$\boxed{= 4\pi Z_p e^2 \hbar^2 \frac{\bar{\beta}_\tau \cdot}{q^2-(E/c)^2} \int \bar{\alpha}\, e^{i\bar{q}\cdot\bar{r}'/\hbar} \varphi_f^*(\bar{r}')\varphi_i(\bar{r}') \, d^3r'} \quad \checkmark$$

Análisis de los factores de forma atómicos

$$F_n(\bar{q}) = Z^{-1/2} \sum_j (n| e^{i\bar{q}\cdot\bar{r}_j/\hbar} |0) \qquad |n\rangle = \varphi_f(\bar{r}')$$

$$\bar{G}_n(\bar{q}) = Z^{-1/2} \sum_j (n| \bar{\alpha}_j e^{i\bar{q}\cdot\bar{r}_j/\hbar} |0) \qquad |0\rangle = \varphi_i(\bar{r}')$$

Bajo momento transferido $\hbar/q$ $\hbar/q >> a_0$

$$F_n(\bar{q}) \approx Z^{-1/2} \sum_j (n| \cancel{1} + i\bar{q}\cdot\bar{r}_j/\hbar + \cdots |0)$$

$$\approx Z^{-1/2} \sum_j (n| i\bar{q}\cdot\bar{r}_j/\hbar |0)$$

$$\boxed{|F_n(\bar{q})|^2 \approx Z^{-1} q^2/\hbar^2 \left(\sum_j x_j\right)_{n0}} \quad ; \quad \bar{x}_j // \bar{q}$$

$$Q = \frac{q^2}{2m} \quad \text{(no relativista)}$$

$$q^2 = \frac{E_n^2}{v^2} + p^2\theta^2 \quad ; \; q \text{ bajo } \theta \approx 0$$

$$q^2 \approx \frac{E_n^2}{v^2}$$

$$2mQ \approx \frac{E_n^2}{v^2}$$

$$|F_n(\bar{q})|^2 \approx Z^{-1} \frac{E_n^2}{\hbar^2 v^2} \left(\sum_j x_j\right)_{n0} = Q f_n / E_n \quad ; f_n \text{ - DOS}$$

optical dipole
oscillator strength

$$|F_n(\bar{q})|^2 \cdot$$

$$|\bar{\beta}_\tau \cdot \bar{G}_n(\bar{q})|^2 \approx Z^{-1}\left|\bar{\beta}_\tau \cdot \left(\sum_j \bar{\alpha}_j \,\bar{q}\cdot\bar{r}_j/\hbar\right)_{n0}\right|^2$$

$$\approx Z^{-1}\left|\sum_j (\bar{\beta}_\tau \cdot \bar{\alpha}_j)(\bar{q}\cdot\bar{r}_j)/\hbar\right|_{n0}^2$$

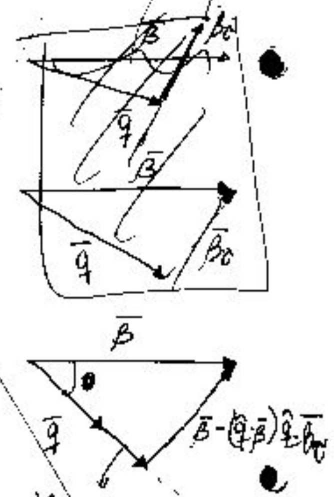
$$\bar{\alpha}_j = \frac{\dot{\bar{r}}_j}{c} = \frac{\dot{y}_j}{c}$$

$$\approx Z^{-1}\left|\sum \beta_\tau \frac{v_\tau}{c}\,\frac{q\,y_j}{\hbar}\right|_{n0}^2$$

$$\approx Z^{-1}\beta_\tau^2 \frac{E_n^2}{\hbar^2 c^2}\left|\left(\sum_j y_j\right)_{n0}\right|^2$$

$; \bar{y}_j \parallel \bar{\beta}_\tau$

$q v_\tau = E_n$



$(\hat{q}\cdot\bar{\beta})\hat{q}$

$$\left(\bar{\beta} - (\hat{q}\cdot\bar{\beta})\hat{q}\right)\cdot\bar{q}$$

$$= \bar{\beta}\cdot\bar{q} - (\hat{q}\cdot\bar{\beta})\hat{q}\cdot\bar{q}$$
$$= \bar{\beta}\cdot\bar{q} - \bar{q}\cdot\bar{\beta} = 0$$

Término magnético

$$|\bar{\beta}_c \cdot \bar{G}(\bar{q})|^2 = \frac{1}{Z}\left|\bar{\beta}_c \sum_j \left(\bar{\alpha}_j e^{i\bar{q}\cdot\bar{r}_j/\hbar}\right)_{n0}\right|^2$$

A cero orden

$$\sum_j \langle n|\bar{\alpha}_j e^{i\bar{q}\cdot\bar{r}_j/\hbar}|0\rangle = \sum_j \langle n|\bar{\alpha}_j|0\rangle$$

$$\hat{\alpha}_j = \frac{1}{c}\left[\frac{\partial \hat{r}_j}{\partial t} + \frac{i}{\hbar}\left(\hat{H}\hat{r}_j - \hat{r}_j\hat{H}\right)\right] ; \quad \frac{\partial \hat{r}_j}{\partial t} = 0$$ (no depende explícitamente del tiempo). Estamos en la cuántica.

$$\langle n|\bar{\alpha}_j|0\rangle = \frac{i}{c\hbar}\left[\langle n|\hat{H}\hat{r}_j|0\rangle - \langle n|\hat{r}_j\hat{H}|0\rangle\right]$$

$$= \frac{i}{c\hbar}\left[E_n\langle n|\hat{r}_j|0\rangle - E_0\langle n|\hat{r}_j|0\rangle\right] \quad E_0 = 0$$ (por convención).

$$= \frac{iE_n}{c\hbar}\langle n|\hat{r}|0\rangle$$

Entonces $|\bar{\beta}_c \cdot \bar{G}(\bar{q})|^2 \approx \frac{1}{Z}\left|\bar{\beta}_c \cdot \left(\sum_j \frac{iE_n}{c\hbar}\langle n|\hat{r}_j|0\rangle\right)\right|^2$

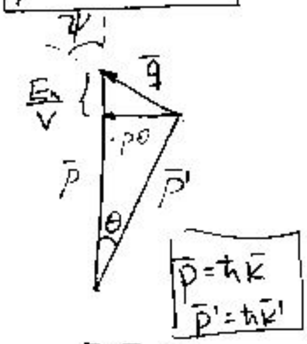
de $\hat{r}_j$ solo sobrevivirá el término paralelo a $\bar{\beta}_c$ (y_j)

$$|\bar{\beta}_c \cdot \bar{G}(\bar{q})|^2 \approx \frac{1}{Z}\frac{\beta_c^2 E_n^2}{c^2\hbar^2}\left|\sum_j (y_j)_{n0}\right|^2 = \frac{\beta_c^2 E_n}{2mc^2} f_n$$

OOS

Calculemos $d\sigma$.

