Elementary probability theory

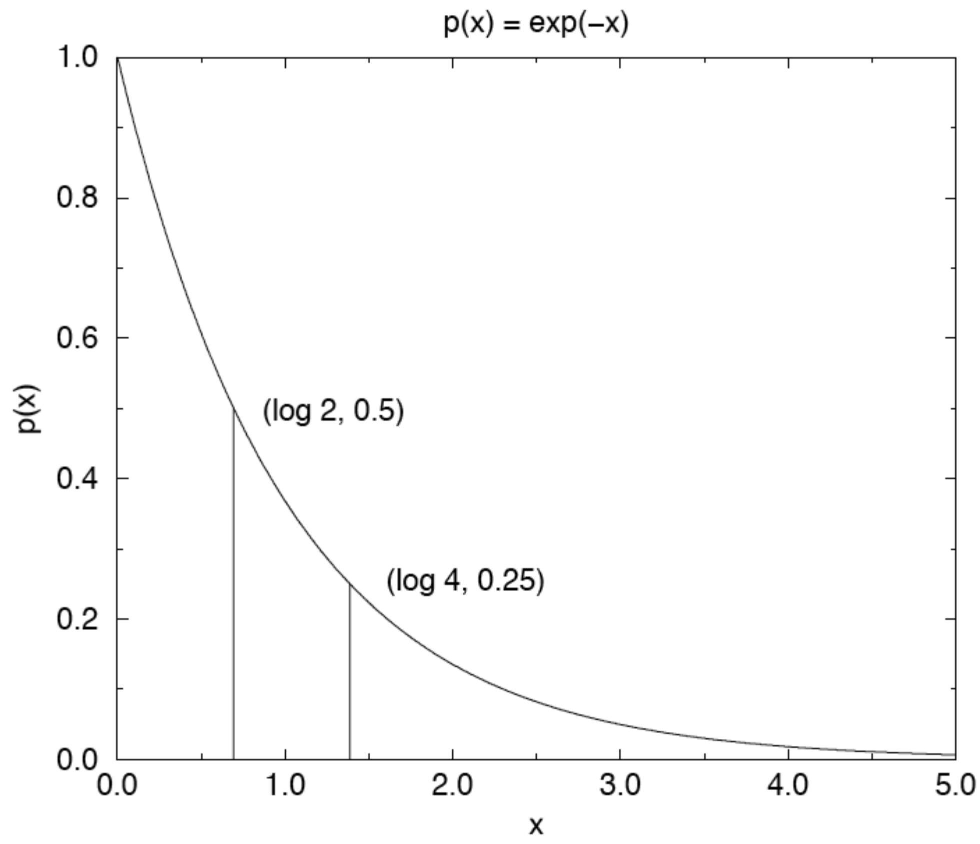
Continuous random variables.
Probability density function

$$p(x) \geq 0$$

$$\int_{x_{\min}}^{x_{\max}} \mathrm{d}x \; p(x) = 1$$

$$-\infty < x_{\min} < x_{\max} < +\infty$$

Probability distribution function



Elementary probability theory

Continuous random variables.
Distribution moments

$$\langle x^n \rangle = \int_{x_{\min}}^{x_{\max}} \mathrm{d}x \; x^n p(x)$$

Variance

$$\mathrm{var}\{x\} = \langle x^2 \rangle - \langle x \rangle^2$$

Elementary probability theory

Continuous random variables.
Examples of distributions.

Cauchy/Lorentz

$$p(x) = \frac{1}{\pi}\frac{\Gamma}{\Gamma^2 + x^2} \qquad -\infty < x < \infty$$

Rutherford/Wentzel

$$p(\mu) = \frac{a(2+a)}{2}\frac{1}{(1-\mu+a)^2} \quad ; \quad -1 \leq \mu \leq 1$$

μ is the cosine of the scattering angle, $\cos\Theta$

$$\langle\Theta\rangle = \pi\sqrt{a/2}$$

Elementary probability theory

Continuous random variables.
Two-variable functions. $f(x, y)$

Moments $$\langle x^n y^m \rangle = \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} \mathrm{d}x\mathrm{d}y \; x^n y^m p(x, y)$$

Co-variance $$\mathrm{cov}\{x, y\} = \langle xy \rangle - \langle x \rangle \langle y \rangle$$

If x and y are independent

$$p(x, y) = p_1(x) p_2(y) \text{ and } \mathrm{cov}\{x, y\} = 0.$$

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Two-variable functions. $f(x, y)$

Correlation coefficient
$$\rho\{x, y\} = \frac{\text{cov}\{x, y\}}{\sqrt{\text{var}\{x\}\text{var}\{y\}}}$$

Interesting relation
$$\text{var}\{x \pm y\} = \text{var}\{x\} + \text{var}\{y\} \pm 2\ \text{cov}\{x, y\}$$

Elementary probability theory

Continuous random variables

Marginal probabilities

$$m(x) = \int_{y_{\min}}^{y_{\max}} \mathrm{d}y \; p(x,y) \quad ; \quad m(y) = \int_{x_{\min}}^{x_{\max}} \mathrm{d}x \; p(x,y)$$

Conditional probability

$$p(y|x) = \frac{p(x,y)}{m(x)}$$

Elementary probability theory

Continuous random variables.

Cumulative probability functions
$$c(x) = \int_{x_{\min}}^{x} \mathrm{d}x' \, p(x')$$

$$p(x) = \frac{\mathrm{d}c(x)}{\mathrm{d}x} \qquad c(x_{\min}) = 0 \qquad c(x_{\max}) = 1$$