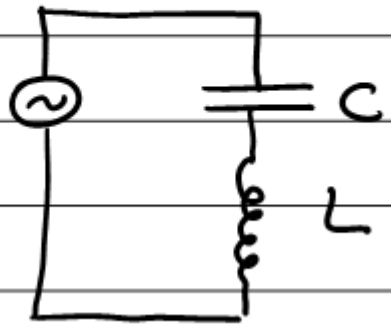
QUANTIZAÇÃO DO CAMPO ELETROMAGNÉTICO

Circuito LC com fonte:

1ª Lei das malhas (Kirchoff): $V(t)$



$$V(t) - V_C - V_L = 0$$

$$V(t) - \frac{q}{C} - L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q(t)}{dt^2} + \omega_0^2\, q(t) = \frac{1}{L}V(t), \quad \omega_0^2 = \frac{1}{LC}$$

↳ O.H. forçado

$q \equiv$ "posição"

$$V_C = q/C$$

$$\frac{dq}{dt} = I_L = \frac{1}{L}\int V_L\, dt$$

$$\frac{d^2q}{dt^2} = \frac{V_L}{L}$$

variável conjugada: $p(t) = L\frac{dq}{dt} = LI(t)$

eq. Hamilton:

$$\begin{cases} L\frac{dq}{dt} = p(t) \\ \frac{dp}{dt} = -\frac{1}{C}q + V(t) \end{cases}$$

Como $\frac{dq}{dt} = \frac{\partial H}{\partial p} = \frac{1}{L}p$; $\frac{dp}{dt} = -\frac{\partial H}{\partial q} = -\frac{1}{C}q + V(t)$

$\hookrightarrow$ $H(t) = \frac{1}{2L}p^2 + \frac{1}{2C}q^2 - V(t)q$

quantização: $q, p \rightarrow$ op. Hermitianos/

$$[q, p] = i\hbar$$

Definindo $a \equiv (2\hbar\omega_0 L)^{-1/2}[\omega_0 L q + ip]$

$a^\dagger \equiv (2\hbar\omega_0 L)^{-1/2}[\omega_0 L q - ip]$

$$p = i\sqrt{\frac{\hbar\omega_0 L}{2}}(a^\dagger - a); \quad q = \sqrt{\frac{\hbar\omega_0 C}{2}}(a + a^\dagger)$$

$\omega_0 L = (\omega_0 C)^{-1} \rightarrow [a, a^\dagger] = 1$

$$H(t) = \hbar\omega_0(a^\dagger a + 1/2) - \frac{\hbar V(t)}{\sqrt{2\hbar\omega_0 L}}(a^\dagger + a)$$

$$f(t) \equiv \frac{-V(t)}{2\hbar\omega_0 L} = f^*(t)$$

$\hookrightarrow$ $$H(t) = \hbar\omega_0 (a^\dagger a + 1/2) + \hbar (f(t)\, a^\dagger + f^*(t)\, a)$$

$\hookrightarrow$ $$|\psi(t)\rangle = e^{A(t)}\, e^{\alpha(t) a^\dagger}\, \mathcal{N}\{e^{\beta(t)\, a^\dagger a}\}\, e^{B(t) a}\, |\psi(0)\rangle$$

$\hookrightarrow$ ordenamiento normal (Louisell 3.11)

$$A(t) = -\int_0^t dt'' f(t'') \int_0^{t''} e^{i\omega_0 (t'-t'')} f(t')\, dt'$$

$$B(t) = -i \int_0^t e^{-i\omega_0 t'} f(t')\, dt'$$

$$\alpha(t) = -e^{i\omega_0 t} B^*(t) \; ; \quad \beta(t) = e^{-i\omega_0 t} - 1$$

p/ $|\psi(0)\rangle = |0\rangle \rightarrow$

$$|\psi(t)\rangle = e^{A(t)} e^{\alpha(t) a^{+}} |0\rangle \equiv \text{Estado coerente}$$