$$\boxed{d\sigma = \frac{2\pi}{\hbar v}\left|a_{if}\right|^2 \frac{dk'}{h^3}\delta(E'+E_n-E)} \qquad \boxed{\bar{q} = \hbar(\bar{k}-\bar{k}')}$$



$$\cancel{\bar{q}\cdot\hat{p} = \frac{1}{p}(q^2+p^2) = \frac{1}{p}(\bar{p}-\bar{p}')\cdot\bar{p} = \frac{1}{p}(p^2-\bar{p}\cdot\bar{p}')}$$

$$\bar{q}\cdot\hat{p} \quad \frac{\bar{q}\cdot\bar{p}}{p} = \frac{q\,p\cos\psi}{p} = q\cos\psi \qquad ; \bar{q} = \bar{p}-\bar{p}'$$

$$\bar{p}' = \bar{p}-\bar{q} \quad ; \quad p'^2 = p^2+q^2-2pq\cos\psi$$

$$\cos\psi = \frac{-p'^2+p^2+q^2}{2pq}$$

$$\bar{q}\cdot\hat{p} = q\frac{(-p'^2+p^2+q^2)}{2pq} = \frac{p^2-p'^2}{2p}+\frac{q^2}{2p} = \frac{(p-p')(p+p')}{2p}+\frac{q^2}{2p}$$

como $q \ll p$ (debido a la gran masa del proyectil)

$$p+p' \approx 2p \quad , \quad q^2/2p \approx 0$$

$$\bar{q}\cdot\hat{p} \approx p-p' \approx \frac{dp}{dE}E_n = \frac{dp}{dE}\Delta E$$

$$E = \frac{p^2}{2m} \qquad dE = \frac{p\,dp}{m} \Rightarrow \frac{dp}{dE} = \frac{1}{v}$$

$$\boxed{\bar{q}\cdot\hat{p} \approx \frac{E_n}{v}} \quad \text{momento mínimo transferido.}$$

Entonces $\quad q^2 = \dfrac{E_n^2}{v^2} + p^2\theta^2$

Definamos $Q(1+Q/2mc^2) = \frac{q^2}{2m}$

$$d\sigma = \frac{2\pi}{\hbar v} |a_{if}|^2 d\bar{p}' \delta(E'+E_n-E) \qquad d\bar{p}' = p'^2 dp' d\Omega$$

$$d\bar{p}' = p'^2 \frac{dp'}{dE'} dE' d\Omega$$

$$= \frac{p'^2}{v'} dE' d\Omega$$

$$d\sigma = \frac{2\pi}{\hbar v v'} |a_{if}|^2 \frac{p'^2 d\Omega}{\hbar^3 (2\pi)^3}$$

$$= \frac{1}{4\pi^2 \hbar^4 v v'} |a_{if}|^2 p'^2 d\cos\theta \, d\varphi \quad ; \quad \int f(E') \delta(E'+E_n-E) dE' = f(E')$$

$$d\sigma = \frac{1}{4\pi^2 \hbar^4 v v'} |a_{if}|^2 q \, dq \, d\varphi \qquad ; \; p'^2 d\cos\theta \simeq p'^2 \theta^2 d\theta = q\,dq$$

$$q^2 = \frac{E_n^2}{v^2} + p^2\theta^2$$

integrando por φ

$$d\sigma = \frac{1}{2\pi \hbar^4 v v'} |a_{if}|^2 q \, dq$$

$$\frac{q\,dq}{m} = dQ + \frac{Q\,dQ}{mc^2}$$

$$q\,dq = m\,dQ(1 + Q/mc^2)$$

$$d\sigma = \frac{m}{2\pi \hbar^4 v v'} |a_{if}|^2 (1+Q/mc^2) dQ.$$

$$a_{if} = 4\pi Z_p e^2 \hbar^{2}\, {}^{2V_2} \left[\frac{F_n(\bar{q})}{2mQ(1+Q/2mc^2)} + \frac{\bar{\beta}_t \cdot \bar{G}(\bar{q})}{2mQ(1+Q/2mc^2) - E_n^2/c^2} \right]$$

$$|a_{if}|^2 = 4^2\pi^2 Z_p^2 Z e^4 \hbar^4 \left[\frac{|F_n(\bar{q})|^2}{4m^2 Q^2 (1+Q/2mc^2)^2} + \frac{|\bar{\beta}_t \cdot \bar{G}(\bar{q})|^2}{4m^2 [Q^2(1+Q/2mc^2) - E_n^2/c^2]^2} \right]$$

el término cruzado se anula ya que [transi]ciones de tipo magnético y eléctrico tienen dife[rentes] reglas de paridad. ~~No pueden quedar~~ [illegible].

$$|a_{if}|^2 = \frac{4\pi^2 z_p^2 Z e^4 \hbar^4}{m^2}\left[\frac{|F_n(\vec q)|^2}{Q^2(1+Q/2mc^2)^2} + \frac{|\vec\beta_t\cdot \vec G_n(\vec q)|^2}{[Q(1+Q/2mc^2) - E_n^2/2mc^2]^2}\right]$$

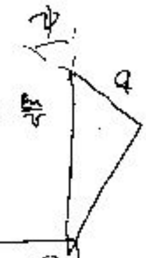
$v' \simeq v$

$$d\sigma = \frac{4\pi}{2\pi\hbar^4 v^2}\,\frac{4\pi^2 z_p^2 Z e^4 \hbar^4}{m^2}\left[\frac{|F_n|^2}{Q^2(1+Q/2mc^2)^2} + \frac{|\vec\beta_t\cdot \vec G_n|^2}{[Q(1+Q/2mc^2) - E_n^2/2mc^2]^2}\right] \times (1+Q/mc^2)\,dQ$$

$$\boxed{d\sigma = \frac{2\pi z_p^2 Z e^4}{m v^2}\left[\frac{|F_n|^2}{Q^2(1+Q/2mc^2)^2} + \frac{|\vec\beta_t\cdot \vec G_n|^2}{[Q(1+Q/2mc^2) - E_n^2/2mc^2]^2}\right]\left(1+\frac{Q}{mc^2}\right)dQ}$$

NO RELATIVISTA: $Q/2mc^2 \ll 1$ $\quad \vec\beta_t\cdot\vec G_n \ll |F_n|^2$

$$\boxed{d\sigma = \underbrace{\frac{2\pi z_p^2 Z e^4}{m v^2}\,\frac{dQ}{Q^2}}_{\text{Rutherford}}\,|F_n(q)|^2}$$



AHORA $\quad Q_{min} = \frac{E_n}{v}$

$$\cos\psi =$$

$$\cos^2\psi = \frac{q^2_{min}}{q^2} = \frac{Q_{min}(Q_{min}+2mc^2)}{Q(Q+2mc^2)}$$

COMBINANDO CON $|F_n|^2$ y $(\vec\beta_t\cdot\vec G_n)^2$ PARA $\boxed{\text{low-}Q}$

$$* \; d\sigma = \frac{2\pi z_p^2 Z e^4}{m v^2}\left[\frac{Q f_n}{E_n Q^2(1+Q/2mc^2)^2} + \frac{\beta_t^2 f_n E_n}{2mc^2[Q(1+Q/2mc^2) - E_n^2/2mc^2]^2}\right]\left(1+\frac{Q}{mc^2}\right)dQ$$

$$= \frac{2\pi z_p^2 Z e^4}{m v^2}\left[\frac{f_n\,dQ}{E_n Q} + \frac{\beta_t^2 f_n E_n\, dQ\,(mc^2+Q)}{2m^2c^4[Q(1+Q/2mc^2) - E_n^2/2mc^2 \cdots]^2}\right]$$

$$\vec\beta_t = \vec\beta - \vec\beta\cos\psi\cdot\hat q$$

$$\beta_t^2 = \beta^2 + \beta^2\cos^2\psi - 2\beta^2\cos^2\psi = \beta^2(1-\cos^2\psi)$$

$$= \beta^2\sin^2\psi$$

* VER DOS HOJAS ADELANTE.

