GABARITO TESTE 3

(1)

a) $m\ddot{x} = -kx - b\dot{x}$

b) $\boxed{\beta = \frac{b}{2m} > \omega_0 = \sqrt{\frac{k}{m}}}$

REESCREVENDO A EQUAÇÃO

$$\ddot{x} + \frac{k}{m}x + \frac{b}{m}\dot{x} = 0 \qquad 2\beta = \frac{b}{m}$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

SOLUÇÃO:

$$x(t) = Ae^{\lambda t}$$

$$\dot{x}(t) = A\lambda e^{\lambda t}$$

$$\ddot{x} = A\lambda^2 e^{\lambda t}$$

$$A(A\lambda^2 + 2\beta\lambda + \omega_0^2)e^{\lambda t} = 0 \longrightarrow \lambda^2 + 2\beta\lambda + \omega_0^2 = 0$$

$$\lambda = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

COMO É DITO

$\beta > \omega_0$ ENTÃO

TEMOS DUAS RAÍZES REAIS.

SOLUZIONE

$$x(t) = A_1 e^{(-\beta + \sqrt{\beta^2 - \omega_0^2})t} + A_2 e^{(-\beta - \sqrt{\beta^2 - \omega_0^2})t}$$

DEFINIAMO $\omega_1 = \sqrt{\beta^2 - \omega_0^2}$

$$\boxed{x(t) = A_1 e^{(-\beta + \omega_1)t} + A_2 e^{(-\beta - \omega_1)t}}$$

c) CONDIZIONI INIZIALI

① $x(t=0) = x_0 = 0 \quad \dot{x}(t=0) = v_0 \neq 0$

② $x(t=0) = x_0 \neq 0 \quad \dot{x}(t=0) = v_0 = 0$

① PONIAMO $x(t=0) = 0$

$$x(t=0) = A_1 + A_2 = 0 \quad (I)$$

$$\dot{x}(t) = (-\beta + \omega_1) A_1 e^{(-\beta + \omega_1)t} + A_2 (-\beta - \omega_1) e^{(-\beta - \omega_1)t} \quad (IIa)$$

$$\dot{x}(t=0) = (-\beta + \omega_1) A_1 + A_2 (-\beta - \omega_1) = v_0 \quad (IIb)$$

DA EQ (I),

$A_1 = -A_2$ SOSTITUENDO NA EQ. (IIb)

$$(-\beta + \omega_1)(-A_2) + A_2 (-\beta - \omega_1) = v_0$$

$$A_2 [\beta - \omega_1 - \beta - \omega_1] = v_0 \qquad A_2 = \frac{-v_0}{2\omega_1}$$

E PORTANTO $A_1 = \frac{v_0}{2\omega_1}$

A SOLUÇÃO É,

$$x(t) = \frac{v_0}{2\omega_1}\left[e^{(-\beta+\omega_1)t} - e^{(-\beta-\omega_1)t}\right]$$

$$\boxed{x(t) = \frac{v_0}{\omega_1} e^{-\beta t} \operatorname{senh}(\omega_1 t)}$$

NO CASO (2), DA EQ. (II a)

$$\dot{x}(t=0) = 0 \Rightarrow (-\beta+\omega_1)A_1 + (-\beta-\omega_1)A_2 = 0$$

$$\boxed{A_1 = \left(\frac{\beta+\omega_1}{-\beta+\omega_1}\right) A_2} \quad (III)$$

CONDIÇÃO $x(t=0) = x_0$

$$x_0 = A_1 + A_2 \quad (IV)$$

SUBSTITUINDO (III) EM (IV),

$$x_0 = \left(\frac{\beta+\omega_1}{-\beta+\omega_1}\right) A_2 + A_2 = A_2\left[\frac{\beta+\omega_1 - \beta+\omega_1}{-\beta+\omega_1}\right]$$

$$A_2 = \frac{X_0(-\beta+\omega_1)}{2\omega_1}$$

COMO PARA EQ. (III)

$$A_1 = \frac{(\beta+\omega_1)}{(-\beta+\omega_1)}\left[\frac{X_0(-\beta+\omega_1)}{2\omega_1}\right]$$

$$A_1 = \frac{(\beta+\omega_1)X_0}{2\omega_1}$$

ENTÃO

$$X(t) = \frac{X_0}{2\omega_1}\left[(\beta+\omega_1)e^{-\beta t}e^{\omega_1 t} + (-\beta+\omega_1)e^{-\beta t}e^{-\omega_1 t}\right]$$

$$X(t) = \frac{X_0 e^{-\beta t}}{2\omega_1}\left[2\beta \operatorname{senh}\omega_1 t + \omega_1 2\cosh\omega_1 t\right]$$

COMPORTAMENTO

$$X_1(t) = \frac{v_0}{\omega_1} e^{-\rho t} \sin(\omega_1 t)$$

$X_1(t=0) = 0$

$\dot{X}_1(t=0) = v_0$

$\lim_{t \to \infty} X_1(t) = 0$

EXISTEM MÁXIMOS OU MÍNIMOS

$$\dot{X}_1(t) = \frac{v_0}{\omega_1} \left[(-\rho + \omega_1) e^{(-\rho+\omega_1)t} - (\rho + \omega_1) e^{(-\rho-\omega_1)t} \right]$$

$$\dot{X}_1(t=0) = \frac{v_0}{\omega_1} \left[(-\rho + \omega_1) + (\rho + \omega_1) \right]$$

$$\dot{X}_1(t) = \left(\frac{v_0}{\omega_1} \right) \left[\underbrace{(-\rho + \omega_1)}_{<0} e^{(-\rho+\omega_1)t} + \underbrace{(\rho + \omega_1)}_{>0} e^{(-\rho-\omega_1)t} \right]$$

COMO $-\rho + \omega_1 < 0$ ENTÃO EXISTE UM MÁXIMO PARA $t \neq 0$ E $t \to \rho$.

$$\dot{X}_1(t) = 0 \Rightarrow \underbrace{(-\rho + \omega_1)}_{<0} e^{\omega_1 t} + (\rho + \omega_1) e^{-\omega_1 t} = 0$$

PARA O CASO $x(t=0) \neq 0$ $\dot{x}(t=0) = 0$

$$x(t) = \frac{x_0}{2\omega_1}\left[(\beta+\omega_1)e^{-\beta t}e^{\omega_1 t} + (-\beta+\omega_1)e^{-\beta t}e^{-\omega_1 t}\right]$$

MÁXIMOS OU MÍNIMOS

$$\dot{x}(t) = \frac{x_0}{2\omega_1}\left[(\beta+\omega_1)(-\beta+\omega_1)e^{-\beta t}e^{\omega_1 t}\right.$$

$$\left. + (-\beta+\omega_1)(-\beta-\omega_1)e^{-\beta t}e^{-\omega_1 t}\right]$$

$$\dot{x}(t) = \left(\frac{x_0}{2\omega_1}\right)\left[(\omega_1^2-\beta^2)e^{-\beta t}e^{\omega_1 t} - (\omega_1^2-\beta^2)e^{-\beta t}e^{-\omega_1 t}\right.$$

$$\dot{x}(t) = \left(\frac{x_0}{2\omega_1}\right)(\omega_1^2-\beta^2)e^{-\beta t}\left[2\,\mathrm{senh}\,\omega_1 t\right]$$

COMO $\beta > \omega_1 \Rightarrow \omega_1^2 - \beta^2 < 0$

O MÁXIMO OU MÍNIMO $t = 0$ e $t = \infty$. EM t_c

A FORMA DO GRÁFICO É