Como $A(t) + A^{*}(t) = -|\alpha(t)|^2$

$\hookrightarrow \langle \psi(t) | \psi(t) \rangle = 1$

$$\rightarrow e^{\alpha a^{+}} |0\rangle = \sum_{\ell=0}^{\infty} \frac{\alpha^{\ell} a^{+\ell}}{\ell!} |0\rangle = \sum_{\ell=0}^{\infty} \frac{\alpha^{\ell}\sqrt{\ell!}}{\ell!} |\ell\rangle$$

$$= \sum_{\ell=0}^{\infty} \frac{\alpha^{\ell}}{\sqrt{\ell!}} |\ell\rangle$$

$\rightarrow$ Prob de medida de $\underline{a^{+}a} \equiv$ nº fótons/

$$a^{+}a|m\rangle = m|m\rangle$$

$$P_{m,0}(t) = \mathrm{Tr}\{ |m\rangle\langle m| \psi(t)\rangle\langle\psi(t)| \}$$

$$= |\langle m|\psi(t)\rangle|^2 = \langle 0| e^{A^*(t)} e^{\alpha(t)a} |m\rangle\langle m| e^{A(t)} e^{\alpha(t)a^+} |0\rangle$$

$$= e^{-|\alpha(t)|^2} \langle 0| e^{\alpha^*(t)a} |m\rangle\langle m| e^{\alpha(t)a^+} |0\rangle$$

$$= e^{-|\alpha(t)|^2} |\langle m| e^{\alpha(t)a^+} |0\rangle|^2$$

como $\langle l|m\rangle = \delta_{l,m}$

$\hookrightarrow$ $$P_{m,0}(t) = e^{-|\alpha(t)|^2} \frac{[|\alpha(t)|^2]^m}{m!}$$

$\hookrightarrow$ dist. Poisson

$$\langle a^\dagger a \rangle = \sum_{m=0}^{\infty} m P_{m,0}(t) = |\alpha(t)|^2$$

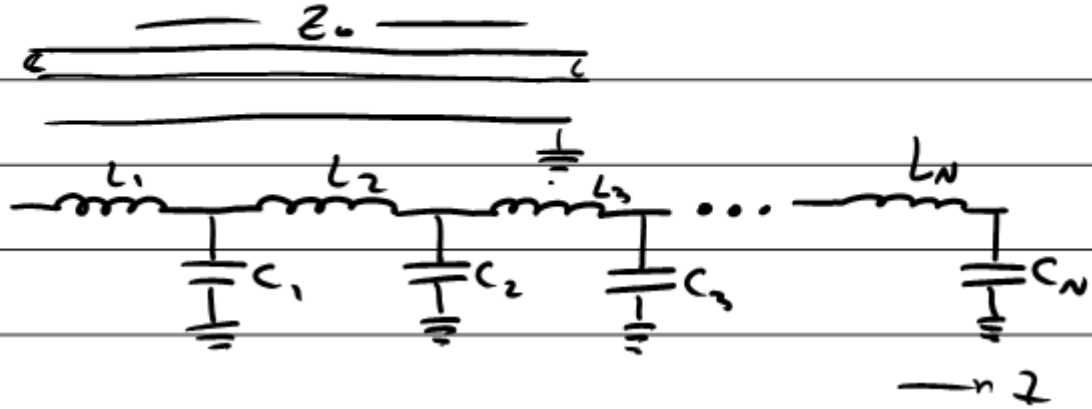
$$\langle (a^\dagger a)^2 \rangle = \sum_{m=0}^{\infty} m^2 P_{m,0}(t) = \langle a^\dagger a \rangle [\langle a^\dagger a \rangle + 1]$$

$$\therefore \Delta^2 \langle a^\dagger a \rangle = \langle (a^\dagger a)^2 \rangle - \langle a^\dagger a \rangle^2 = \langle a^\dagger a \rangle = |\alpha(t)|^2$$

$$|\alpha(t)\rangle = e^{-\frac{1}{2}|\alpha^2(t)|^2} \sum_{\ell=0}^{\infty} \frac{\alpha^\ell(t)}{\sqrt{\ell!}} |\ell\rangle$$

$\Rightarrow$ mínima incerteza de corrente e carga.

