Models for transport

Surviving probability

The fractional change of the surviving probability as a function of the distance traveled by the particle z is:

$$\frac{\mathrm{d}p_{\mathrm{s}}(z)}{p_{\mathrm{s}}(z)} = -\mu(z)\mathrm{d}z$$

μ is the interaction probability per unit path-length and is equivalent to the macroscopic cross section and the Inverse mean free path

Models for transport

After having traveled a distance z, the probability for the particle survive is:

$$p_s(z) = \exp\left(-\int_0^z dz' \mu(z')\right)$$

And that for interacting $c(z) = 1 - p_s(z) = 1 - \exp\left(-\int_0^z dz' \mu(z')\right)$

Notice that $c(\infty) = 1$ $p_s(\infty) = 0$

Models for transport

After having traveled a distance z, the interaction probability per unit pathlength is

$$p(z) = \frac{\mathrm{d}}{\mathrm{d}z}\left[1 - \exp\left(-\int_0^z \mathrm{d}z'\mu(z')\right)\right] = \mu(z)\exp\left(-\int_0^z \mathrm{d}z'\mu(z')\right) = \mu(z)p_{\mathrm{s}}(z)$$

First, ther particle has to survive after traveling a distance z (p_s),
Then, it has a probability of interacting per unit path length μ.

Models for transport

Infinite, isotropic and homogeneous media:

$$p_s(z) = e^{-\mu z}$$

$$c(z) = 1 - p_s(z) = 1 - e^{-\mu z}$$

$$p(z) = \frac{d}{dz}[1 - e^{-\mu z}] = \mu e^{-\mu z} = \mu p_s(z)$$

Models for transport

Sampling of the free path between successive interactions:

$$s = -\frac{1}{\mu}\ln(1-r) = -\frac{1}{\mu}\ln r$$

Notice that r is as ramdon as (1-r)

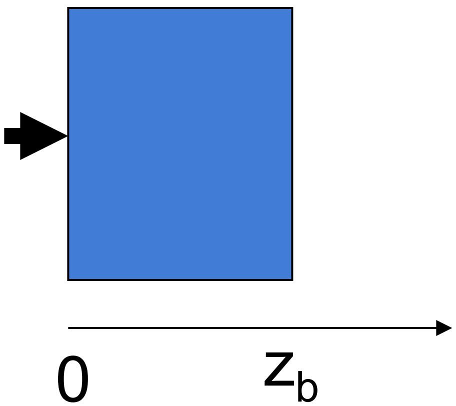
Models for transport

Transport in finite media

Interaction probability

$$c(z) = 1 - p_s(z) = 1 - \exp\left(-\int_0^z dz' \mu(z')\right) + \exp\left(-\int_0^{z_b} dz' \mu(z')\right)\theta(z - z_b)$$

Condition: $\mu(z) = 0 \text{ for } z \geq z_b$



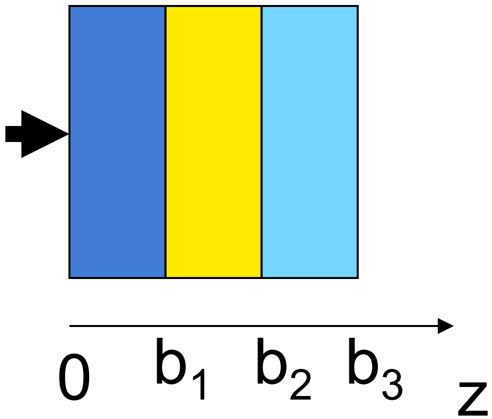
Interaction probability per unit path length:

$$p(z) = \mu(z) \exp\left(-\int_0^z dz' \mu(z')\right) + p_s(z_b)\delta(z - z_b)$$

Models for transport

Transport in medium with different regions.

Interaction probability:

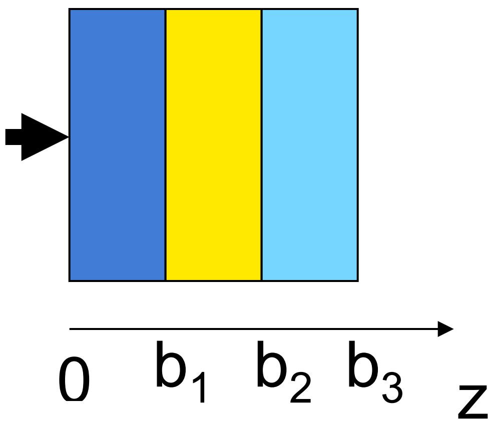


$$
\begin{aligned}
p(z) &= \theta(z)\theta(b_1 - z)\mu_1(z)e^{-\int_0^z \mathrm{d}z'\mu_1(z')} \\
&+ \theta(z - b_1)\theta(b_2 - z)\mu_2(z)e^{-\int_0^{b_1} \mathrm{d}z'\mu_1(z')}e^{-\int_{b_1}^{z} \mathrm{d}z'\mu_2(z')} \\
&+ \theta(z - b_2)\theta(b_3 - z)\mu_3(z)e^{-\int_0^{b_1} \mathrm{d}z'\mu_1(z')}e^{-\int_{b_1}^{b_2} \mathrm{d}z'\mu_2(z')}e^{-\int_{b_2}^{z} \mathrm{d}z'\mu_3(z')} \\
&\vdots
\end{aligned}
$$

