# Geometrical modeling

The regions within a given geomtrical model are defined by surfaces, which represent interfaces that particle will traverse during its transport.

# Geometrical modeling

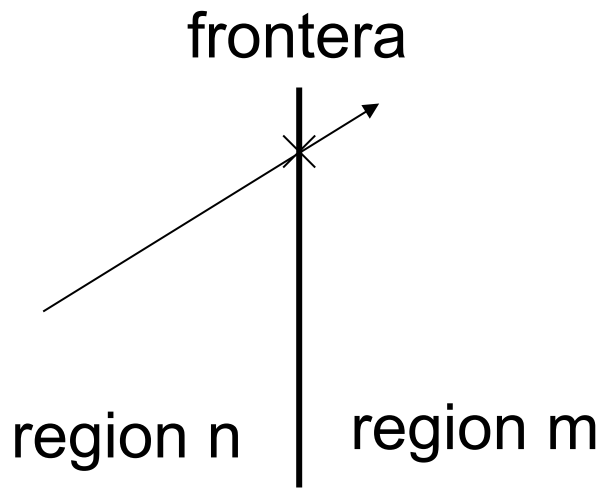
The geometrical problem in Monte Carlo simulations

- If a particle travels a distance s from position $X_0$ with direction μ, does it intersect any frontier?
- If the answer to the last question is positive, what is the distance to the point of intersection between the particle trajectory and the frontier?
- Furthermore, what is the new region which the particle enters in?

# Geometrical modeling

Interface crossing

- Determination of the distance to the interface.
- Determination of the new region which the particle enters in.



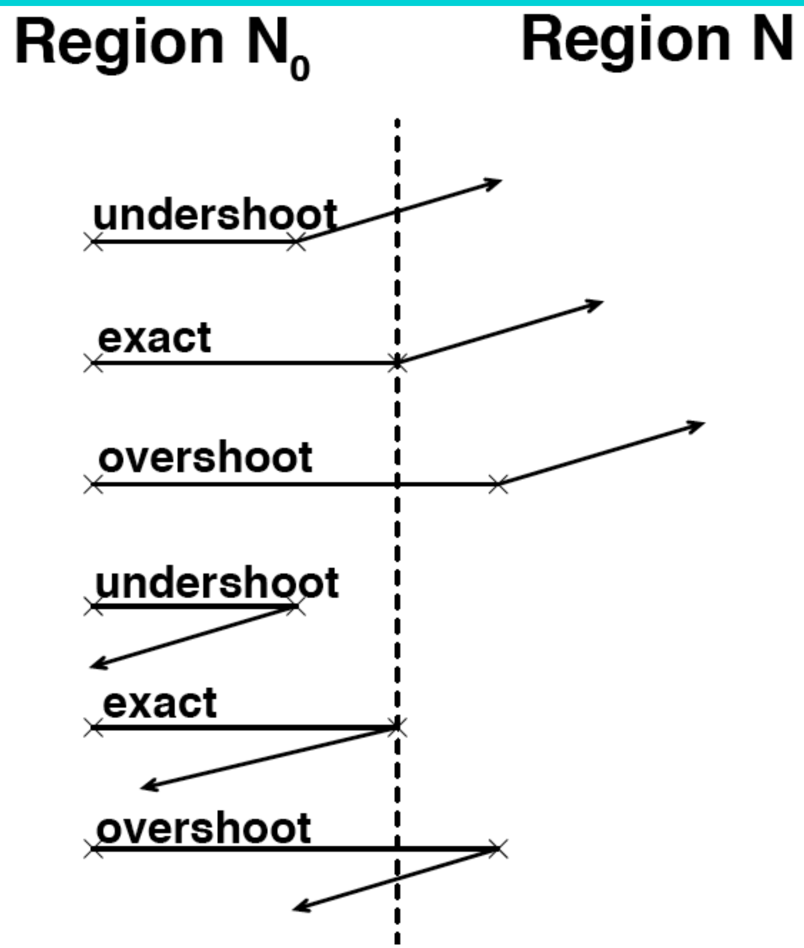
# Geometrical modeling

Precision problems

- It is important to use double precision floating point variables to represent all quantities associated to the geometrical problem (positions, directions, surface description, etc.).
- The precision is not infinitely small so it is possible that the particle falls before (undershoot) or after (overshoot) the surface when updating its position.