$$\cos^2\psi = \frac{E_n^2}{v^2}\,\frac{1}{2mQ(1+Q/2mc^2)}$$

$$= \frac{E_n^2}{v^2}\,\frac{2\cancel{m}c^2}{2\cancel{m}Q(2mc^2+Q)} = \frac{E_n^2}{\beta^2 Q(2mc^2+Q)}$$

$$\cos^2\psi = \frac{E_n^2}{\beta^2 Q(2mc^2+Q)}$$

$$\frac{\beta\tau^2 f_n E_n (mc^2+Q)\, 4m^2c^4}{2m^2c^4\left[Q(2mc^2+Q)-E_n^2\right]^2} = \frac{2\beta\tau^2 f_n E_n (Q+mc^2)}{\left[Q(2mc^2+Q)-E_n^2\right]^2}$$

$$\sin^2\psi = 1-\cos^2\psi = \frac{\beta^2 Q(2mc^2+Q)-E_n^2}{\beta^2 Q(2mc^2+Q)}$$

$$= \frac{2\beta\tau^2 f_n E_n (Q+mc^2)}{Q^2(2mc^2+Q)^2\left[1-\underbrace{E_n^2/Q(2mc^2+Q)}_{\beta^2\cos^2\psi}\right]^2}$$

$$= \frac{2\beta\tau^2 f_n E_n (Q+mc^2)}{Q^2(2mc^2+Q)^2\left[1-\beta^2\cos^2\psi\right]^2}$$

$$\cancel{= \frac{mc^2\beta\tau^2 f_n E_n (Q/mc^2+1)}{Q^2(2mc^2+Q)^2\left[1-\beta^2\cos^2\psi\right]^2}} \quad \cancel{= \frac{\beta\tau^2\beta^2\cdot 2mc^2 E_n (Q/mc^2+1)}{\beta^2 Q^2(2mc^2+Q)^2\left[1-\beta^2\cos^2\psi\right]^2}}\, f_n$$

$$\cancel{= \frac{\beta\tau^2\beta^2 E_n (Q/mc^2+1)}{\beta^2}}$$

$$= \frac{2\beta^2 \sin^2\psi\, f_n E_n (Q+mc^2)}{Q^2(2mc^2+Q)^2\left[1-\beta^2\cos^2\psi\right]^2} = \frac{2\beta^2 E_n (Q+mc^2)}{Q^2(2mc^2+Q)^2}\,\frac{f_n \sin^2\psi}{\left[1-\beta^2\cos^2\psi\right]^2}$$

AHORA BIEN

$$d\cos^2\psi = d\left(\frac{E_n^2}{v^2q^2}\right) = d\left(\frac{q_{min}}{q}\right)^2$$

$$= -\frac{E_n^2}{v^2}\frac{2}{q^3} = -2\left(\frac{q_{min}}{q}\right)^2\frac{dq}{q} = -2\cos^2\psi\frac{dq}{q}$$

$$\frac{q^2}{2m} = Q(1+Q/2mc^2)$$

$$q\,dq = m(1+Q/mc^2)dQ$$

$$\frac{dq}{q} = \frac{m(1+Q/mc^2)}{q^2}dQ = \frac{\cancel{m}(1+Q/mc^2)dQ}{2\cancel{m}\,Q(1+Q/2mc^2)}$$

$$= \frac{\cancel{2}(mc^2+Q)}{\cancel{2}Q(2mc^2+Q)}dQ = \frac{(mc^2+Q)}{Q(2mc^2+Q)}dQ$$

$$d\cos^2\psi = -2\cos^2\psi\frac{(mc^2+Q)}{Q(2mc^2+Q)}dQ$$

$$= -\frac{2E_n^2(mc^2+Q)}{\beta^2Q^2(2mc^2+Q)^2}dQ$$

$\therefore$

$$\frac{2\beta^2E_n(mc^2+Q)}{Q^2(2mc^2+Q)^2}\frac{f_n\sin^2\psi\,dQ}{(1-\beta^2\cos^2\psi)^2} = \frac{\beta^4}{E_n}\underbrace{\frac{2E_n^2(mc^2+Q)dQ}{\beta^2Q^2(2mc^2+Q)^2}}_{d\cos^2\psi}\frac{f_n\sin^2\psi}{(1-\beta^2\cos^2\psi)^2}$$

$$= \frac{\beta^4 f_n\sin^2\psi\,d\cos^2\psi}{E_n(1-\beta^2\cos^2\psi)^2}$$

Finalmente.

$$\boxed{d\sigma_n = \frac{2\pi Z_p^2 e^4 Z f_n}{m v^2 E_n}\left[\frac{dQ}{Q} + \frac{\beta^4 \sin^2\vartheta \, d\cos^2\vartheta}{(1-\beta^2\cos^2\vartheta)^2}\right]}$$

Este resultado es para colisiones suaves, bajo Q. ($f_n \equiv$ OOS).

$Q/mc^2 << 1$

Q intermedio: En este intervalo las excitaciones transversales son despreciables. Esto ocurre porque $\frac{\bar{\beta}_t \cdot \bar{G}_n}{[Q(1+Q/2mc^2)-(E/c)^2]^2} \xrightarrow[Q \gg Q_{min}]{} 0$

($\bar{G}_n$ no depende explicitamente de Q.)

$$\therefore \quad d\sigma_n = \frac{2\pi Z_p^2 e^4 Z}{m v^2 E_n} \frac{dQ}{Q} \frac{|F_n(\bar{q})|^2}{(1+Q/2mc^2)^2}$$

$$\sum E_n d\sigma_n = \frac{2\pi Z_p^2 e^4 Z}{m v^2} \frac{dQ}{Q(1+Q/2mc^2)^2} \underbrace{\sum E_n |F_n(\bar{q})|^2}_{Q}$$

$\therefore$ El poder de frenado no dependerá de $|F_n(\bar{q})|^2$ o f_n.

$Q >> E_b$; E_b - energía de enlace

$q >> p_0$; p_0 - momento propio en estado ligado.

Aquí se puede considerar q' la partícula está libre $\therefore$ $\bar{q} = \bar{p}_n$; $\bar{p}_0 = 0$

En esto caso debemos usar funciones de onda para el e^- tipo Dirac.