Linha de transmissão sem perdas



$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \quad ; \quad \frac{\partial I}{\partial t} = -C\frac{\partial V}{\partial t}$$

L, C = ind., cap. / unidade de comprimento

$$\Rightarrow \frac{\partial^2 V}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2 V}{\partial t^2} \quad ; \quad \frac{\partial^2 I}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2 I}{\partial t^2}$$

$$\Rightarrow \quad C^2 = \frac{1}{LC}$$

$\hookrightarrow$ $V(z,t) = \sqrt{\frac{L}{C}}\, I(z,t) = \sqrt{\frac{\hbar\omega}{2C z_0}}\left(a e^{-i\omega t + ikz} + a^* e^{i\omega t - ikz}\right)$

z_0 = comp. linha transm. , $a, a^* = \underline{cte}$

Energia:

$$H = \frac{1}{2}\int_0^{z_0} dz\, \left[C V^2(z,t) + L I^2(z,t)\right]$$

$$= \int_0^{z_0} dz\, C V^2(z,t)$$

$$= \hbar\omega\, a^* a \; ; \quad z_0 = m\lambda = 2\pi \frac{c}{\omega} m$$

quant. $\hookrightarrow$ $a \to \hat{a}$ e $a^* \to \hat{a}^+ \,/\, [\hat{a}, \hat{a}^+] = 1$

$$H = \hbar\omega\, \hat{a}^+ \hat{a}$$

Eq. Maxwell (sem fontes)

$$\nabla \cdot \mathbb{B} = 0 \quad , \quad \nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t}$$

$$\nabla \cdot \mathbb{D} = 0 \quad \nabla \times \mathbb{H} = \frac{\partial \mathbb{D}}{\partial t}$$

$$\mathbb{D} = \epsilon_0 \mathbb{E} \quad ; \quad \mathbb{B} = \mu_0 \mathbb{H} \quad , \quad \mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\nabla \cdot \mathbb{B} = 0 \rightarrow \mathbb{B} = \nabla \times \mathbb{A}$$

$$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \rightarrow \mathbb{E} = -\frac{\partial \mathbb{A}}{\partial t} - \nabla V$$

<u>Gauge de Coulomb</u> : $\nabla \cdot \mathbb{A} = 0$, $V = 0$

$$\left(\rightarrow \mathbb{B} = \nabla \times \mathbb{A} ; \mathbb{E} = -\frac{\partial \mathbb{A}}{\partial t} \right.$$

$\hookrightarrow$ $$\nabla^2 \mathbb{A} = \frac{1}{c^2}\frac{\partial^2 \mathbb{A}}{\partial t^2}$$ $\rightarrow$ eq. onda (vácuo)

Cavidade de volume V:

Energia:

$$H = \frac{1}{2}\int_V \left(\epsilon_0 \mathbb{E}^2 + \frac{1}{\mu_0}\mathbb{B}^2\right) dx\,dy\,dz$$

$$= \frac{1}{2}\int_V \left[\epsilon_0 \left(\frac{\partial \mathbb{A}}{\partial t}\right)^2 + \frac{1}{\mu_0}\left(\nabla \times \mathbb{A}\right)^2\right] d^3r$$

$\hookrightarrow$ seja $V = L^3$ $\Rightarrow$ (cavidade cúbica)

$$\mathbb{A}(\vec{r},t) = \frac{1}{\sqrt{\epsilon_0}} \sum_{\ell} q_{\ell}(t)\, \vec{u}_{\ell}(\vec{r})$$

ℓ ≡ modo normal.

$$\hookrightarrow \begin{cases} \nabla^2 \vec{u}_{\ell}(\vec{r}) + \dfrac{\omega_{\ell}^2}{c}\, \vec{u}_{\ell}(\vec{r}) = 0 \\[2ex] \dfrac{d^2 q_{\ell}}{dt^2} + \omega_{\ell}\, q_{\ell}^2 = 0 \end{cases}$$

ω_{ℓ} ≡ cte de separação p/ cada ℓ.

Soluções estacionárias se comp. tangencial $\mathbb{E} = 0$ e comp. normal $\mathbb{B} = 0$ nas paredes da cavidade.

