CHAPTER 8

Spontaneously Broken Symmetry and the Higgs Mechanism

One of the most important developments in quantum field theory during the past three decades has been the realization that there is more than one way that symmetries and broken symmetries can manifest themselves in physical systems. In this chapter we discuss three important modes of symmetry realization: (1) the *Wigner–Weyl mode (Wigner mode)*, (2) the *Nambu–Goldstone mode (Goldstone mode)*, and (3) the *Higgs mode.* In the next chapter we will elaborate on the Higgs mode and introduce yet another method of symmetry realization: (4) *anomalous symmetries*, which are symmetries realized at the classical level but broken at the quantum level.

8.1 Classical Symmetry Modes

The Wigner mode is the usual form of symmetry and symmetry breaking that is encountered in elementary quantum mechanics. Its characteristic signature is degenerate multiplet structure for the spectrum [see the comments following eq. (5.6)], and the violation of this kind of symmetry involves explicit symmetry-breaking terms in the Hamiltonian that lift the multiplet degeneracies. It is typical of the Wigner implementation of symmetry that the original multiplet structure is easily recognized for small symmetry breaking perturbations. A good example is provided by a spherical quantum-mechanical system such as an atom. In the absence of external fields the states form degenerate $SU(2)$ multiplets as a consequence of the conservation of angular momentum. If we now place a magnetic field along the z-axis the rotational symmetry is lost since a preferred direction has been selected in space; the corresponding nondegenerate multiplet structure is well known from the Zeeman effect, as Fig. 8.1 illustrates. In group-theoretical language the original $SU(2)$ symme-

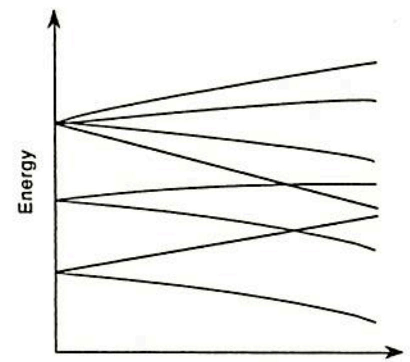

Fig. 8.1 Loss of symmetry by addition of an explicit symmetry-breaking field to the Hamiltonian.

try has been broken down to $U(1)$, since the system is still invariant under rotations about a single axis (chosen to be the z-axis in this example).

Another familiar illustration is the $SU(2)$ multiplet structure for isospin or, more generally, the $SU(N)$ flavor symmetry discussed in Ch. 5. For example, $SU(3)$ flavor multiplet symmetry is broken to $SU(2)$ isospin by terms in the Hamiltonian that depend on hypercharge. The isospin symmetry is further broken to $U(1)$ charge symmetry by terms such as Coulomb interactions that select a preferred direction in isospace, but the $U(1)$ symmetry remains intact since all known interactions conserve charge. By such an analysis a ***hierarchy of symmetry breakings*** may be established for symmetries implemented in the Wigner mode.

Of more interest in the present discussion are the other two symmetry modes. The manifestation of a symmetry in the Goldstone or Higgs mode is commonly termed ***spontaneous symmetry breaking.*** This is a picturesque but somewhat misleading expression. A more descriptive name is ***hidden symmetry:*** in spontaneous symmetry breaking the original symmetry is still present, but nature manages to camouflage the symmetry in such a way that its presence can be glimpsed only indirectly through relations among coupling constants, or by the unexpected appearance of massless bosons. The difference between the Goldstone and Higgs modes is simply that the spontaneous symmetry breaking occurs in the presence of a *local* gauge symmetry for the Higgs mode; as we shall see, this simple proviso has enormous consequences for the particle spectrum of such theories.

The crucial distinction between symmetry implementation in the Wigner, Goldstone, and Higgs modes lies in the structure of the vacuum (the lowest energy state). The Lagrangian of a system may be invariant under transformation by some unitary representation U of a symmetry group. However, for a perturbative quantum field theory we build states from the vacuum and the symmetry properties of such a theory require specification of the symmetry for the vacuum state, as well as that of the Lagrangian. If the Lagrangian is invariant under a set of transformations the symmetry is implemented in the

1. *Wigner mode* if the vacuum $|0\rangle$ is also invariant:

$$U\,|0\rangle = |0\rangle\,,$$

2. The *Goldstone mode* if the vacuum is not invariant and the Lagrangian symmetry is global:

$$U\,|0\rangle \neq |0\rangle \qquad \text{(global symmetry)},$$

3. The *Higgs mode* if the vacuum is not invariant and the Lagrangian symmetry is a *local* gauge symmetry:

$$U\,|0\rangle \neq |0\rangle \qquad \text{(local symmetry)}.$$

We now present some illustrations.