Para el estado inicial $\bar{p}_0 = 0$ (libre)
final $\bar{p}$

$$\psi_0 = \sqrt{\frac{E+mc^2}{2E}} \begin{pmatrix} \chi^s \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+m}\chi^s \end{pmatrix} e^{i\bar{p}\cdot\bar{x}} \quad ; \quad \chi^1 = \underset{s=1/2}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \; \chi^2 = \underset{s=-1/2}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

entonces

$$\psi_i = \sqrt{\frac{\sqrt{p^2+m^2c^4}+mc^2}{2\sqrt{p^2+m^2c^4}}} \begin{pmatrix} \chi^s \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+m}\chi^s \end{pmatrix} e^{i\bar{p}\cdot\bar{x}}$$

$$\psi_i = \begin{pmatrix} \chi^s \\ 0 \end{pmatrix}$$

$$\psi_f = \sqrt{\frac{E+mc^2}{2E}} \begin{pmatrix} \chi^s \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+m}\chi^s \end{pmatrix} e^{i\bar{p}\cdot\bar{x}} \quad ; \bar{p} \text{ - momento final}$$

$$|F_n(\bar{q})|^2 = \frac{1}{Z^2}\left|\sum_j \langle \psi_f | e^{i\bar{q}\cdot\bar{x}_j/\hbar} | \psi_i \rangle\right|^2$$

$$= \frac{1}{Z^2}\left|\sum_j \int \left(\chi^{s+} \; \frac{\bar{\sigma}\cdot\bar{p}}{E+m}\chi^{s+}\right)\begin{pmatrix} \chi^s \\ 0 \end{pmatrix}\sqrt{\frac{E+mc^2}{2E}}\, \underbrace{e^{i(\bar{q}-\bar{p})\cdot\bar{x}_j/\hbar}\, dx_j}_{\delta_{qp}}\right|^2$$

$$= \frac{1}{Z^2}\left| Z\sqrt{\frac{E+mc^2}{2E}}\right|^2 \delta_{qp}$$

$$= \frac{E+mc^2}{2E} = \frac{Q+mc^2+mc^2}{2(Q+mc^2)} = \frac{1+Q/2mc^2}{1+Q/mc^2}$$

$$\boxed{|F_n(\bar{q})|^2 = \frac{1+Q/2mc^2}{1+Q/mc^2}}$$ 2.

continúa dos pág después.

* CORRESPONDE AL CASO DE BAJO Q.

$$F_n(\bar{q}) = \frac{1}{Z^{1/2}} \sum_j (n| e^{i\bar{q}\cdot\bar{r}_j/\hbar} |0)$$

$$e^{i\bar{q}\cdot\bar{r}_j/\hbar} \approx 1 + \frac{i\,\bar{q}\cdot\bar{r}_j}{\hbar} \quad ; \bar{q} - \text{PEQUEÑO}$$

$$\sum_j (n| 1 + \frac{i\bar{q}\cdot\bar{r}_j}{\hbar} |0) = \sum_j (n| \frac{i\bar{q}\cdot\bar{r}_j}{\hbar} |0)$$

$$= \frac{q}{\hbar} \left(\sum_j i x_j\right)_{n0} \quad ; x \parallel q$$

$$\therefore \boxed{|F_n(\bar{q})|^2 = \frac{1}{Z} \frac{q^2}{\hbar^2} |(\Sigma x_j)_{n0}|^2 = \frac{Q f_n}{E_n}}$$

$$f_n \equiv \frac{1}{Z} \frac{E_n q^2}{\hbar^2 Q} |(\Sigma x_j)_{n0}|^2$$

$$Q(1 + Q/2mc^2) = \frac{q^2}{2m}$$

$$Q \ll mc^2 : Q = \frac{q^2}{2m}$$

$$q \approx \frac{E_n}{v}$$

VEAMOS $\bar{\beta}_t \cdot \bar{G}(\bar{q})$

$$|\bar{\beta}_t \cdot \bar{G}(\bar{q})|^2 = \frac{1}{Z} \left| \bar{\beta}_t \cdot \sum_j (n| \bar{\alpha}_j e^{i\bar{q}\cdot\bar{r}_j/\hbar} |0) \right|$$

$$\bar{\alpha}_j = \frac{\dot{\bar{r}}_j}{c} = \frac{1}{c}\left[\cancel{\frac{\partial \bar{r}_j}{\partial t}} + \frac{i}{\hbar}\left(\hat{H}\bar{r}_j - \bar{r}_j\hat{H}\right)\right]$$

; COMO EL OPERADOR $\bar{r}_j$ NO DEPENDE EXPLÍCITAMENTE DEL TIEMPO $\frac{\partial \bar{r}_j}{\partial t} = 0$

$$(n|\bar{\alpha}_j|0) = \frac{i}{\hbar c}\left[(n|H\bar{r}_j|0) - (n|\bar{r}_j \hat{H}|0)\right]$$

$$(n|\bar{\alpha}_j|0) = \frac{i}{\hbar c}\left[E_n (n|\bar{r}_j|0) - E_0 (n|\bar{r}_j|0)\right]$$

como $E_0 = 0$ por convención

$$(n|\bar{\alpha}_j|0) = \frac{i E_n}{\hbar c} (n|\bar{r}_j|0)$$

A orden como, $e^{i\bar{q}\cdot\bar{r}_j/\hbar} \approx 1$

$$|\bar{\beta}_t \cdot \bar{G}(q)|^2 = \left|\frac{1}{Z}\frac{E_n^2}{\hbar^2 c^2}\, \bar{\beta}_t \cdot \sum_j (n|\bar{r}_j|0)\right|^2$$

$$= \frac{\beta_t^2 E_n^2}{Z\, \hbar^2 c^2} \left|\sum_j (n|y_j|0)\right|^2 \qquad ;\ y_j \parallel \bar{\beta}_t\ ;\ y_j \perp \bar{q}$$

$$|\bar{\beta}_t \cdot \bar{G}(\bar{q})|^2 = \frac{\beta_t^2 E_n}{c^2}\,\frac{E_n\left|\sum_j (n|y_j|0)\right|^2}{Z\hbar^2}$$

$$= \frac{\beta_t^2 E_n}{c^2}\,\frac{Q f_n}{q^2} \qquad q^2 \approx 2mQ$$

$$= \frac{\beta_t^2 E_n}{c^2}\,\frac{\cancel{Q} f_n}{2m\cancel{Q}}$$

$$\boxed{|\bar{\beta}_t \cdot \bar{G}(\bar{q})|^2 = \frac{\beta_t^2 E_n}{2mc^2} f_n}$$

Seguimos con el caso Q grande (e^- relativista).

$\boxed{\text{Dispersión de un } e^- \text{ relativista en un campo E.M.}}$

$$\psi_i = \begin{pmatrix}\chi^s\\ 0\end{pmatrix} \quad ; \quad \psi_f = \sqrt{\frac{E+mc^2}{2E_n}} \begin{pmatrix}\chi^s \\ \frac{\vec\sigma\cdot\vec q}{E+m}\chi^s\end{pmatrix} e^{i\vec p\cdot\vec r/\hbar}$$

$$|\vec\beta_t\cdot \vec G_n(\vec q)|^2 = \frac{1}{Z}\left|\vec\beta_t \cdot \sum_j (\psi_f|\vec\alpha_j e^{i\vec q\cdot\vec r_j/\hbar}|\psi_i)\right|^2$$

$$\psi_f^*\psi_i = \sqrt{\frac{E+mc^2}{2E_n}}\, e^{-i\vec p\cdot\vec r_j/\hbar}$$

$$|\vec\beta_t\cdot\vec G_n(\vec q)|^2 = \frac{1}{Z\hbar^2}\left|\vec k\cdot\sum_j\left[(\psi_f|\hat H\vec r_j e^{i\vec q\cdot\vec r_j/\hbar}|\psi_i) - \underbrace{(\psi_f|\vec r_j \hat H e^{i\vec q\cdot \vec r_j/\hbar}|\psi_i)}_{\to 0}\right]\right|^2$$

