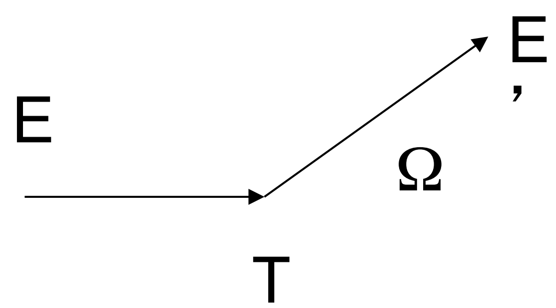
Differential cross sections

σ depends on the incident particle energy and the target material



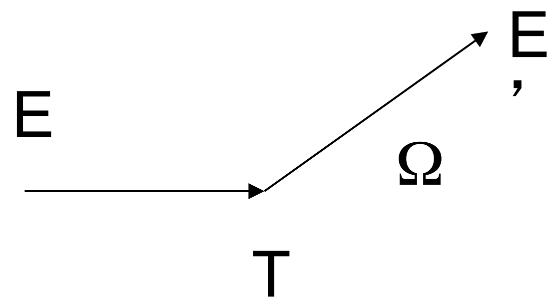
$$\sigma(E) = \int \mathrm{d}E' \, \sigma(E, E') = \int \mathrm{d}\Omega \, \sigma(E, \Theta, \Phi) = \int \mathrm{d}E' \int \mathrm{d}\Omega \, \sigma(E, E', \Theta, \Phi)$$

Simulation process

Simulation steps

- Determination of μ

$$\mu(E) = n \int \mathrm{d}E' \int \mathrm{d}\Omega\, \sigma(E, E', \Theta, \Phi)$$



$$n = \frac{\rho}{A} N_{\mathrm{A}}$$

Simulation process

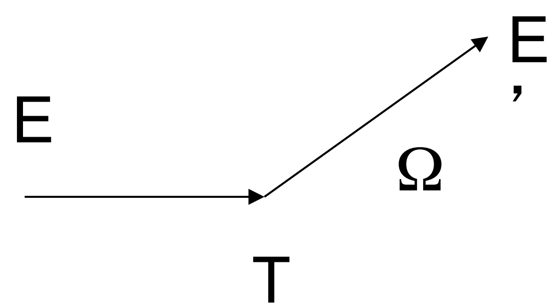
Simulation steps (cont.)

- Step length sampling

$$s = -\frac{1}{\mu(E)} \log(1 - r)$$

- Particle is displaced

$$\vec{x} = \vec{x}_0 + \vec{u}_0 s,$$

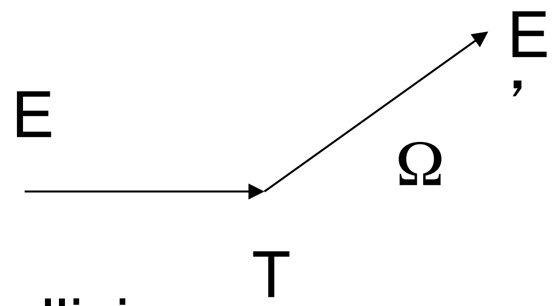


Simulation process

Simulation steps (cont.)

- Sampling of the particle energy after collision E’

$$m(E,E') = \int \mathrm{d}\Omega \, p(E,E',\Theta,\Phi)$$



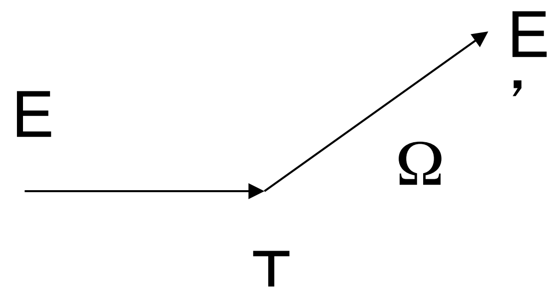
- Sampling of particle direction after collision

$$p(E,\Theta,\Phi|E') = \frac{p(E,E',\Theta,\Phi)}{m(E,E')}$$

Simulation process

Simulation steps (cont.)