8.2 A Simple Example

Consider a self-interacting real scalar field ϕ with a Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4 \qquad (\lambda > 0). \tag{8.1}$$

(Omitting powers of ϕ higher than four ensures a renormalizable perturbation theory—see Exercise 6.3b; the coefficient λ is required to be positive so that the energy is bounded from below.) This Lagrangian is invariant under the discrete transformation

$$\phi \to -\phi. \tag{8.2}$$

Two qualitatively different cases may be distinguished, depending on the sign of the coefficient μ^2. The potential for $\mu^2 > 0$ is shown in Fig. 8.2a, and that for $\mu^2 < 0$ is shown in Fig. 8.2b. The case (a) with $\mu^2 > 0$ corresponds to the usual situation (Wigner mode). From Fig. 8.2a we have for the vacuum expectation value of the field

$$\langle\phi\rangle_0 \equiv \langle 0|\,\phi\,|0\rangle = 0 \qquad (\mu^2 > 0).$$

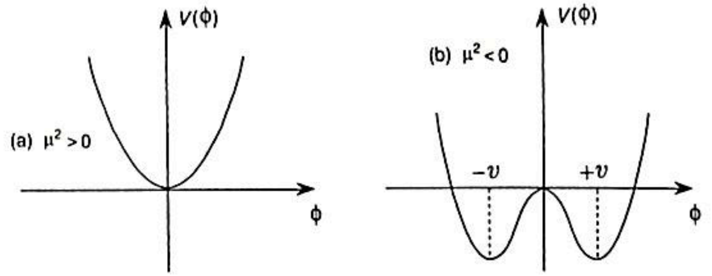

Fig. 8.2 Effective potentials $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$ for the Lagrangian density (8.1) with differing signs for the coefficient μ^2.

Expanding about $\langle\phi\rangle_0$ to second order

$$\mathcal{L} \simeq \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}\mu^2\phi^2, \tag{8.3}$$

which is the Lagrangian density of a free scalar field of mass μ [see eq. (2.41)] Thus we may interpret small quantized oscillations of the field about the origi as particles, and for the symmetric case shown in Fig. 8.2a the parameter μ plays the role of a mass.

Now consider the case $\mu^2 < 0$. This situation is depicted in Fig. 8.2b, an the potential has minima at

$$\langle\phi\rangle_0 = \pm\sqrt{\frac{-\mu^2}{\lambda}} \equiv \pm v. \tag{8.4}$$

Now there are ***two degenerate vacuum states***. The difference between th situations in Fig. 8.2a and 8.2b is characteristic of a *phase transition*, wit μ^2 playing the part of an order parameter. The minima at $\langle\phi\rangle_0 = \pm v$ ar equivalent, and either may be chosen as the classical ground (vacuum) state c the system.‡ The original Lagrangian is invariant under (8.2), so the physica results must be independent of this choice; however, once the vacuum is chose as either $+v$ or $-v$ it is no longer invariant under the transformation (8.2) This is a typical case of spontaneous symmetry breaking or, more descriptivel

‡For ordinary particle mechanics the degeneracy of the vacua in Fig. 8.2b would b lifted by tunneling between the two minima (see §8.5.1 and §13.1.7). However, i the present case the abscissa is the field ϕ, not a spatial coordinate. The tunnelin probability is $P \simeq \exp(iS)$ where S is the action (see Exercise 13.7c). For a tunnelin process S is imaginary; further, in this example it is *infinite*, since it is found b integrating over all space. Thus the tunneling probability between the field vacu in Fig. 8.2b is $P \simeq \exp(-\infty) = 0$.

hidden symmetry: the Lagrangian is invariant under a symmetry operation, the vacuum is not.

Let us choose

$$\langle \phi \rangle_0 = +v \tag{8.5}$$

as the vacuum state on which to construct our quantum theory. In the previous example ($\mu^2 > 0$) we examined the particle spectrum by expanding the field about the minimum at $\langle \phi \rangle_0 = 0$. In the present case of a spontaneously broken symmetry this is no longer suitable as an expansion point since it is a maximum of the potential energy (it is also obvious that the interpretation of μ as a mass is untenable since $\mu^2 < 0$ when the symmetry is spontaneously broken); an infinitesimal fluctuation is sufficient to drive the system into either of the minima at $\pm v$, and it is clear that the corresponding particle spectrum should be examined by expansion about the minima at $\pm v$. To facilitate this let us define a *shifted field*

$$\xi(x) \equiv \phi(x) - \langle \phi \rangle_0 = \phi(x) - v. \tag{8.6}$$

In terms of this new variable the vacuum state is $\langle \xi \rangle_0 = 0$, and the Lagrangian density is (neglecting constant terms)