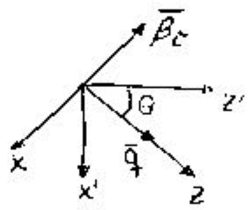
$$= \frac{1}{Z\hbar^2}\left|\vec k\cdot\sum_j E_n\, (\psi_f|\vec r_j e^{i\vec q\cdot\vec r_j/\hbar}|\psi_i)\right|^2$$

$$\dot{\vec r}_j = \frac{\partial \vec r_j}{\partial t} + \frac{i}{\hbar}\left[\hat H \vec r_j - \vec r_j \hat H\right]$$

Emplearemos el formalismo QED para calcular esta amplitud de transición

$$\psi_i = u(p)\, e^{i\vec p\cdot\vec r/\hbar}\sqrt{\frac{E+m}{2E}}$$

$$\psi_f = u(p')\, e^{i\vec p'\cdot\vec r/\hbar}\sqrt{\frac{E'+m}{2E'}}$$



$$M_{if} = \bar\psi_f\, \hat A(q)\,\psi_i = \bar\psi_f\gamma^0A_0\psi_i + \bar\psi_f\,\vec\gamma\, \vec A_\mu\,\psi_i$$

$$\hat A = \gamma^\mu A_\mu$$

$$A_\mu = (\underset{\downarrow\, 0}{A_0}, \vec A)$$ $\longrightarrow$ Componentes de Fourier del campo externo

$\bar{A}$ $\quad A_\mu = \left(\frac{4\pi Z e^2}{K^2}, \ \frac{4\pi Z_p e^2 \bar{\beta}_c}{K^2 - (w/c)^2} \right)$

$\bar{q} = \hbar \bar{k}$

Término Coulombiano longitudinal

$$\bar{\psi}_f \gamma^0 A_0 \psi_i = \psi_f^* \psi_i A_0 = \sqrt{\frac{(E+m)(E'+m)}{4EE'}} A_0 \ u(p')^* u(p)$$

$$\psi_f^* = \bar{\psi}_f \gamma^0$$

$$u(p')^* u(p) = \begin{pmatrix} \chi_s'^* & \frac{\bar{\sigma}\cdot\bar{p}'\chi_s'^*}{E'+m} \end{pmatrix} \begin{pmatrix} \chi_s \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+m}\chi_s \end{pmatrix}$$

$$= \chi_s'^* \chi_s + \frac{(\bar{\sigma}\cdot\bar{p}')}{E'+m}\chi_s'^* \frac{(\bar{\sigma}\cdot\bar{p})}{E+m}\chi_s$$

Si no hay spin-flipping $\chi_s'^* \chi_s = 1$

$$u(p')^* u(p) = 1 + \chi_s^* \frac{\bar{\sigma}\cdot\bar{p}'}{E'+m} \frac{(\bar{\sigma}\cdot\bar{p})}{E+m}\chi_s$$

$$= 1 + \chi_s^* \frac{\bar{p}'\cdot\bar{p}}{(E'+m)(E+m)} \chi_s$$

$$= 1 + \frac{\chi_s^*(\bar{p}'\cdot\bar{p} + i\bar{\sigma}\cdot(\bar{p}'\times\bar{p}))\chi_s}{(E'+m)(E+m)}$$

$$= 1 + \frac{\chi_s^* \bar{p}'\cdot\bar{p}\,\chi_s}{(E'+m)(E+m)} + i\frac{\chi_s^* \bar{\sigma}\cdot(\bar{p}'\times\bar{p})\chi_s}{(E'+m)(E+m)}$$

(1 c) $\quad \bar{q} = \bar{p}' - \bar{p} \quad ; \ \bar{p} = \bar{p}' - \bar{q} \quad ; \ \bar{p}' = \bar{q} + \bar{p}$

$$= 1 + \frac{\chi_s^*(\bar{p}^2 + \bar{q}\cdot\bar{p})\chi_s}{(E'+m)(E+m)} + i\frac{\chi_s^* \bar{\sigma}\cdot(\bar{q}\times\bar{p})\chi_s}{(E'+m)(E+m)}$$

Si $\bar{p}=0$ (e^- en reposo)

$u^*(p')u(p) = 1$

$$\therefore \quad M_{fi_0} = \bar{\psi}_f \gamma^\mu A_0 \psi = \sqrt{\frac{(E'+m)\cancel{m}}{4E'\cancel{m}}} \frac{4\pi Z_p e^2}{K^2}$$

$$M_{fi_0} = \sqrt{\frac{E'+m}{4E'}} \cdot \frac{4\pi Z_p e^2}{K^2} = \sqrt{\frac{Q+2m}{4(Q+m)}} \frac{4\pi Z_p e^2}{K^2}$$

$$\boxed{M_{fi_0} = \frac{4\pi Z_p e^2}{K^2} \sqrt{\frac{Q/2m+1}{2(Q/m+1)}}} \quad \text{Lqqd}$$

$$d\sigma = \frac{1}{16\pi^2} |M_{fi}|^2 d\Omega$$

Con spin flipping $\chi_0'^* \chi_0 = 0$

$\boxed{\therefore \; M_{fi_0} = 0}$ No hay spin flipping en transición eléctrica

Término magnético (transversal)

$$\bar{\psi}_f \gamma^\mu A_\mu \psi_i = \bar{\psi}_f \gamma^\mu \psi_i A_\mu \quad (A_\mu \text{ no actúa sobre el espinor})$$

$$= \psi_f^* \gamma^0 \gamma^\mu \psi_i A_\mu \quad ; \bar{\alpha} \equiv \gamma^0 \gamma^\mu \equiv \alpha^\mu$$

$$= \sqrt{\frac{(E+m)(E'+m)}{4EE'}} \quad u(p')^* \alpha^\mu u(p) A_\mu$$

$$A_\mu = \frac{4\pi Z_p e^2 \beta_{\mu}}{K^2 - (\omega/c)^2} ;$$

Sin spin flipping.

$$u(p')^* \bar\alpha\, u(p) = \left(\chi_s^* \quad \frac{\bar\sigma\cdot\bar p'}{E'+m}\chi_s^*\right)\begin{pmatrix}0 & \bar\sigma\\ \bar\sigma & 0\end{pmatrix}\begin{pmatrix}\chi_s\\ \frac{\bar\sigma\cdot\bar p}{E+m}\chi_s\end{pmatrix}$$

$$= \left(\chi_s^* \quad \frac{\bar\sigma\cdot\bar p'}{E'+m}\chi_s^*\right)\begin{pmatrix}\bar\sigma\,\frac{(\bar\sigma\cdot\bar p)\chi_s}{E+m}\\ \bar\sigma\,\chi_s\end{pmatrix} = \chi_s^*\,\frac{\bar\sigma(\bar\sigma\cdot\bar p)}{E+m}\chi_s + \frac{(\bar\sigma\cdot\bar p')}{E'+m}\chi_s^*\bar\sigma\chi_s$$

$$= \chi_s^*\left(\frac{\bar\sigma(\bar\sigma\cdot\bar p)}{E+m} + \frac{(\bar\sigma\cdot\bar p')\bar\sigma}{E'+m}\right)\chi_s$$

$$= \chi_s^+\left(\frac{\bar p + i(\bar p\times\bar\sigma)}{E+m} \quad \frac{\bar p' + i(\bar\sigma\times\bar p')}{E'+m}\right)\chi_s$$

si el e^- está en reposo inicialmente $\bar p = 0$

$$= \frac{\chi_s^+\left(\bar p' + i(\bar\sigma\times\bar p')\right)\chi_s}{E'+m}$$