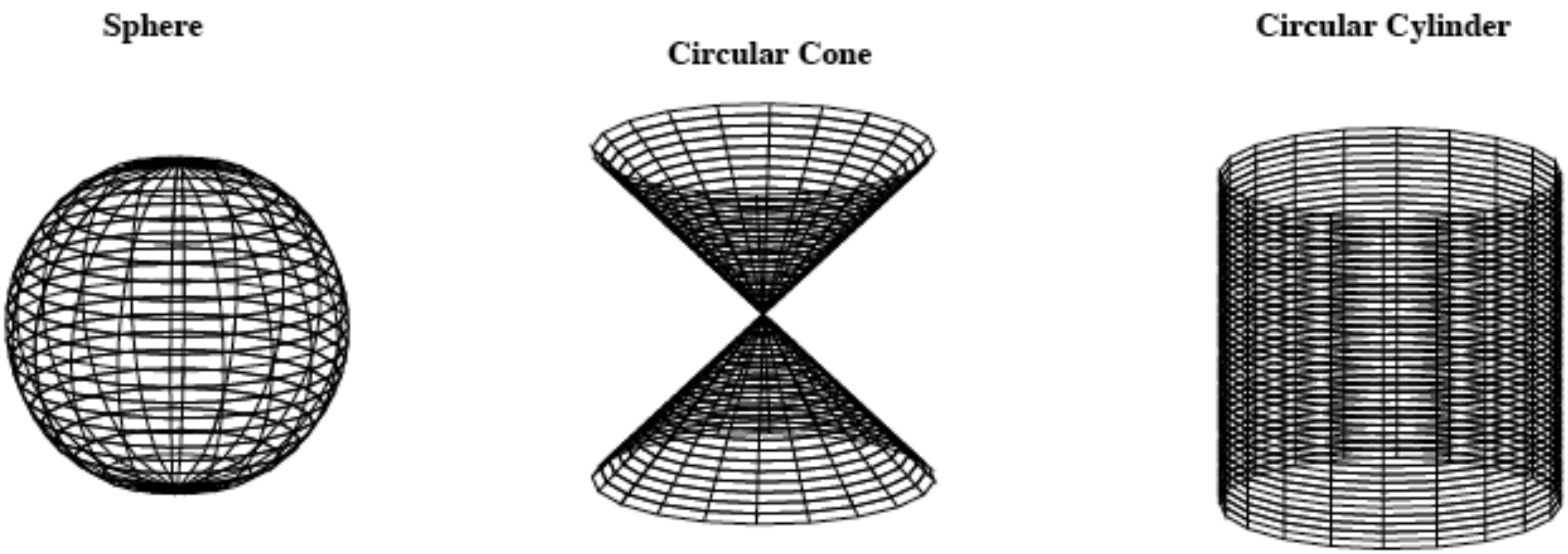
# Geometrical modeling

Possible approaches to the surface



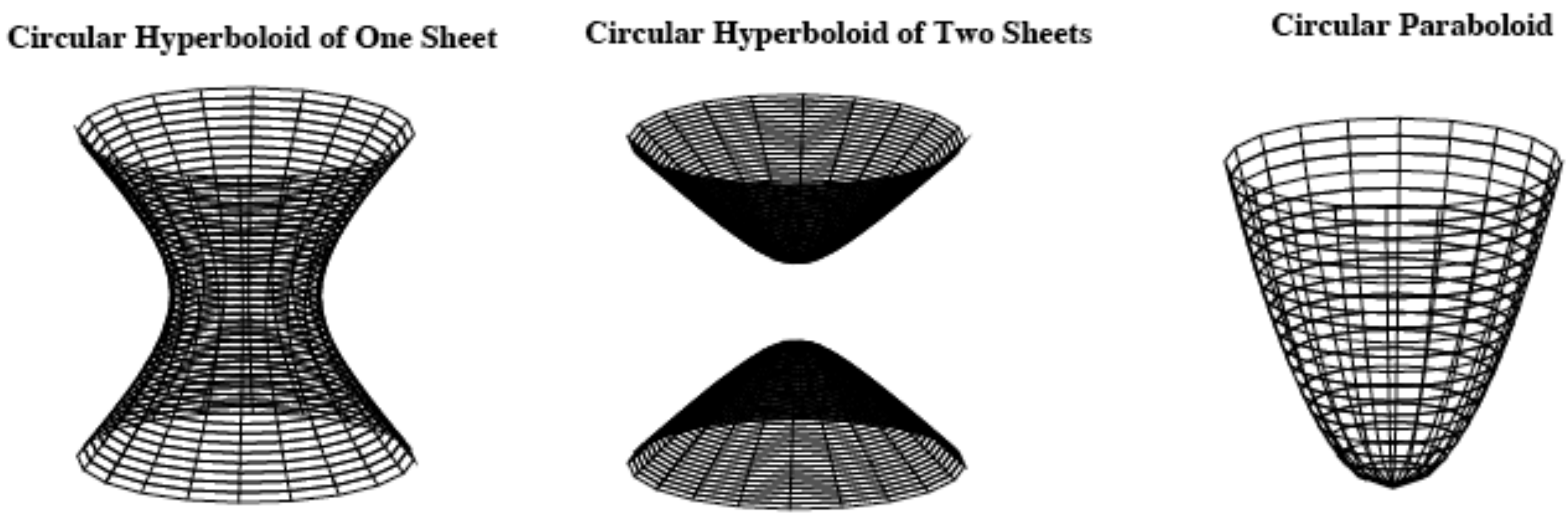
# Geometrical modeling

Most common surfaces in Monte Carlo simulations



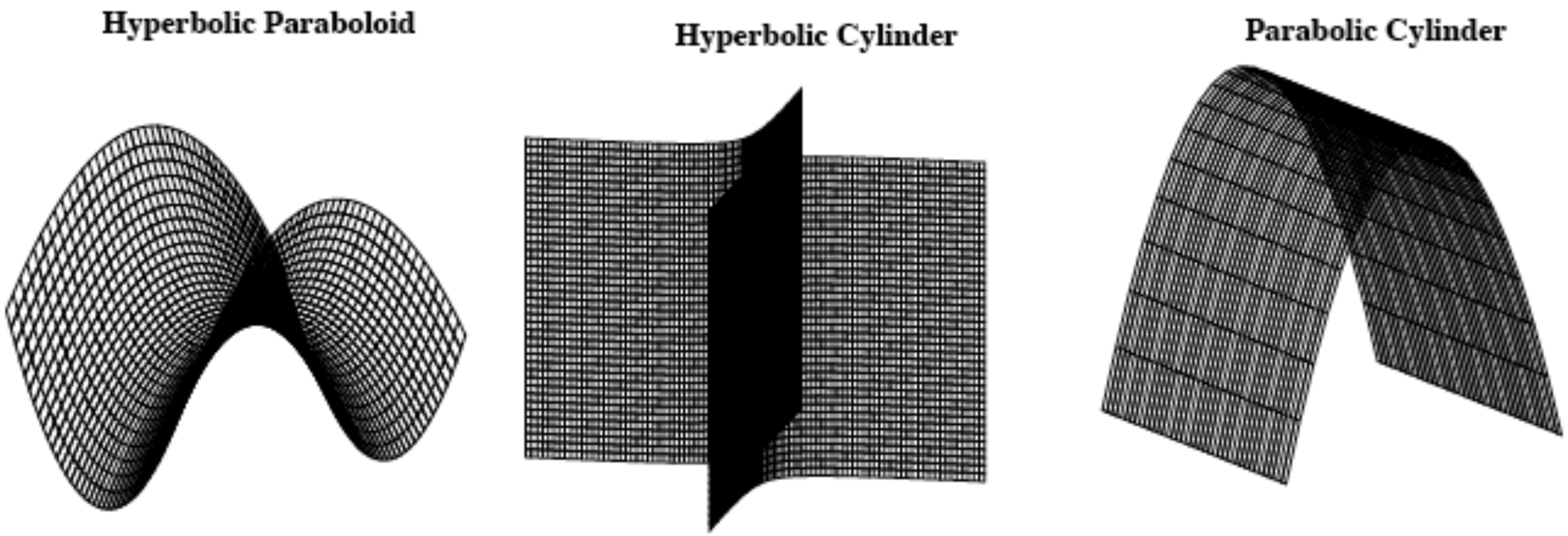
# Geometrical modeling

Most common surfaces in Monte Carlo simulations



# Geometrical modeling

Most common surfaces in Monte Carlo simulations



# Geometrical modeling

Mathematical expressions for surface definition

1. **ellipsoids:** $a_1^2 x_1^2 + a_2^2 x_2^2 + a_3^2 x_3^2 - c^2 = 0.$

2. **cones:** $a_1^2 x_1^2 + a_2^2 x_2^2 - a_3^2 x_3^2 = 0.$

3. **cylinders:** $a_1^2 x_1^2 + a_2^2 x_2^2 - c^2 = 0.$

4. **hyperboloids of one sheet:** $a_1^2 x_1^2 + a_2^2 x_2^2 - a_3^2 x_3^2 - c^2 = 0.$

5. **hyperboloids of two sheets:** $a_1^2 x_1^2 + a_2^2 x_2^2 - a_3^2 x_3^2 + c^2 = 0.$

# Geometrical modeling

Mathematical expressions for surface definition

6. elliptic paraboloids: $a_1^2 x_1^2 + a_2^2 x_2^2 + a_3 x_3 = 0.$