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - \lambda v^2 \xi^2 - \lambda v \xi^3 - \tfrac{1}{4}\lambda \xi^4, \tag{8.7}$$

which has no apparent reflection symmetry. In fact, the symmetry is there because the original Lagrangian possessed such a symmetry, but it has been hidden. For small oscillations about the classical vacuum

$$\mathcal{L} \simeq \tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - \lambda v^2 \xi^2, \tag{8.8}$$

which is the Lagrangian density of a free scalar field of mass $m_\xi = \sqrt{-2\mu^2}$ [see (2.41) and (8.4)]. The mass is real and positive since $\mu^2 < 0$.

This is an exceedingly simple example, but it contains most of the features that characterize spontaneous symmetry breaking:

1. There is a nonzero expectation value of some field in the vacuum state.
2. The resulting classical theory has a degenerate vacuum, with the choice among the equivalent vacua completely arbitrary.
3. The transition from a symmetric vacuum to a degenerate vacuum typically occurs as a phase transition as some order parameter (μ^2 in the above example) is varied.
4. The chosen vacuum state does not possess the same symmetry as the Lagrangian.
5. On expansion around the chosen vacuum the original symmetry of the Lagrangian is no longer apparent. The degenerate vacua are related to each other by symmetry operations [eq. (8.2)], which tells us that the symmetry is still there, but it is not manifest; it is hidden.
6. The masses of the particles appearing in the theory with and without the spontaneous symmetry breaking may differ substantially. We say that the masses have been acquired spontaneously in the latter case.
7. Once the theory develops degenerate vacua the origin becomes an unstable point. Thus the symmetry may be "broken spontaneously" in the absence of external intervention.

However, there are two important aspects of spontaneous symmetry breaking that do not appear in this simple model. They will occur only when the symmetry that is spontaneously broken is a *continuous* one. Briefly stated,

8. If the spontaneously broken symmetry is a *continuous global symmetry,* one massless scalar field (*Goldstone boson*) must appear in the theory for each group generator that has been broken.
9. If a *continuous local gauge symmetry* is spontaneously broken no Goldstone bosons are produced, and the gauge bosons may acquire a mass without spoiling gauge invariance (*Higgs mechanism*).

To appreciate the importance of the new features (8) and (9), we now examine the spontaneous breaking of continuous symmetries.

8.3 Goldstone Bosons

Consider a Lagrangian density involving a complex scalar field

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2, \tag{8.9}$$

where $\lambda > 0$. This Lagrangian density is invariant under the group $U(1)$ of global phase transformations,

$$\phi(x) \to \phi'(x) = e^{i\theta}\phi(x), \tag{8.10}$$

where θ is independent of x (see Exercise 2.15). Defining

$$\rho = \phi^\dagger \phi, \tag{8.11}$$

we may identify a potential

$$V(\rho) = \mu^2 \rho + \lambda \rho^2. \tag{8.12}$$

As before, we may distinguish two cases: (a) $\mu^2 > 0$, the minimum is at $\rho = \phi = 0$, and the classical ground state is symmetric, as illustrated in

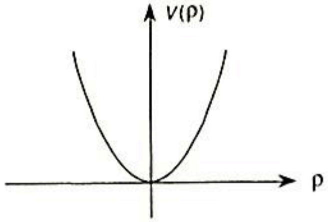

Fig. 8.3 Manifest symmetry for the Lagrangian density (8.9).

Fig. 8.3. (b) $\mu^2 < 0$, and the minima occur in the complex ϕ plane on a circle of radius

$$|\phi| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}, \tag{8.13}$$

as illustrated in Fig. 8.4. Case (b) is the one of primary interest, and we recognize immediately the characteristic features of a spontaneously broken symmetry. There is now an *infinity of degenerate ground states*, corresponding to different positions on the ring of minima in the complex ϕ plane, and the symmetry operation (8.10) relates one to another. Proceeding as before we choose as the vacuum the point in the minimum on the real ϕ axis, $\mathrm{Re}(\phi) = v/\sqrt{2}$, and expand around it to investigate the spectrum. We may write [see (8.26)]

$$\phi(x) = \tfrac{1}{\sqrt{2}}\left[v + \xi(x) + i\chi(x)\right]. \tag{8.14}$$

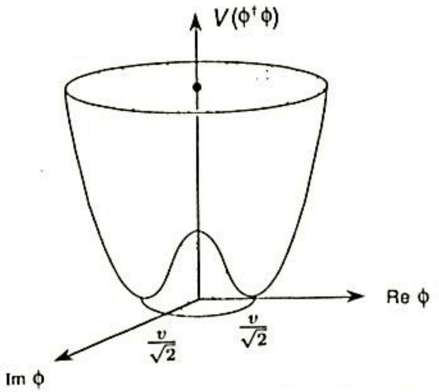

Fig. 8.4 Spontaneously broken symmetry for the Lagrangian density (8.9).