$\chi_s\begin{pmatrix}1\\0\end{pmatrix}$; $\chi_s^*(1 \;\; 0)$

$$\bar\sigma\times\bar p'\chi_s = -\bar p'\times\bar\sigma = -\hat\imath\begin{pmatrix}p_2\\ -ip_3\end{pmatrix} + \hat\jmath\begin{pmatrix}p_1\\ -p_3\end{pmatrix} + k\begin{pmatrix}0\\ -ip_1+p_2\end{pmatrix}$$

$\therefore$

$$\bar\sigma\times\bar p' = \begin{vmatrix}\hat\imath & \hat\jmath & \hat k\\ \sigma_1 & \sigma_2 & \sigma_3\\ p_1' & p_2' & p_3'\end{vmatrix} = \hat\imath\left(\sigma_2 p_3' - \sigma_3 p_2'\right) + \hat\jmath\left(\sigma_3 p_1' - p_3'\sigma_1\right) + \hat k\left(\sigma_1 p_2' - \sigma_2 p_1'\right)$$

$$= \hat\imath\left[\begin{pmatrix}0 & -ip_3'\\ ip_3' & 0\end{pmatrix} - \begin{pmatrix}p_2' & 0\\ 0 & -p_2'\end{pmatrix}\right] + \hat\jmath\left[\begin{pmatrix}p_1' & 0\\ 0 & -p_1'\end{pmatrix} - \begin{pmatrix}0 & p_3'\\ p_3' & 0\end{pmatrix}\right]$$

$$\hat k\left[\begin{pmatrix}0 & p_2'\\ p_2' & 0\end{pmatrix} - \begin{pmatrix}0 & -ip_1'\\ ip_1' & 0\end{pmatrix}\right] = \hat\imath\begin{pmatrix}-p_2' & -ip_3'\\ ip_3' & p_2'\end{pmatrix} + \hat\jmath\begin{pmatrix}p_1' & p_3'\\ -p_3' & -p_1'\end{pmatrix} +$$

$$= \hat k\begin{pmatrix}0 & p_2' + ip_1'\\ p_2' - ip_1' & 0\end{pmatrix}$$

$$u(p')^{*}\bar{\alpha}\,u(p) = \frac{X_s^{*}\bar{p}'X_s}{E'+m} + \frac{i\,X_s^{*}(\bar{\sigma}\times\bar{p}')\,X_s}{E'+m}$$

$$= \frac{\bar{p}'}{E'+m} + \frac{(1\quad 0)}{E'+m}\left[\hat{i}\begin{pmatrix}-iP_2'\\ -P_3'\end{pmatrix} + \hat{j}\begin{pmatrix}iP_1'\\ -iP_3'\end{pmatrix} + \hat{k}\begin{pmatrix}0\\ P_1'+iP_2'\end{pmatrix}\right]$$

$$= \frac{\bar{p}'}{E'+m} + \left(-iP_2'\,\hat{i} + iP_1'\,\hat{j}\right)$$

SIENDO $\bar{p}' = \bar{q} = q(0,0,1)$

$$\therefore\quad u(p')^{*}\bar{\alpha}\,u(p) = \frac{\bar{p}'}{E'+m} = \frac{\bar{q}}{E'+m}$$

COMO $\bar{\beta}_T \perp \bar{q}$; $u(p')^{*}\alpha^{\mu}u(p)A_{\mu} = 0$

VEAMOS QUÉ PASA CUANDO HAY SPIN FLIPPING

$\left(X_{s_0} = \begin{pmatrix}1\\0\end{pmatrix}\quad X_{s_f} = \begin{pmatrix}0\\1\end{pmatrix}\right.$

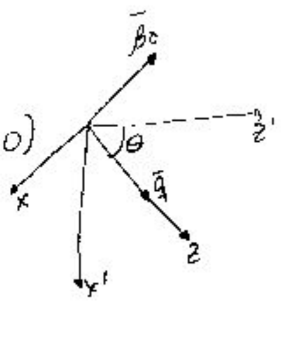
$$u(p')^{*}\bar{\alpha}\,u(p) = \frac{X_{s_f}^{*}\bar{p}'X_{s_0}}{E'+m} + \frac{i\,X_{s_f}^{*}(\bar{\sigma}\times\bar{p}')\,X_{s_0}}{E'+m}$$

$$= \frac{(0\quad 1)\,\bar{p}'}{E'+m}\begin{pmatrix}1\\0\end{pmatrix} + \frac{(0\quad 1)}{E'+m}\left[\hat{i}\begin{pmatrix}-iP_2'\\ -P_3'\end{pmatrix} + \hat{j}\begin{pmatrix}iP_1'\\ -iP_3'\end{pmatrix} + \hat{k}\begin{pmatrix}0\\ P_1'+iP_2'\end{pmatrix}\right]$$

$$= -P_3'\,\hat{i} - iP_3'\,\hat{j} + (P_1'+iP_2')\,\hat{k}$$

COMO $\bar{p}' = \bar{q} = q(0,0,1)$

$$u(p')^{*}\bar{\alpha}\,u(p) = \frac{q}{E'+m}(-\hat{i} - i\hat{j}) = \frac{q}{E'+m}(-1, -i, 0)$$



Ahora rotamos el vector $\mathcal{U}(p')^{*}\tilde{\alpha}\,\mathcal{U}(p)$ respecto a $\hat{y}$ θ grados

$$\bar{r}' = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \bar{r}$$

$$\left(\mathcal{U}(p')^{*}\bar{\alpha}\,\mathcal{U}(p)\right)' = \frac{q}{E'+m}\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}\begin{pmatrix} -1 \\ -i \\ 0 \end{pmatrix} = -\cos\theta\,\hat{i} - i\,\hat{j} - \sin\theta\,\hat{k}$$

$$= q(-\cos\theta, -i, \sin\theta)$$

$$\bar{\beta}_c' = (-\cos\theta, 0, \sin\theta)$$

$$\therefore (\mathcal{U}\bar{\alpha}\mathcal{U})^{\mu}\beta_{c\mu} = \frac{\beta_c \cdot q}{E'+m}(\cos^2\theta + \sin^2\theta) = \beta_c\, q$$