7. hyperbolic paraboloids: $a_1^2 x_1^2 - a_2^2 x_2^2 + a_3 x_3 = 0.$

8. hyperbolic cylinders: $a_1^2 x_1^2 - a_2^2 x_2^2 + c^2 = 0.$

9. parabolic cylinders: $a_1^2 x_1^2 + a_3 x_3 = 0.$

10. simple planes: $a_3 x_3 + c = 0.$

# Geometrical modeling

Intersection between the particle trajectory and a surface

General quadratic equation for a surface

$$f(\vec{x}) = \sum_{i,j=0}^{3} a_{ij} x_i x_j = 0.$$

Trajectory equation

$$\vec{x} = \vec{p} + \vec{\mu} s$$

Equation to be solved

$$s^2 \left( \sum_{i,j=0}^{3} a_{ij} \mu_i \mu_j \right) + 2s \left( \sum_{i,j=0}^{3} a_{ij} p_i \mu_j \right) + \left( \sum_{i,j=0}^{3} a_{ij} p_i p_j \right) = 0.$$

# Geometrical modeling

Intersection between the particle trajectory and a surface

Equation to be solved

$$A(\vec{\mu})s^2 + 2B(\vec{\mu},\vec{p})s + C(\vec{p}) = 0$$

$$A(\vec{\mu}) = \sum_{i,j=0}^{3} a_{ij}\mu_i\mu_j$$
$$B(\vec{\mu},\vec{p}) = \sum_{i,j=0}^{3} a_{ij}p_i\mu_j$$