$\Leftrightarrow$ $\vec{u}_\ell \big|_{tang} = 0$ e $\nabla \times \vec{u}_\ell \big|_{norm} = 0$

nas paredes da cavidade

$$\mathbb{E} = -\frac{\partial \mathbb{A}}{\partial t} = -\frac{1}{\sqrt{\epsilon_0}} \sum_\ell \frac{\partial q_\ell(t)}{\partial t} \vec{u}_\ell(\vec{r})$$

$$\mathbb{B} = \nabla \times \mathbb{A} = \frac{1}{\sqrt{\epsilon_0}} \sum_\ell q_\ell(t) \nabla \times \vec{u}_\ell(r)$$

sendo que $\nabla \cdot \vec{u}_\ell = 0$ em toda a cavidade

$\therefore$ soluções devem satisfazer:

$$\int_V \vec{u}_\ell(\vec{r}) \cdot \vec{u}_m(\vec{r})\, d^3r = \delta_{\ell m}$$

$\vec{u}_\ell(\vec{r})$ depende da geometria:

Cúbica: $\vec{u}_\ell(\vec{r}) = \operatorname{sen} \vec{k}\cdot\vec{r}$; $(\cos \vec{k}\cdot\vec{r})$.

$q_\ell \to$ eq. oscilador harmônico $\to$

Cada modo campo $\to$ 1 o.H. freq. ω_ℓ

Energia: $H = \frac{1}{2}\int d^3r \left[\epsilon_0 \left(\frac{\partial A}{\partial t}\right)^2 + \frac{1}{\mu_0}(\nabla\times A)^2 \right]$

$$= \frac{1}{2}\sum_{\ell,m} \dot{q}_\ell \dot{q}_m \int_V d^3r\, \vec{u}_\ell \cdot \vec{u}_m$$

$$+ \frac{c}{2}\sum_{\ell,m} q_\ell q_m \int_V d^3r\, (\nabla\times\vec{u}_\ell)\cdot(\nabla\times\vec{u}_m)$$

condição de ortonormalidade:

① $\rightarrow \quad \frac{1}{2} \sum_{\ell} (\dot{q}_\ell)^2$

②

$$(\nabla \times \vec{u}_\ell) \cdot (\nabla \times \vec{u}_m) = \vec{u}_m \cdot \nabla \times \nabla \times \vec{u}_\ell + \nabla \cdot (\vec{u}_m \times \nabla \times \vec{u}_\ell)$$

mas

$$\int_V d^3r \quad \nabla \cdot (\vec{u}_m \times \nabla \times \vec{u}_\ell)$$

$$= \int_S (\vec{u}_m \times \underline{\underline{\nabla \times \vec{u}_\ell}}) \cdot d\vec{S}$$

$$= 0 \qquad \left[\nabla \times \vec{u}_\ell \Big|_{\text{norm}} = 0 \right]$$

$$-\int dV \; \vec{u}_m \cdot \left(\nabla(\nabla \cdot \vec{u}_\ell) - \nabla^2 \vec{u}_\ell \right) dV$$

$$= \frac{\omega_\ell^2}{c^2} \int \vec{u}_\ell \cdot \vec{u}_m \, d^3r \qquad \left\{ \begin{array}{l} \nabla \cdot \vec{u}_\ell(\vec{r}) = 0 \\ \nabla^2 \vec{u}_\ell + \frac{\omega_\ell^2}{c^2} \vec{u}_\ell(\vec{r}) = 0 \end{array} \right.$$

$$= \frac{\omega_\ell^2}{c^2} \delta_{\ell m}$$

$$\therefore \qquad H = \frac{1}{2} \sum_\ell \left(\dot{q}_\ell^2 + \omega_\ell^2 q_\ell^2 \right) = \sum_\ell H_\ell$$

H_ℓ = osc. harmônico freq. $\frac{\omega_\ell}{2}$.