Finalmente

$$\bar{\Psi}_f\, \gamma^{\mu}\Psi_i A_{\mu} = \sqrt{\frac{(E+m)(E'+m)}{4EE'}}\;\frac{\beta_c q}{E'+m}\left(\frac{4\pi Z_p e^2}{K^2-(\omega/c)^2}\right); \quad \bar{q} = \hbar\bar{K}$$

como $\beta' = 0$

$$= \sqrt{\frac{(E'+m)\cdot 2m}{2\,4E\cdot m}}\left(\frac{4\pi Z_p e^2}{K^2-(\omega/c)^2}\right)\frac{\beta_c q}{(E'+m)}$$

promedio por espín inicial

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{1}{16\pi^2}\left|\bar{\Psi}_f\gamma^{\mu}\Psi_i A_{\mu}\right|^2$$

$$= \frac{16\pi^2 Z_p^2 e^4 \beta_c^2 q^2}{16\pi^2\left[K^2-(\omega/c)^2\right]^2}\cdot\frac{\quad}{2E(E+m)} = \frac{Z_p^2 e^4 \beta_c^2}{\left[K^2-(\omega/c)^2\right]^2}\frac{(E'^2-m^2)}{2E'(E'+m)}$$

$$= \frac{1}{2}\frac{Z_p^2 e^4 \beta_c^2}{\left[K^2-(\omega/c)^2\right]^2}\cdot\frac{E'-m}{2E'} = \frac{Z_p^2 e^4 \beta_c^2}{\left[K^2-(\omega/c)^2\right]^2}\left(\frac{Q/2m}{Q/m+1}\right)$$

R

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\underbrace{\frac{Z_p^2 e^4 \beta_t^2}{\left[k^2-(\omega/c)^2\right]^2}\left(\frac{Q/2mc^2}{Q/mc^2+1}\right)}_{\text{término transversal}} + \underbrace{\frac{Z_p^2 e^2}{k^4}\left(\frac{1+Q/2mc^2}{1+Q/mc^2}\right)}_{\text{término longitudinal}}$$

$$\frac{d\sigma}{d\Omega} = \frac{4Z_p^2 e^4}{2}\left[\frac{1+Q/2mc^2}{k^4(1+Q/mc^2)} + \frac{\beta_t^2 (Q/2mc^2)}{\left[k^2-(\omega/c)^2\right]^2(1+Q/mc^2)}\right]$$

$$Q(1+Q/2mc^2) = \frac{q^2}{2m} = \frac{\hbar k^2}{2m}$$

~~Ahora~~

$$d\Omega = 2\pi\, d\cos\theta \qquad \theta - \text{ángulo de dispersión del } e^-.$$

$$= \frac{2\pi E_n\, dq}{v\, q^2}$$

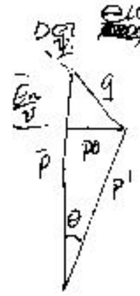
$$\cos\theta = \frac{E_n}{vq}$$

$$d\cos\theta = \frac{E_n}{vq^2}\,dq$$

$$\frac{d\sigma}{dq} = \frac{2\pi Z_p^2 e^4}{v^2}$$

$$\beta_t^2 = \beta^2(1-\cos^2\theta) = \beta^2\left(1-\frac{q_{min}^2}{q^2}\right)$$

θ - ángulo de dispersión electrón



$$q_{min} = \frac{E_n}{v}$$

Como inicialmente el e^- estaba con reposo $E_n = Q$

$$\beta_t^2 = \beta^2 - \frac{\beta^2 Q^2}{v^2 q^2} = \beta^2 - \frac{\cancel{\beta^2} Q^2}{\cancel{v^2/c^2}\,(Q+2mc^2)}$$

$$= -1 + \beta^2 + 1 - \frac{Q}{Q+2mc^2} = -(1-\beta^2) + \frac{2mc^2}{Q+2mc^2} =$$

$$\boxed{\beta_t^2 = -(1-\beta^2) + \frac{1}{(1+ \ldots)}}$$

Hagamos el mismo cálculo mediante el formalismo de la matriz de densidad

Término longitudinal (coulombiano).

$= -e\,\bar{u}(p')\hat{A}u(p)$

$$M_A = -e\,\bar{u}(p')\gamma^0 u(p)\,A_0(\vec{q}) = -e\,u^+(p')u(p)\,A_0(\vec{q})$$

Promediando por ambas polarizaciones

$$\frac{1}{2}\sum_{pol}|M_A|^2 = e^2\, 2\,Tr(\rho\hat{A}\rho'\hat{A}) = e^2 2\,Tr(\rho\gamma^0\rho'\gamma^0)|A_0(\vec{q})|^2$$

$$\rho = \frac{1}{2}(m+\hat{p})\ ,\quad \rho' = \frac{1}{2}(m+\hat{p}')$$

$$= e^2 2|A|^2 Tr\ \frac{1}{2}(m+\hat{p})\gamma^0\frac{1}{2}(m+\hat{p}')\gamma^0 = \frac{1}{2}|A|^2 e^2\,Tr[(m+\hat{p})\gamma^0(m+\hat{p}')\gamma^0]$$

$$\gamma^0\hat{p}'\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\mathcal{E}' & -\vec{p}'\cdot\vec{\sigma}\\ \vec{p}'\vec{\sigma} & -\mathcal{E}'\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\mathcal{E}' & \vec{p}'\cdot\vec{\sigma}\\ \vec{p}'\cdot\vec{\sigma} & -\mathcal{E}'\end{pmatrix}$$

$$= \begin{pmatrix}\mathcal{E}' & \vec{p}'\cdot\vec{\sigma}\\ -\vec{p}'\cdot\vec{\sigma} & -\mathcal{E}'\end{pmatrix}$$

$$= \begin{pmatrix}\mathcal{E}' & \vec{p}'\cdot\vec{\sigma}\\ -\vec{p}'\cdot\vec{\sigma} & -\mathcal{E}'\end{pmatrix} \equiv \boxed{\hat{\bar{p}}' \quad \text{donde}\ \bar{p}' = (\mathcal{E}', -\vec{p}')}$$

$\hat{p} = \mathcal{E}\gamma^0 - \vec{p}\cdot\vec{\gamma}$

$= \begin{pmatrix}\mathcal{E} & 0\\ 0 & -\mathcal{E}\end{pmatrix} - \begin{pmatrix}\vec{p} & 0\\ 0 & \vec{p}\end{pmatrix}\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix}$

$= \begin{pmatrix}\mathcal{E} & 0\\ 0 & -\mathcal{E}\end{pmatrix} - \begin{pmatrix}0 & \vec{p}\cdot\vec{\sigma}\\ \vec{p}\vec{\sigma} & 0\end{pmatrix}$