Substituting in the Lagrangian density (8.9) yields

$$\begin{aligned}\mathcal{L} =& \tfrac{1}{2}(\partial_\mu\xi)^2 + \tfrac{1}{2}(\partial_\mu\chi)^2 - \lambda v^2\xi^2 - \lambda v\xi(\xi^2+\chi^2) \\ &- \tfrac{1}{4}\lambda(\xi^2+\chi^2)^2 + \text{constants}.\end{aligned} \tag{8.15}$$

Now this resembles a Lagrangian density for a quantum field theory with two basic fields, ξ and χ. If interpreted in this way the χ field is massless, but the ξ field is massive by virtue of the term $-\lambda v^2\xi^2$:

$$m_\xi = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \qquad (\mu^2 < 0). \tag{8.16}$$

The physical interpretation of these degrees of freedom is illustrated in Fig. 8.5. The massive mode ξ corresponds to "radial oscillations" against a restoring potential; we may say that the field ξ has acquired its mass spontaneously. The massless mode χ corresponds to angular motion about the bottom of the circular valley, for which there is no restoring force.

The appearance of the massless scalar field χ is a specific example of a general phenomenon in the spontaneous breaking of global symmetries that is important enough to have achieved the status of a theorem:

> ***Goldstone Theorem***: *If a continuous global symmetry is broken spontaneously, for each broken group generator there must appear in the theory a massless particle.*

(Goldstone, 1961; Nambu, 1960; Nambu and Jona-Lasinio, 1961; Goldstone, Salam, and Weinberg, 1962; Bludman and Klein, 1962).

The massless particles for the theories that interest us here are quanta of

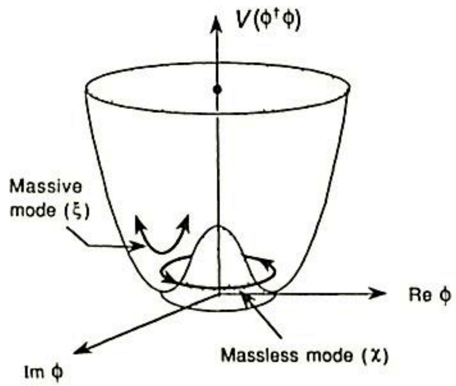

Fig. 8.5 Modes of a complex scalar field with spontaneously broken symmetry.

scalar or pseudoscalar fields that are termed *Goldstone bosons.*† In the example just discussed $U(1)$ has a single generator that is broken. That is, once the symmetry is hidden by choosing a particular vacuum state from the infinite number of equivalent possibilities the manifest symmetry under phase rotations by the $U(1)$ generator is obscured. As a consequence, there appears in the theory a massless field associated with the same motion as that induced by the generator that was broken (circular motion in the valley of the potential). We say that the corresponding massless particle (the Goldstone boson) carries the quantum numbers of the broken generator.

In this simple example only a single generator is broken, but the symmetry breaking generally may involve more than one group generator, and a Goldstone particle associated with each. For example, let us consider a Lagrangian density containing n real scalar fields ϕ_i in the form (Abers and Lee, 1973)

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi_i)(\partial^\mu\phi_i) - \tfrac{1}{2}\mu^2\phi_i\phi_i - \tfrac{1}{4}\lambda(\phi_i\phi_i)^2. \tag{8.17}$$