Eq. Hamilton

$$\frac{\partial H_\ell}{\partial q_\ell} = -\dot{p}_\ell = \omega_\ell^2 q_\ell \; ; \; \frac{\partial H_\ell}{\partial p_\ell} = \dot{q}_\ell = p_\ell$$

Seja $\quad q_k = \sqrt{\frac{\hbar}{2\omega_k}}\,(a_k^* + a_k)$

$$p_k = i\sqrt{\frac{\hbar\omega_k}{2}}\,(a_k^* - a_k)$$

$$a = \frac{1}{\sqrt{2\hbar\omega_k}}\,(\omega_k q_k + i p_k)$$

$$a^* = \frac{1}{\sqrt{2\hbar\omega_k}}\,(\omega_k q_k - i p_k)$$

$$\rightarrow \frac{da_k(t)}{dt} = -i\omega_k a_k(t) \; ; \; \frac{da_k^*(t)}{dt} = i\omega_k a_k^*(t)$$

Então $\quad a_k(t) = a_k e^{-i\omega_k t}$

$$a_k^*(t) = a_k^* e^{i\omega_k t}$$

$$H = \frac{1}{2}\sum_k \hbar\omega_k (a_k^* a_k + a_k a_k^*) = \sum_k \hbar\omega_k a_k^* a_k$$

$$\mathbb{A} = \frac{1}{\sqrt{\epsilon_0}} \sum_\ell q_\ell(t)\, \vec{u}_\ell(\vec{r})$$

$$\mathbb{E} = -\frac{1}{\sqrt{\epsilon_0}} \sum_\ell p_\ell(t)\, \vec{u}_\ell(\vec{r})$$

$$\mathbb{B} = \frac{1}{\mu_0} \frac{1}{\sqrt{\epsilon_0}} \sum_\ell q_\ell(t)\, \vec{\nabla} \times \vec{u}_\ell(\vec{r})$$

quantizacja: $a^* \to a^+$

$$[a, a^+] = 1$$

$$H = \sum_\ell \hbar \omega_\ell \left(a_\ell^+ a_\ell + \frac{1}{2} \right)$$

Representação em termos de ondas planas

$$\mathbb{A}(\vec{r},t) = \sum_{\ell} \sum_{\sigma=1}^{2} \sqrt{\frac{\hbar}{2\omega_{\ell}\epsilon_0 V}}\, \hat{e}_{\ell\sigma} \left\{ a_{\ell\sigma}\, e^{i(\vec{k}\cdot\vec{r}-\omega_{\ell}t)} + a^{*}_{\ell\sigma}\, e^{-i(\vec{k}\cdot\vec{r}-\omega_{\ell}t)} \right\}$$

σ: polarização $\to$ $\hat{e}_{\ell 1}$, $\hat{e}_{\ell 2}$:

$$\hat{e}_{\ell 1}\cdot\hat{e}_{\ell 2} = 0$$

$$\vec{k}_{\ell}^{\,2} = \frac{\omega_{\ell}^{2}}{c^{2}} \quad , \quad \nabla\cdot\mathbb{A} = 0 \iff \hat{e}_{\ell\sigma}\cdot\vec{k}_{\ell} = 0$$

Cond. periódicas de contorno: Cavidade Cúbica.

$$\mathbb{A}(\vec{r}+L\hat{i},t) = \mathbb{A}(\vec{r}+L\hat{j},t) = \mathbb{A}(\vec{r}+L\hat{k},t) = \mathbb{A}(\vec{r},t)$$

$$\Rightarrow \quad \vec{k}_l = \frac{2\pi}{L}\left(l_1\hat{i} + l_2\hat{j} + l_3\hat{k}\right)$$

$$\hookrightarrow \quad H = \frac{1}{2}\sum_{l\sigma} \hbar\omega_l \left(a_{l\sigma}a^*_{l\sigma} + a^*_{l\sigma}a_{l\sigma}\right)$$

$$= \frac{1}{2}\sum_{l,\sigma}\left(p^2_{l\sigma} + \omega_l^2 q^2_{l\sigma}\right)$$

quant. $\hookrightarrow$

$$H = \sum_{l,\sigma} \hbar\omega_l \left(a^+_{l\sigma}a_{l\sigma} + \frac{1}{2}\right)$$

obs:

$$a_{k\sigma}(t) \equiv a_{k\sigma} e^{-i\omega_k t} \quad ; \quad a^{+}_{k\sigma}(t) \equiv a^{+}_{k\sigma} e^{i\omega_k t}$$