$= \begin{pmatrix}\mathcal{E} & -\vec{p}\cdot\vec{\sigma}\\ \vec{p}\vec{\sigma} & -\mathcal{E}\end{pmatrix}$

$$\frac{1}{2}\sum_{pol}|M_A|^2 = \frac{e^2}{2}|A|^2\,Tr(m+\hat{p})(m+\hat{\bar{p}}')$$

$$= \frac{e^2|A|^2}{2}Tr[m^2 + m\hat{\bar{p}}' + \hat{p}m + \hat{p}\hat{\bar{p}}']$$

$$= \frac{e^2|A|^2}{2}\left[4m^2 + Tr(m\hat{\bar{p}}' + \hat{p}m + \hat{p}\hat{\bar{p}}')\right]$$

$$m\hat{p}' = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \mathcal{E}' & \bar{p}'\cdot\bar{\sigma} \\ -\bar{p}'\cdot\bar{\sigma} & -\mathcal{E}' \end{pmatrix} = \begin{pmatrix} m\mathcal{E}' & m\bar{p}'\cdot\bar{\sigma} \\ -m\bar{p}'\cdot\bar{\sigma} & -m\mathcal{E}' \end{pmatrix}$$

$$Tr(m\hat{p}') = 0$$

$$m\,\hat{p} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \mathcal{E} & -\bar{p}\cdot\bar{\sigma} \\ +\bar{p}\cdot\bar{\sigma} & -\mathcal{E} \end{pmatrix} = \begin{pmatrix} m\mathcal{E} & -m\bar{p}\cdot\bar{\sigma} \\ +m\bar{p}\cdot\bar{\sigma} & -m\mathcal{E} \end{pmatrix}$$

$$Tr(m\hat{p}) = 0$$

$$\hat{p}\,\hat{p}' = \begin{pmatrix} \mathcal{E} & -\bar{p}\cdot\bar{\sigma} \\ \bar{p}\cdot\bar{\sigma} & -\mathcal{E} \end{pmatrix}\begin{pmatrix} \mathcal{E}' & \bar{p}'\cdot\bar{\sigma} \\ -\bar{p}'\cdot\bar{\sigma} & -\mathcal{E}' \end{pmatrix}$$

$$= \begin{pmatrix} \mathcal{E}\mathcal{E}' + (\bar{p}\cdot\bar{\sigma})(\bar{p}'\cdot\bar{\sigma}) & \mathcal{E}\bar{p}'\cdot\bar{\sigma} + \mathcal{E}'\bar{p}\cdot\bar{\sigma} \\ \mathcal{E}'\bar{p}\cdot\bar{\sigma} + \mathcal{E}\bar{p}'\cdot\bar{\sigma} & +(\bar{p}\cdot\bar{\sigma})(\bar{p}'\cdot\bar{\sigma}) + \mathcal{E}\mathcal{E}' \end{pmatrix}$$

$$Tr(\hat{p}\hat{p}') = Tr\ \cancel{2}\left[(\bar{p}\cdot\bar{\sigma})(\bar{p}'\cdot\bar{\sigma}) + \mathcal{E}\mathcal{E}'\right]\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1}{2}\sum_{pol}|M_A|^2 = \frac{e^2A^2}{2}\left[\quad m^2 + 2\,\bar{p}\cdot\bar{p}' + 2i\,\bar{\sigma}\cdot[\bar{p}\times\bar{p}']\right]$$

$$= 2\,Tr\left[\bar{p}\cdot\bar{p}' + i\,\bar{\sigma}\cdot[\bar{p}\times\bar{p}'] \cancel{\mathcal{E}\mathcal{E}'}\right]$$

$$= 4\left[\bar{p}\cdot\bar{p}' + \mathcal{E}\mathcal{E}'\right]$$

$Tr\left(\bar{\sigma}\cdot[\bar{p}\times\bar{p}']\right) = 0$ ya que $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Finalmente

$$\frac{1}{2}\sum_{pol}|M_A|^2 = \frac{e^2|A_0|^2}{2}\left[4m^2 + 4\,\bar{p}\cdot\bar{p}' + \mathcal{E}\mathcal{E}'\right]$$

$$\boxed{\frac{1}{2}\sum_{pol}|M_A|^2 = 2e^2|A_0|^2\left[m^2 + \bar{p}\cdot\bar{p}' + \mathcal{E}\mathcal{E}'\right]}$$

Dispersión elástica

$$\mathcal{E} = \mathcal{E}'$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\, 2e^2|A_0|^2\left[m^2 + \bar{p}\cdot\bar{p}' + \mathcal{E}^2\right]$$

$$\bar{q} = \bar{p}' - \bar{p} \qquad \text{(momento transferido)}$$

$$= \frac{1}{16\pi^2}\, 2e^2|A_0|^2\left[m^2 + \bar{p}(\bar{q}+\bar{p}) + \mathcal{E}^2\right]$$

$$\cancel{= \frac{e^2|A_0|^2}{8\pi^2}\left[m^2 + \mathcal{E}^2 + \right]}$$

$$\bar{q}^2 = \bar{p}'^2 + \bar{p}^2 - 2\bar{p}\cdot\bar{p}'$$

$$\bar{p}\cdot\bar{p}' = \frac{-\bar{q}^2 + \bar{p}'^2 + \bar{p}^2}{2} = \frac{-\bar{q}^2}{2} + \frac{\mathcal{E}^2 - m^2 + \mathcal{E}^2 - m^2}{2}$$

$$\boxed{\bar{p}\cdot\bar{p}' = -\frac{\bar{q}^2}{2} + \mathcal{E}^2 - m^2}$$

Sustituyendo

$$\frac{d\sigma}{d\Omega} = \frac{1}{8\pi^2}\, e^2|A_0|^2\left[\cancel{m^2} + \mathcal{E}^2 - \frac{\bar{q}^2}{2} + \mathcal{E}^2 - \cancel{m^2}\right]$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\, e^2|A_0|^2\mathcal{E}^2\left[1 - \frac{\bar{q}^2}{4\mathcal{E}^2}\right]}$$

Para una carga puntual estática

$$A_0(\bar{q}) = \frac{4\pi Z e}{\bar{q}^2} \qquad \left[A_0(\bar{q}) = \frac{4\pi\, \rho(\bar{q})}{\bar{q}^2} \; ; \text{ de forma general} \right]$$

$$\therefore \quad \frac{d\sigma}{d\Omega} = \frac{(4\pi Z e)^2}{\bar{q}^4} \frac{e^2}{4\pi^2} \mathcal{E}^2 \left[1 - \frac{q^2}{4\mathcal{E}^2} \right]$$

$$= \underbrace{\frac{4 (Z e^2)^2}{q^4} \mathcal{E}^2}_{\frac{d\sigma_{rutherford}}{d\Omega}} \left[1 - \frac{q^2}{4\mathcal{E}^2} \right] \longrightarrow q^2 = 4 p^2 \sin^2 \theta/2$$

$$\boxed{\frac{d\sigma_{rutherford}}{d\Omega} = \frac{(Z e^2)^2 \mathcal{E}^2}{4 p^4 \sin^4 \theta/2}} \xrightarrow{\mathcal{E}^2 \to m^2} \boxed{\frac{(Z e^2)^2}{(2 m v^2 \sin^2 \theta/2)^2}}$$

No relativista.

$$d\sigma = d\sigma_{RUTH} \cdot \left(1 - \frac{q^2}{4\mathcal{E}^2} \right)$$

$$= d\sigma_{ruth} \left(1 - \frac{\cancel{4} p^2 \sin^2 \theta/2}{\cancel{4} \mathcal{E}^2} \right)$$

$$\boxed{d\sigma = d\sigma_{RUTH} \left(1 - \frac{p^2 \sin^2 \theta/2}{\mathcal{E}^2} \right)}$$

Ahora $\frac{\bar{p}}{\mathcal{E}} = \bar{\beta}$

$$\therefore \quad \frac{d\sigma}{d\Omega} = \frac{(Z e^2)^2}{4 p^2 v^2 \sin^4 \theta/2} \left(1 - \beta^2 \sin^2 \theta/2 \right)$$

$$\boxed{\frac{d\sigma_{RUTH}}{d\Omega} = \left(\frac{Z_p e^2}{2 p v \sin^2 \theta/2} \right)^2}$$

este término es 1 si spin = 0