Models for transport

Transport in medium with different regions

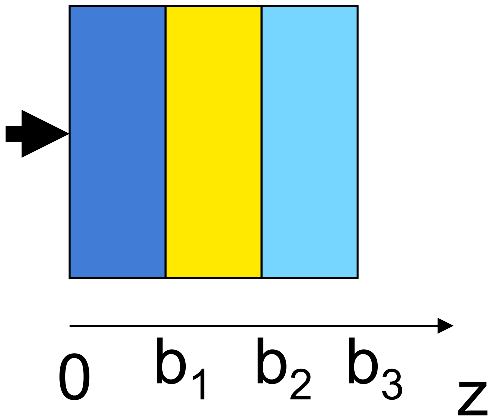
Interaction probability written in a different form:



$$
\begin{aligned}
p(z) &= \theta(z)\theta(b_1 - z)\mu_1(z)e^{-\int_0^z \mathrm{d}z'\mu_1(z')} \\
&+ p_s(b_1)\theta(z - b_1)\theta(b_2 - z)\mu_2(z)e^{-\int_{b_1}^z \mathrm{d}z'\mu_2(z')} \\
&+ p_s(b_2)\theta(z - b_2)\theta(b_3 - z)\mu_3(z)e^{-\int_{b_2}^z \mathrm{d}z'\mu_3(z')} \\
&\vdots
\end{aligned}
$$

Models for transport

Transport in medium with different regions



Condition probability for the particle to arrive in $z=b_i$ without interacting in region b_{i-1}

$$p_s(b_i|b_{i-1})$$

Probability for arriving in $z=b_i$

$$p_s(b_i) = p_s(b_{i-1}) p_s(b_i|b_{i-1})$$

Interaction probability after having traveled a distance z

$$p(z) = p(z_1, z_2, z_3 \cdots) = p(z_1) + p_s(b_1)[p(z_2) + p_s(b_2|b_1)[p(z_3) + p_s(b_3|b_2)[\cdots$$

Models for transport

Interaction probability after having traveled a distance z

$$\begin{aligned} p(z) &= \theta(z)\theta(b_1 - z)\mu_1(z)e^{-\int_0^z \mathrm{d}z'\mu_1(z')} \\ &+ p_\mathrm{s}(b_1)\theta(z - b_1)\theta(b_2 - z)\mu_2(z)e^{-\int_{b_1}^z \mathrm{d}z'\mu_2(z')} \\ &+ p_\mathrm{s}(b_2)\theta(z - b_2)\theta(b_3 - z)\mu_3(z)e^{-\int_{b_2}^z \mathrm{d}z'\mu_3(z')} \\ &\vdots \end{aligned}$$

$$p(z) = p(z_1, z_2, z_3 \cdots) = p(z_1) + p_\mathrm{s}(b_1)[p(z_2) + p_\mathrm{s}(b_2|b_1)[p(z_3) + p_\mathrm{s}(b_3|b_2)[\cdots$$

Probabilities now depend on a single variable in each region and they are independent

Models for transport

Interacting probability in zone 1

$$p(z_1) = \mu_1(z_1) \exp\left(-\int_0^{z_1} \mathrm{d}z' \mu_1(z')\right) + p_\mathrm{s}(b_1)\delta(z - b_1)$$

Interacting probability in zone 3

$$p(z_2) = \mu_2(z_2) \exp\left(-\int_0^{z_2} \mathrm{d}z' \mu_2(z')\right) + p_\mathrm{s}(b_2|b_1)\delta(z - b_2)$$

Interacting probability in zone 3

$$p(z_3) = \mu_3(z_3) \exp\left(-\int_0^{z_3} \mathrm{d}z' \mu_3(z')\right) + p_\mathrm{s}(b_3|b_2)\delta(z - b_3)$$

Models for transport

Example with two zones divided by plane z=b

Interaction coefficient $$\mu(z) = \mu_1\theta(b-z) + \mu_2\theta(z-b)$$

Global interaction probability

$$p(z) = \theta(b-z)\mu_1 e^{-\mu_1 z} + \theta(z-b)\mu_2 e^{-\mu_1 b} e^{\mu_2 (z-b)}$$

For each zone:

$$p(z_1) = \mu_1 e^{-\mu_1 z} + e^{-\mu_1 b}\delta(z-b)$$

$$p(z_2) = \mu_2 e^{-\mu_2 z_2}$$

Models for transport

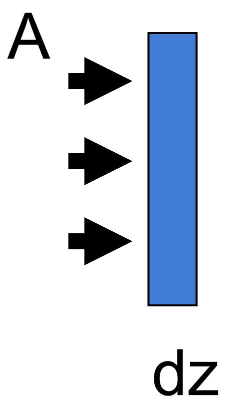
By zone:

For z<b $$p(z_1) = \mu_1 e^{-\mu_1 z} + e^{-\mu_1 b}\delta(z-b)$$

For z>b $$p(z_2) = \mu_2 e^{-\mu_2 z_2}$$

Relation between μ and σ

Consider a beam with cross section area A that impinges a foil with thickness dz containing n target per unit volume



Interaction probability

$$dp = \frac{n A_b dz \sigma}{A_b}$$

$$\frac{dp}{dz} = n\sigma \qquad \mu = \frac{dp}{dz} \qquad \mu = \Sigma = n\sigma$$