

# GABARITO TESTE 3

1)

a)  $m \ddot{x} = -kx - b \dot{x}$

b)  $\beta = \frac{b}{2m} > \omega_0 = \sqrt{\frac{k}{m}}$

RESOLVENDO A EQUAÇÃO

$$\ddot{x} + \frac{k}{m}x + \frac{b}{m}\dot{x} = 0 \quad \text{e } \beta = \frac{b}{2m}$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

SOLUÇÃO:

$$x(t) = A e^{\lambda t}$$

$$\dot{x}(t) = A \lambda e^{\lambda t}$$

$$\ddot{x} = A \lambda^2 e^{\lambda t}$$

$$A(A\lambda^2 + 2\beta\lambda + \omega_0^2)e^{\lambda t} = 0 \quad \Rightarrow \quad \lambda^2 + 2\beta\lambda + \omega_0^2 = 0$$

$$\lambda = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2} \quad (\text{COMO É ALIÍO})$$

$$\beta > \omega_0 \quad \text{O.M.T.}$$

TOMAMOS O CASO REAL.

SOLUSI

$$X(t) = A_1 e^{(-\beta + \sqrt{\beta^2 - \omega_c^2})t} + A_2 e^{(-\beta - \sqrt{\beta^2 - \omega_c^2})t}$$

DITAMBAH  $\omega_1 = \sqrt{\beta^2 - \omega_c^2}$

$$X(t) = A_1 e^{(\beta + \omega_1)t} + A_2 e^{(-\beta - \omega_1)t}$$

c) kondisi awal

①  $X(t=0) = X_0 = 0$      $\dot{X}(t=0) = v_0 \neq 0$

②  $X(t=0) = X_0 \neq 0$      $\dot{X}(t=0) = v_0 \neq 0$

① PMA  $X(t=0) = 0$

$$X(t=0) = A_1 + A_2 = 0 \quad (\text{I})$$

$$\dot{X}(t) = (-\beta + \omega_1) A_1 e^{(\beta + \omega_1)t} + A_2 (-\beta - \omega_1) e^{(-\beta - \omega_1)t} \quad (\text{IIa})$$

$$\dot{X}(t=0) = (-\beta + \omega_1) A_1 + A_2 (-\beta - \omega_1) = v_0 \quad (\text{IIb})$$

DA EQ (I),

$$A_1 = -A_2 \quad \text{SUBSTITUSI DA EQ (IIb)}$$

$$(-\beta + \omega_1)(-A_2) + A_2(-\beta - \omega_1) = v_0$$

$$A_2 [\beta - \omega_1 - \beta - \omega_1] = v_0$$

$$A_2 = \frac{-v_0}{2\omega_1}$$

E PORTATO  $A_1 = \frac{N_0}{2\omega_1}$

A SOLUZIONE  $\bar{E}$ ,

$$X(t) = \frac{N_0}{2\omega_1} \left[ e^{(-\beta + \omega_1)t} - e^{(-\beta - \omega_1)t} \right]$$

$$X(t) = \frac{N_0}{\omega_1} e^{-\beta t} \sinh(\omega_1 t)$$

NO CASO (2), ~~DA EQ (IIa)~~ DA EQ (IIa)

$$\dot{X}(t=0) = 0 \Rightarrow (-\beta + \omega_1)A_1 + (-\beta - \omega_1)A_2 = 0$$

$$A_1 = \left( \frac{\beta + \omega_1}{-\beta + \omega_1} \right) A_2 \quad (\text{III})$$

CONDIZIONE  $X(t=0) = X_0$

$$X_0 = A_1 + A_2 \quad (\text{IV})$$

SOSTITUISCIAMO (III) IN (IV),

$$X_0 = \left( \frac{\beta + \omega_1}{-\beta + \omega_1} \right) A_2 + A_2 = A_2 \left[ \frac{\beta + \omega_1 - \beta + \omega_1}{-\beta + \omega_1} \right]$$

$$A_2 = \frac{X_0(-\beta + \omega_1)}{2\omega_1}$$

COMO PORA EQ. (III)

$$A_1 = \frac{(\beta + \omega_1)}{(-\beta + \omega_1)} \left[ \frac{X_0(-\beta + \omega_1)}{2\omega_1} \right]$$

$$A_1 = \frac{(\beta + \omega_1)X_0}{2\omega_1}$$

EM TAC

$$X(t) = \frac{X_0}{2\omega_1} \left[ (\beta + \omega_1) e^{-\beta t} e^{\omega_1 t} + (-\beta + \omega_1) e^{-\beta t} e^{-\omega_1 t} \right]$$

$$X(t) = \frac{X_0 e^{-\beta t}}{2\omega_1} \left[ 2\beta \sinh \omega_1 t + \omega_1 2 \cosh \omega_1 t \right]$$

ENTÃO A FORMA É  
(COM PORTAMENTO)

$$X_1(t) = \frac{N_0}{\omega_1} e^{-\rho t} \sin(\omega_1 t)$$

$$X_1(t=0) = 0$$

$$\dot{X}_1(t=0) = N_0$$

Se  $X_1(t) = 0$   
 $t \rightarrow \infty$

EXISTEM MÁXIMOS OU MÍNIMOS

$$\dot{X}_1(t) = \frac{N_0}{\omega_1} \left[ (-\rho + \omega_1) e^{-(\rho - \omega_1)t} \right]$$