- Particle direction is rotated



$$\mathcal{R}(\{\vec{x}_0, \vec{u_0}\}, \Theta, \Phi) = \{\vec{x}_0, \Re(\theta_0, \phi_0)\Re(\Theta, \Phi)\Re^{-1}(\theta_0, \phi_0)\vec{u}_0\} = \{\vec{x}_0, \vec{u}\}$$

Compounds and mixtures

Determination of attenuation coeficients in compounds and mixtures

$$\mu(E) = \sum_i n_i \sigma_i$$

n_i is the volumetric atomic density for the material with microscopic cross section σ_i.

Compounds and mixtures

Determination of attenuation coefficients according to to the relative weight w_i of the element in question

$$\mu(E) = \sum_i \frac{\tilde{\rho}_i}{A_i} N_A \sigma_i \ . = \sum_i \tilde{\rho}_i \left(\frac{\mu_i}{\rho_i}\right) = \rho \sum_i w_i \left(\frac{\mu_i}{\rho_i}\right)$$

$\tilde{\rho}_i$ is the actual density of the i-th elment in the compound
μ/ρ is the mass attenuation coefficient

$$\frac{\mu(E)}{\rho} = \sum_i w_i \left(\frac{\mu_i}{\rho_i}\right) \qquad w_i \doteq \tilde{\rho}_i/\rho$$

Attenuation coefficients for each interaction

For instance, a photon can undergo several competitive interaction processes (photo electric effect, Compton scattering, pair production, etc.)

The total attenuation coefficient is determined as

$$\mu(E) = \sum_i \mu_i(E)$$

where i runs for all kind of possible processes

Attenuation coefficients for each interaction

Interation probability per unit pathlenth

$$p(z) = \sum_i p_i(z) = \sum_i \mu_i(E) e^{-z\mu(E)} = \sum_i \mu_i(E) p_{\mathrm{s}}(z)$$

Fractional probability for i-th event

$$P_i = \frac{p_i(z)}{p(z)} = \frac{\mu_i(E)}{\mu(E)}$$

Attenuation coefficients for each interaction

Sampling of the current event

- A random number r is generated

$r \leq \mu_1(E)/\mu(E)$ Event type 1 occurs

- Else

$r \leq [\mu_1(E) + \mu_2(E)]/\mu(E)$ Event type 2 occurs 2

- Else

$r \leq [\mu_1(E) + \mu_2(E) + \mu_3(E)]/\mu(E)$ Event type 3 occurs

And so on…

Interaction models

Isotropic scattering

$$p(\Theta, \Phi)\, \mathrm{d}\Theta\, \mathrm{d}\Phi = \frac{1}{4\pi} \sin\Theta\, \mathrm{d}\Theta\, \mathrm{d}\Phi$$

Expressions for sampling

$$\begin{aligned} \cos\Theta &= 1 - 2r_1 \\ \Phi &= 2\pi r_2 \end{aligned}$$

Interaction models

Semi-isotropic scattering

$$p(\Theta,\Phi)\,\mathrm{d}\Theta\,\mathrm{d}\Phi = \frac{1}{4\pi}(1+a\cos\Theta)\sin\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi$$

Expression for sampling

$$\begin{aligned}\cos\Theta &= \frac{2-a-4r_1}{1+\sqrt{1-a(2-a-4r_1)}}\\ \Phi &= 2\pi r_2\end{aligned}$$

Interaction models

Small angle Rutherford scattering

$$p(\Theta,\Phi)\,\mathrm{d}\Theta\,\mathrm{d}\Phi = \frac{2a}{\pi}\frac{\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi}{(\Theta^2+2a)^2} \qquad 0 \le a < \infty\ , 0 \le \Theta < \infty$$

Expressions for sampling

$$\begin{aligned} \Theta &= \sqrt{\frac{2ar}{1-r}} \\ \Phi &= 2\pi r_2 \end{aligned}$$

If $\theta > \pi$, the angle is rejected and the process is repeated