This Lagrangian density is invariant under the group $O(n)$ of orthogonal transformations in n dimensions, which has $\frac{1}{2}n(n-1)$ generators (we deal with orthogonal transformations rather than unitary ones because the fields are real—see Exercise 2.15). If the symmetry is spontaneously broken by choosing $\mu^2 < 0$, a ring of minima appears satisfying $\phi_i\phi_i = -\mu^2/\lambda \equiv v$. The fields may be viewed as the components of a vector ϕ, in which case the equation for the minimum defines the magnitude but not the direction of ϕ. The vacuum state can be chosen as

$$\langle\phi\rangle_0 \equiv \begin{pmatrix}\phi_1\\ \phi_2\\ \vdots\\ \phi_n\end{pmatrix}_{\text{vac}} = \begin{pmatrix}0\\ 0\\ \vdots\\ -\mu^2/\lambda\end{pmatrix} \tag{8.18}$$

and all other vacuum states are related to this one by $O(n)$ rotations. In contrast to our earlier example, this vacuum state is *invariant under a subgroup of the original group*: the group $O(n-1)$, which does not mix the last field with the others. The group $O(n)$ has $\frac{1}{2}n(n-1)$ generators, so $O(n-1)$ has $\frac{1}{2}(n-1)(n-2)$ generators and the difference between the number of generators for the original group $O(n)$ and the residual group $O(n-1)$ is $n-1$; thus, there are $n-1$ broken generators. An analysis similar to that given for the previous example then shows that only one field acquires a mass, and $n-1$ scalar fields appear in the theory with no mass (see Exercise 8.2). These are

†Goldstone particles are not always bosons: in spontaneously broken supersymmetries there are spin-$\frac{1}{2}$ Goldstone fields. However, we will only consider theories for which the Goldstone quanta are bosons.

the Goldstone bosons, corresponding to the $n-1$ broken symmetry generators of the original group.

The largest subgroup of G that leaves the vacuum invariant is termed the *little group* or the *stability subgroup* (O'Raifeartaigh, 1986, Chs. 8 and 11). If that subgroup is denoted by H, we say that G has been spontaneously broken down to H. For example, in Exercise 8.2 we consider a Lagrangian invariant under $G = SO(3)$, with a ground state that is only invariant under the subgroup $H = SO(2)$. The resulting theory has $n - 1 = 2$ Goldstone bosons.

EXERCISE 8.1 (a) Show that when there is spontaneous symmetry breaking at least one of the generators for the symmetry group fails to annihilate the vacuum. (b) Derive the minima of Fig. 8.2 from eq. (8.1). Obtain the result (8.7) for $\mu^2 < 0$ and show that

$$m_\xi = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \qquad (\mu^2 < 0)$$

is the mass of the ξ field after spontaneous symmetry breaking.

EXERCISE 8.2 Investigate the Lagrangian density (8.17) for an isovector Lorentz scalar field $\phi_i (i = 1, 2, 3)$, invariant under the global internal group $SO(3)$. Show that for $\mu^2 < 0$ the symmetry is spontaneously broken from $SO(3)$ to $SO(2)$, and that the particle spectrum corresponds to two massless Goldstone fields and one massive scalar.

EXERCISE 8.3 A concept that will be important when we consider chiral symmetry and the nature of the pion in subsequent chapters is that of an *approximate Goldstone boson.* Consider a complex scalar field with a potential

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 - \epsilon\phi_1 \equiv V_0(\phi) - \epsilon\phi_1,$$

where $\lambda > 0$, the parameter ϵ is small and positive, and $\phi \equiv (\phi_1 + i\phi_2)/\sqrt{2}$. First, set $\epsilon = 0$, choose the μ and λ parameters to break the symmetry spontaneously, sketch the potential $V(\phi)$, identify a classical vacuum state, and investigate the free-particle spectrum for small oscillations of the fields about this vacuum. Now, demonstrate that for $\epsilon \neq 0$ the field that would be a Goldstone boson for $\epsilon = 0$ acquires a small mass proportional to $\epsilon^{1/2}$ and proportional to the divergence $\partial_\mu J^\mu$ of a current

$$J_\mu = \phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1$$

that is conserved if $\epsilon = 0$ (*Hint*: see §12.3). These ideas will be employed in Ch. 12 to understand the partially conserved axial current (PCAC). The almost massless pion will be interpreted as an approximate Goldstone boson, with its finite mass related to the divergence of the axial current (Exercise 12.4).

8.4 The Higgs Mechanism

The Goldstone theorem and the paucity of massless scalar or pseudoscalar particles in nature would seem to preclude the use of spontaneous symmetry breaking in realistic quantum field theories. However, there is a loophole in this argument: the Goldstone theorem applies to any field theory obeying the "normal postulates" such as locality, Lorentz invariance, and positive definite norm on the Hilbert space. But gauge field theories do not fit into that category: there is no single gauge in which such theories simultaneously fulfill each of these conditions! For example, in Ch. 2 it was shown that the quantization of the electromagnetic field (the archetypical gauge field) is nontrivial. We chose to quantize in a manner sacrificing manifest Lorentz covariance; conversely, if the Maxwell field is quantized in a manifestly covariant fashion the notion of a positive definite metric must be sacrificed [see the Gupta–Bleuler mechanism below and the discussion in Ryder (1985), §4.4].