e $\vec{u}_{k\sigma} = \hat{e}_{k\sigma} e^{i\vec{k}\cdot\vec{r}}$; $\vec{u}^{*}_{k\sigma} = \hat{e}_{k\sigma} e^{-i\vec{k}\cdot\vec{r}}$ /

$$\int d^3r \; \vec{u}^{*}_{k\sigma}(\vec{r}) \cdot \vec{u}_{k'\sigma'}(\vec{r}) = \delta_{kk'}\delta_{\sigma\sigma'}$$

agora: $a_{k\sigma} = \frac{1}{\sqrt{2\hbar\omega_k}} (\omega_k q_{k\sigma} + i p_{k\sigma})$

$$a^{+}_{k\sigma} = \frac{1}{\sqrt{2\hbar\omega_k}} (\omega_k q_{k\sigma} - i p_{k\sigma})$$

$$\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}}{\partial t} = i \sum_{k\sigma} \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}} \left[a_{k\sigma}(t) e^{i\vec{k}_k\cdot\vec{r}} - a^{+}_{k\sigma}(t) e^{-i\vec{k}_k\cdot\vec{r}} \right]$$

$$\mathbb{H}(\vec{r},t) = \nabla \times \mathbb{A} = \frac{-i}{c\mu_0} \sum_{k\sigma} \sqrt{\frac{\hbar \omega_k}{2\epsilon_0 V}} \, (\hat{e}_{k\sigma} \times \hat{k}_k) \left[a_{k\sigma}(t) \, e^{i\vec{k}_{k\sigma}\cdot\vec{r}} - a^{+}_{k\sigma} \, e^{-i\vec{k}_k\cdot\vec{r}} \right]$$

$$\hookrightarrow \quad H = \frac{1}{2} \int_V \left(\epsilon_0 |\mathbb{E}|^2 + \mu_0 |\mathbb{H}|^2 \right) d^3r$$

$$= \sum_{k\sigma} \hbar \omega_k \left(a^{+}_{k\sigma} a_{k\sigma} + \frac{1}{2} \right)$$

$$= \frac{1}{2} \sum_{k\sigma} \left(p^2_{k\sigma} + \omega_k^2 q^2_{k\sigma} \right)$$

DENSIDADE DE MODOS

$$\vec{k}_\ell = \frac{2\pi}{L}\left(\ell_1 \hat{i} + \ell_2 \hat{j} + \ell_3 \hat{k}\right)$$

2 polarizações, $\sigma = 1, 2$: $\quad dN = 2\, d\ell_1 d\ell_2 d\ell_3$

$$= 2\left(\frac{L}{2\pi}\right) dk_x\, dk_y\, dk_z$$

$\lim_{L \to \infty}$ (espaço livre) $\to \frac{\ell_i}{L} \to$ contínuo

$$\therefore \quad \frac{1}{L^3} \sum_\ell (\quad) \xrightarrow[L \to \infty]{} \frac{1}{(2\pi)^3} \int dk_x\, dk_y\, dk_z (\quad)$$

$$dk_x\, dk_y\, dk_z = k^2 dk \sin\theta\, d\theta\, d\varphi = k^2 dk\, d\Omega$$

$$\rightsquigarrow dN = 2\left(\frac{L}{2\pi}\right)^3 k^2 dk\, d\Omega.$$

Ou # modos:

$$N = \int dN = 2\left(\frac{L}{2\pi}\right)^3 \int_0^\infty k^2 dk \int_0^{4\pi} d\Omega \to \infty$$

Ou mas # modos entre ω e $\omega + d\omega$ ($\omega^2 = c^2 \vec{k}^2$):

$$k^2 dk = \frac{\omega^2}{c^2} d\omega \to dN = 2\left(\frac{L}{2\pi c}\right)^3 \omega^2 d\omega\, d\Omega$$

$$= 2\left(\frac{L}{c}\right)^3 \nu^2 \, d\nu \, d\Omega \quad ; \quad \nu = \frac{\omega}{2\pi}$$

$\therefore$ # de modos entre ω e $\omega + d\omega$ em um ângulo sólido $d\Omega$

$$g(\omega)\, d\omega = \frac{2\omega^2}{(2\pi c)^3}\, d\omega$$

$\hookrightarrow$ densidade de modos.