$$C(\vec{p}) = \sum_{i,j=0}^{3} a_{ij}p_ip_j$$

# Geometrical modeling

Possible solutions

**IF** $\underline{B^2 - AC < 0}$ Particle does not intersect the surface.

**ELSEIF** <u>The particle thinks it is outside</u>

**IF** $\underline{B \geq 0}$

**IF** $\underline{A \geq 0}$ No solution.

**ELSE** $s = -(B + \sqrt{B^2 - AC})/A.$

**ELSE** $s = \max(0, C/[\sqrt{B^2 - AC} - B]).$

**ELSE** <u>The particle thinks it is inside</u>

**IF** $\underline{B \leq 0}$

**IF** $\underline{A > 0}$ $s = (\sqrt{B^2 - AC} - B)/A.$

**ELSE** No solution.

**ELSE** $s = \max(0, -C/[\sqrt{B^2 - AC} + B]).$

# Geometrical modeling

Sphere example

General sphere equation

$$(\vec{x}-\vec{X})^2 - R^2 = (x-X)^2 + (y-Y)^2 + (z-Z)^2 - R^2 = 0$$

Equation to be solved $As^2 + 2Bs + C = 0$

$$\begin{aligned} A &= 1 \\ B &= \vec{\mu}\cdot(\vec{x}_0-\vec{X}) \\ &= u(x_0-X)+v(y_0-Y)+w(z_0-Z) \\ C &= (\vec{x}_0-\vec{X})^2 - R^2 \\ &= (x_0-X)^2+(y_0-Y)^2+(z_0-Z)^2-R^2 \end{aligned}$$