Thus we are led to investigate whether the Goldstone theorem is operative in a theory possessing a local gauge invariance. We will find a remarkable result: there is an unexpected collusion between the massless gauge fields and the Goldstone bosons produced by the spontaneous symmetry breaking that can be arranged so as to eliminate the massless Goldstone bosons and give a mass to the gauge quanta without spoiling the gauge invariance or renormalizability of the theory. This (seemingly) miraculous state of affairs is called the *Higgs mechanism* (Higgs, 1964; Guralnik, Hagen, and Kibble, 1964; Englert and Brout, 1964; Anderson, 1963).

The simplest example of the Higgs mechanism is the extension of the global $U(1)$ symmetry just discussed to a local $U(1)$ symmetry. This gauge-invariant $U(1)$ theory is often called the *Abelian Higgs Model*. In the absence of spontaneous symmetry breaking it would describe the ordinary electrodynamics of charged scalars; when the symmetry is broken spontaneously it will describe something quite different.

The Lagrangian density is (Quigg, 1983; Leader and Predazzi, 1982)

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{8.19}$$

where λ is positive and

$$\phi = \tfrac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \tag{8.20}$$

$$D^\mu = \partial^\mu + iqA^\mu \tag{8.21}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{8.22}$$

The Lagrangian is invariant under global $U(1)$ rotations and under the local gauge transformations

$$\phi(x) \to e^{iq\alpha(x)}\phi(x) \tag{8.23a}$$

$$A_\mu(x) \to A_\mu(x) - \partial_\mu\alpha(x). \tag{8.23b}$$

As in the example with global $U(1)$ symmetry, two possibilities may be distinguished: (a) $\mu^2 > 0$, and the potential has a minimum at $\phi = \phi^\dagger = 0$ that is unique. The symmetry of the Lagrangian is also the symmetry of the ground state, and the spectrum consists of a massless photon A^μ and a pair of scalar fields ϕ and $\phi^\dagger$ with a common mass μ. (b) $\mu^2 < 0$ corresponds to a spontaneously broken local symmetry. Because spontaneous local symmetry breaking is subtle, we must analyze this situation carefully.

The absolute minima (degenerate vacua) occur at

$$|\phi|^2 = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}. \tag{8.24}$$

Choosing the vacuum as

$$\langle\phi\rangle_0 = \frac{v}{\sqrt{2}} \tag{8.25}$$

with v real and positive, and expanding in polar coordinates,

$$\begin{aligned}\phi(x) &= \tfrac{1}{\sqrt{2}}\left[v + \eta(x)\right]e^{i\xi(x)/v} \\ &= \tfrac{1}{\sqrt{2}}\left[v + \eta(x) + i\xi(x) + \ldots\right].\end{aligned} \tag{8.26}$$

Substituting in (8.19) and retaining low order terms gives

$$\begin{aligned}\mathcal{L} \simeq\, &\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \mu^2\eta^2 + \tfrac{1}{2}(\partial^\mu\xi)(\partial_\mu\xi) \\ &+ qvA_\mu(\partial^\mu\xi) + \tfrac{1}{2}q^2v^2A_\mu A^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ldots\end{aligned} \tag{8.27}$$

Now this looks like the Lagrangian density of a quantum field theory with three fields: η, ξ, and A^μ. By inspection, the η field has a mass [eq. (2.41)]

$$m_\eta = \sqrt{-2\mu^2}, \tag{8.28}$$

as implied by the term $\mu^2\eta^2$. Surprisingly, the photon appears to have gained a mass

$$m_A = qv, \tag{8.29}$$

as implied by the term $\frac{1}{2}q^2v^2A_\mu A^\mu$ [see eq. (2.49)], and the ξ field seems to be massless. However, we should count degrees of freedom. Originally we had

2	(complex scalar field)
+ 2	(transverse field for massless photon)
4	

After the spontaneous symmetry breaking we have

$$\begin{array}{rl} 1 & (\eta \text{ field}) \\ 1 & (\xi \text{ field}) \\ +\ 3 & (\textit{massive} \text{ vector field } A_\mu) \\ \hline 5 & \end{array}$$