Estados de Fock

Autoestados de H:

$$H_k = \hbar\omega_k\left(a_k^\dagger a_k + \frac{1}{2}\right) \; ; \quad a_k^\dagger a_k = \hat{n}_k$$

$$a_k^\dagger a_k |n_k\rangle = n_k |n_k\rangle$$

$$H = \sum_k H_k \rightarrow \quad H_k |n_k\rangle = \hbar\omega_k\left(n_k + \frac{1}{2}\right)|n_k\rangle$$

$$\bigotimes_{k=1}^{\infty} |n_k\rangle = |n_1\rangle \otimes |n_2\rangle \otimes \cdots \otimes |n_k\rangle \otimes \cdots$$

$$\equiv |n_1, n_2, n_3, \ldots, n_k, \ldots\rangle$$

$$a_k |n_k\rangle = \sqrt{n_k}\, |n_k - 1\rangle$$

$$a_k^\dagger |n_k\rangle = \sqrt{n_k + 1}\, |n_k + 1\rangle$$

$$\hookrightarrow |n_k\rangle = \frac{(a_k^\dagger)^{n_k}}{(n_k!)^{1/2}} |0\rangle$$

$$\langle n_k | n_{k'}\rangle = \delta_{mn}\, \delta_{kk'}$$

$$\sum_{n_k=0}^{\infty} |n_k\rangle\langle n_k| = 1$$

Estados Coerentes

$$D(\alpha) = e^{\alpha a^{+} - \alpha^{*} a}$$

Se 2 op. lineares A, B : $[A[A,B]] = [B,[A,B]] = 0$

$$\hookrightarrow \quad e^{A+B} = e^{A} e^{B} e^{-\frac{1}{2}[A,B]}$$

$$\therefore \quad D(\alpha) = e^{-\frac{1}{2}|\alpha|^2} e^{\alpha a^{+}} e^{-\alpha^{*} a}$$

$$D(\alpha)|0\rangle = e^{-\frac{1}{2}|\alpha|^2} e^{\alpha a^{+}} e^{-\alpha^{*} a} |0\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{l=0}^{\infty} \frac{1}{l!} \alpha^{l} a^{+l} |0\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{l=0}^{\infty} \frac{\alpha^l \sqrt{l!}}{l!} |l\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{l=0}^{\infty} \frac{1}{\sqrt{l!}} \alpha^l |l\rangle \equiv |\alpha\rangle$$

Cn propriedade: $|\alpha\rangle$

$$a|\alpha\rangle = e^{-\frac{1}{2}|\alpha|} \sum_{l=0}^{\infty} \frac{\alpha^l}{\sqrt{l!}} a|l\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{l=0}^{\infty} \frac{\sqrt{l}\,\alpha^l}{\sqrt{l!}} |l-1\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{l=1}^{\infty} \frac{1}{\sqrt{(l-1)!}} \alpha^l |l-1\rangle$$

$l-1 \to l$

$$= e^{-\frac{1}{2}|\alpha|} \sum_{l=0}^{\infty} \frac{\alpha^{l+1}}{\sqrt{l!}} |l\rangle = \alpha|\alpha\rangle$$

$\rightarrow$ $a|\alpha\rangle = \alpha|\alpha\rangle$ — $|\alpha\rangle$ é autoestado de a

obs: $D^{\dagger}(\alpha) = D^{-1}(\alpha) = D(-\alpha)$

$\hookrightarrow$ <u>mostrar</u> $D^{\dagger}(\alpha)\, a\, D(\alpha) = a + \alpha$

— $D^{\dagger}(\alpha)\, a^{\dagger} D(\alpha) = a^{\dagger} + \alpha^{*}$

$$D(\alpha+\beta) = D(\alpha)\, D(\beta)\, e^{-i\,\mathrm{Im}(\alpha\beta^{*})}$$

$|\langle\alpha|\alpha\rangle|^2 = 1$ $\rightarrow$ Normalizado.