~~$$X_1(t=0) = \frac{N_0}{\omega_1} \left[ (-\rho + \omega_1) + \beta + \omega_1 \right]$$~~

$$\dot{X}_1(t) = \left( \frac{N_0}{\omega_1} \right) \left[ \underbrace{(-\rho + \omega_1)}_{< 0} e^{-(\rho - \omega_1)t} + \underbrace{(\beta + \omega_1)}_{> 0} e^{-(\rho + \omega_1)t} \right]$$

COMO  $-\rho + \omega_1 < 0$  ENTÃO EXISTE UMA MÁXIMO

PARA  $t \neq 0$  e  $t \rightarrow \infty$ .

$$\dot{X}_1(t) = 0 \Rightarrow \underbrace{(-\rho + \omega_1)}_{< 0} e^{\omega_1 t} + (\rho + \omega_1) e^{-\omega_1 t} = 0$$

Part 0 caso  $x(t=0) \neq 0$   $x(t=0) = 0$

$$X(t) = \frac{X_0}{2\omega_1} \left[ (\beta + \omega_1) e^{-\beta t} e^{\omega_1 t} + (-\beta + \omega_1) e^{-\beta t} e^{-\omega_1 t} \right]$$

Με  $\hat{x}(t)$  ού μίλις

$$\hat{x}(t) = \frac{X_0}{2\omega_1} \left[ (\beta + \omega_1)(-\beta + \omega_1) e^{-\beta t} e^{\omega_1 t} + (-\beta + \omega_1)(-\beta - \omega_1) e^{-\beta t} e^{-\omega_1 t} \right]$$

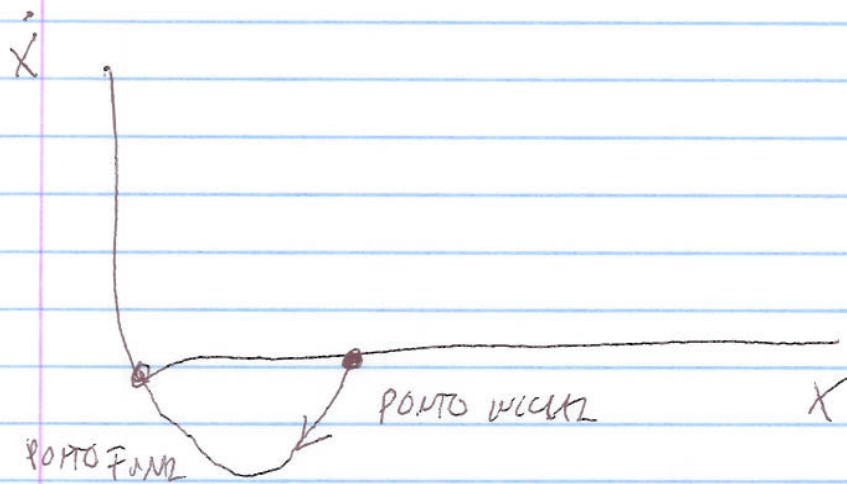
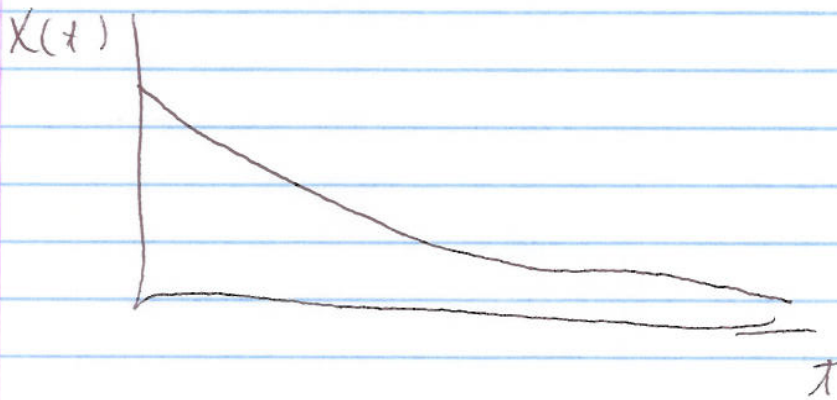
$$\dot{x}(t) = \left( \frac{X_0}{2\omega_1} \right) \left[ (\omega_1^2 - \beta^2) e^{-\beta t} e^{\omega_1 t} - (\omega_1^2 - \beta^2) e^{-\beta t} e^{-\omega_1 t} \right]$$

$$\dot{x}(t) = \left( \frac{X_0}{2\omega_1} \right) (\omega_1^2 - \beta^2) e^{-\beta t} [2 \mu \eta \omega_1 t]$$

$$\text{ομο } \beta > \omega_1 \Rightarrow \omega_1^2 - \beta^2 < 0$$

ο μίλις ού μίλις  $t=0$  e  $t \rightarrow \infty$ . τμήτε

A FORMA DO GRÁFICO É



A point on the curve is