So all is not as it appears—a degree of freedom seems to have been gained in the spontaneous symmetry breaking. This is an illusion, as can be made obvious by an appropriate change of gauge. Implementing the local transformation

$$\begin{aligned} \phi(x) &\rightarrow e^{-i\xi(x)/v}\phi(x) = \frac{v+\eta(x)}{\sqrt{2}} \\ A_\mu(x) &\rightarrow A_\mu(x) + \frac{1}{qv}\partial_\mu\xi(x) \equiv A'_\mu(x), \end{aligned} \tag{8.30}$$

and dropping the primes on A_μ and $F_{\mu\nu}$, the Lagrangian density takes the form

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \mu^2\eta^2 + \tfrac{1}{2}q^2v^2A_\mu A^\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{8.31}$$

Now the particle spectrum is clear: a scalar particle η with mass $\sqrt{-2\mu^2}$ and a massive vector field A_μ with mass qv. The ξ field has disappeared—we say that it has been gauged away—and the number of degrees of freedom has been reduced to the required four: one for η and three for A_μ.

Thus no massless fields appear, and *the Goldstone theorem does not apply to a local gauge theory.* What has happened to the massless field ξ that was gauged away? It contrives to reappear effectively as a *longitudinal polarization degree of freedom for the vector field*, giving it a mass. The massive scalar field η is called the physical *Higgs field*, and the special gauge in which the particle spectrum is manifest for spontaneous breaking of a local gauge symmetry is called the *Unitary Gauge* or *U-gauge.*

It is common to say that the gauge field absorbs the Goldstone boson and becomes massive, or that the Goldstone field becomes the third state of polarization for the massive vector boson. One way to visualize this is the following. If one attempts to quantize a vector field A^μ in a manifestly covariant fashion (retaining all four components), the indefinite Minkowski metric $g_{\mu\nu}$ leads to a *negative norm* for the timelike component A^0. For massless fields the A^0 contribution exactly cancels with the longitudinal spacelike component $\mathbf{p}\cdot\mathbf{A}$, leaving two physical transverse components $\mathbf{p}\times\mathbf{A}$. This is called the *Gupta–Bleuler mechanism.* The Higgs mechanism may be viewed as a kind of generalized Gupta–Bleuler mechanism in which the *Goldstone scalar* cancels the timelike component of the gauge field, leaving the three spacelike components of A_μ intact, so A_μ behaves like a massive vector boson (O'Raifeartaigh, 1986, §8.5).

Although unitary gauge exhibits the particle spectrum clearly, it is not a good gauge for tasks such as demonstrating renormalizability. For that purpose a set of gauges called *R*-gauges is normally used (see, e.g., Cheng and Li, 1984, Ch. 9). In *U*-gauges the spurious degrees of freedom are transformed away, but the propagators are ill-behaved at high energy; in *R*-gauges the particle spectrum is confused by spurious degrees of freedom, but the propagators have a more benign high-energy behavior.

This type of symmetry realization is called the *Higgs mode.* The Wigner mode was characterized by degenerate multiplet structure and the Goldstone mode by the appearance of one massless Goldstone boson for each spontaneously broken generator. The distinctive signature of the Higgs mode is the acquisition of mass by gauge bosons at the expense of would-be Goldstone bosons, which vanish from the theory.

This analysis may be adapted to other Abelian or non-Abelian gauge theories. For the gauge fields to acquire masses we must break the vacuum symmetry with scalar fields. Some pieces of the scalar fields disappear, only to reappear as the longitudinal polarization states for the gauge bosons that acquire an effective mass; the remaining pieces become physical scalar fields, the Higgs bosons.[‡] In a theory with multiple gauge bosons it is possible to arrange for some to acquire a mass and for others to remain massless by this procedure.

A necessary consequence of symmetry realization in this mode will be the appearance of the physical Higgs bosons, but the Higgs particles have a distinct advantage over massless gauge quanta or Goldstone bosons in our theory: their appearance may violate our sense of economy, but they enter with adjustable masses at the present level of understanding and the failure to observe them can be attributed to their masses being too large.

As a preview of subsequent developments, we note that there appear to be two ways that nature has contrived to reduce the number of massless bosons that might have been expected from our initial discussions of Yang–Mills fields and the Goldstone theorem. The first we have just met: the *Higgs mechanism* does double duty in converting the Goldstone bosons into effective longitudinal polarization states for the gauge bosons, which become massive in the process. The second we shall encounter in Ch. 10: the nonlinear interactions of the Yang–Mills gauge quanta, which themselves carry the charges of the gauge field, may in some cases cause an absolute *confinement* of the gauge fields and the matter fields coupled to them. This appears to be the case in QCD, where the $SU(3)$ color symmetry is unbroken so that the eight associated gauge bosons (gluons) are massless, but are confined to the interior of hadrons along with the quark fields.

[‡]The Higgs field is required to transform as a scalar to prevent spontaneous breaking of the Lorentz invariance, which would not be in accord with observation.

8.5 Some General Remarks

The concept of spontaneous symmetry breaking is an important but subtle one in modern theoretical physics. In this section we make some comments that may help clarify the nature of the phenomenon.

First we emphasize that spontaneous symmetry breaking is not confined to relativistic fields. It was known in many guises in condensed matter and nuclear physics before its introduction into relativistic field theory. If a nonrelativistic, many-body Lagrangian is invariant under some set of transformations G and the ground state is nondegenerate, then the ground state transforms as a singlet under the group G; this is the Wigner mode. However, if the ground state is degenerate it may transform as some finite-dimensional multiplet of the group G. If *one* member of this multiplet is arbitrarily selected as the ground state the symmetry is spontaneously broken; this is the Goldstone mode.

8.5.1 Heisenberg Ferromagnet

A well-known example is the infinite ferromagnet, often termed the Heisenberg ferromagnet. The model corresponds to an infinite crystalline array of spin-$\frac{1}{2}$ dipoles with spin–spin interactions between nearest neighbors. Above the critical temperature there is short-range order because of nearest-neighbor interactions, but long-range order is suppressed by thermal fluctuations; below the critical temperature there is also long-range order with macroscopic alignment of spins. The Hamiltonian for the interaction of neighboring spins is rotationally invariant, but below the critical temperature the ground state has a net spin alignment in a particular direction that breaks rotational invariance.

The infinite degeneracy of the possible ground states, corresponding to an infinite number of possible directions for the aligned ground state spin, depends crucially on the assumption of an unlimited spatial extent for the system. Mathematically, the infinite size of the system means that there is no unitary operator that can connect the different ground states and they lie in different Hilbert spaces. This is because the ferromagnet has an infinite moment of inertia, implying that no finite amount of energy can rotate one ground state into another.

If the ferromagnet is of limited spatial extent the different ground states are separated by finite energy barriers and may tunnel into each other; the situation then resembles band structure in the solid state. The degeneracy of the ground state is lifted since one of the linear combinations resulting from the degenerate vacuum will lie lower in energy than all others and will be the (nondegenerate) physical ground state. For a relativistic field theory exhibiting spontaneous symmetry breaking we may expect a related situation.

The vacua are degenerate only if the universe is of infinite spatial extent, for only then will the vacua be orthogonal (§13.1.7). However, from the radius of the known universe one estimates that the frequency of rotation from one vacuum to another is negligible (Taylor, 1976, §5.3).

8.5.2 Superconductivity and the Meissner Effect

The Higgs mechanism described in §8.4 is sometimes expressed in an alternative way: Goldstone bosons can be made to disappear in the presence of long-range forces (Anderson, 1963; Guralnik, Hagen, and Kibble, 1968). The connection between the two descriptions comes through the Yukawa–Wick interpretation that long-range forces like the Coulomb interaction are mediated by massless exchange particles (gauge fields are massless). In this picture the long-range force is shielded and becomes short-ranged, which is equivalent to the generation of an effective mass for the gauge boson.

One nonrelativistic example of the Higgs mechanism for which this shielding description is illuminating is the Meissner effect in superconductivity. Aitchison and Hey (1982, Ch. 9) give a nice heuristic discussion of how the condensate of electron pairs in the ground state of a superconductor plays the role of a Higgs field, and how this gives photons an effective mass inside the superconductor. As a direct consequence a magnetic field can penetrate only exponentially into the superconductor, with a range proportional to the inverse of the effective photon mass. Thus a superconductor expels a magnetic field from its interior (the Meissner effect—see Fig. 12.3), except for a thin layer at the surface (the London penetration depth) over which the field decreases exponentially.

The microscopic origin of the Higgs phenomenon in this case lies in resistanceless screening currents that are produced in the superconductor to compensate the external field. The finite range of the magnetic field that results is just what would be expected if the photon had acquired a mass inside the superconductor. Furthermore, it can be shown that the superconductor with a "massive" photon is gauge invariant, despite our remarks in connection with eq. (2.149). That is because the photon mass has come by the Higgs mechanism—not by an explicit mass term in the Lagrangian. This is our first example of a real physical system in which gauge bosons acquire an effective mass without breaking gauge invariance. Analogies with this example will have important implications in subsequent chapters, particularly in connection with the gauge theory of the weak interactions.

8.5.3 Multiplets and Coupling Constants

The various schemes of spontaneous symmetry breaking in the Higgs mode lead to renormalizable theories only if the original symmetry before gauging is