

FI255 - Tópicos de Óptica e Fotônica II

Óptica Não-Linear

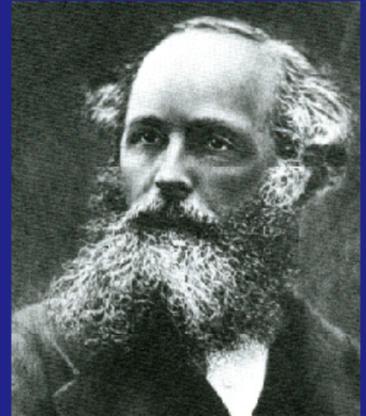
4^a. aula

Prof. Cid B. de Araújo
UNICAMP - 23 de março de 2018

Breve revisão da 3^a. Aula

Equações de Maxwell

REVISÃO



$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

James Clerk Maxwell
(1831-1879)

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\rho = \rho_f + \rho_b = \rho_f - \nabla \cdot \vec{P} \qquad J_f = \frac{\partial \rho_f}{\partial t} \qquad J_b = \frac{dP}{dt}$$

REVISÃO

$$\rho_f = J_f = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho_b}{\epsilon_0} = \frac{-\nabla \cdot \vec{P}}{\epsilon_0}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E} \quad \nabla \cdot \epsilon \vec{E} = 0$$

Meio homogêneo: $\epsilon = constante$

$$\nabla \cdot \vec{E} = 0$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M} = \mu \vec{H}; \quad \vec{M} = 0$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_b + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = \mu_0 \left(\frac{d \vec{P}}{dt} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = \mu_0 \epsilon \frac{d \vec{E}}{dt}$$

REVISÃO

$$\nabla \cdot \vec{E} = 0 \quad \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \mu_0 \epsilon \frac{d \vec{E}}{dt}$$

$$\nabla \times \nabla \times \vec{E} = - \nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = - \frac{\partial}{\partial t} (\nabla \times \vec{B})$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$$

ondas
eletromagnéticas

Analogamente:

$$\boxed{\nabla^2 \vec{B} = \mu_0 \epsilon \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Equação de onda

$$\nabla^2 \vec{E} = \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

REVISÃO

Análoga da equação de uma onda acústica ou ondas na superfície da água.

Velocidade de propagação da onda: $\frac{1}{v^2} = \mu_0 \epsilon$.

No vácuo: $\frac{1}{c^2} = \mu_0 \epsilon_0$

$$c = 3 \times 10^8 \text{ m/s}$$

Solução geral:

$$\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}, t) \exp[i\phi(\vec{r}, t)]$$

↑
amplitude

↑
fase

$$\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}, t) \exp[i\phi(\vec{r}, t)]$$

Mas, o campo elétrico é uma grandeza física real, e então deve ser representado por

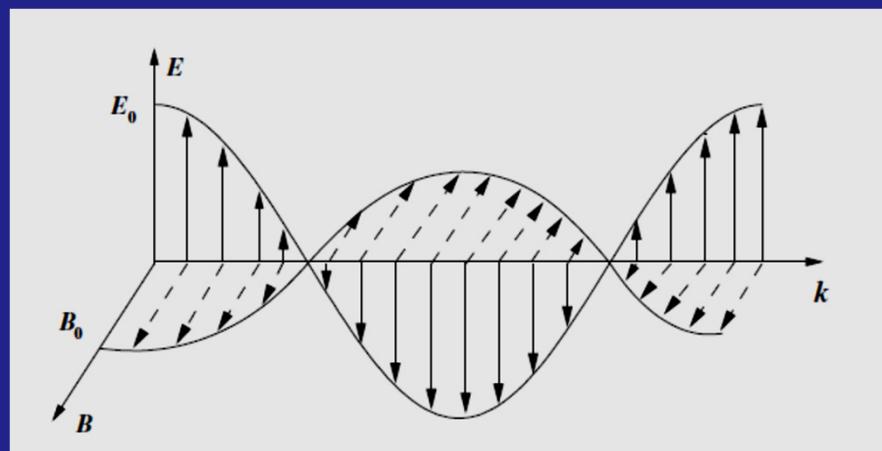
$$\vec{E}(\vec{r}, t) = \frac{1}{2} \{ \vec{E}_0(\vec{r}, t) \exp[i\phi(\vec{r}, t)] + \text{c.c.} \}$$

$$\vec{k} \cdot \vec{E}_0 = 0$$

$$\vec{k} \cdot \vec{B}_0 = 0$$

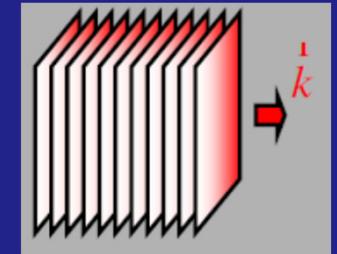
$$\vec{B}_0 = \frac{\vec{k} \times \vec{E}_0}{\omega}$$

$$\vec{E}_0 = - \frac{\vec{k} \times \vec{B}_0}{\omega}$$



$$\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}, t) \exp[i\phi(\vec{r}, t)]$$

$$\phi(\vec{r}, t) = \vec{k} \cdot \vec{r} - \omega t = kr \cos\varphi - \omega t = kr_0 - \omega t$$



Onda Plana: Sentido físico ????????

REVISÃO

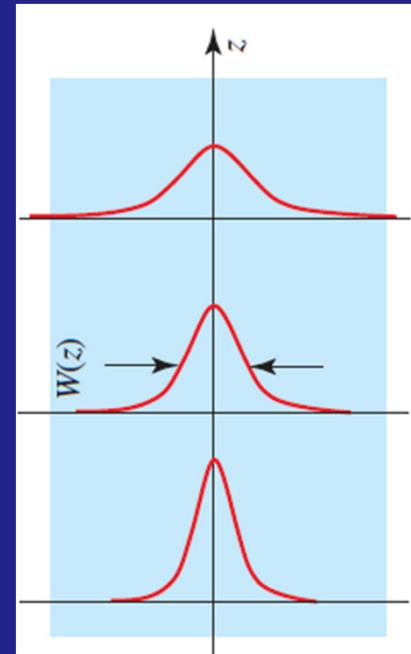
Energia localizada em todas as regiões

Energia total é infinita

Sistema físico real



difração



Solução: feixe óptico

REVISÃO

Livro: YARIV

$$\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}, t) \exp[i\phi(\vec{r}, t)] = \vec{E}(\vec{r}) \exp(-i\omega t)$$

$$\nabla^2 \vec{E} = \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} - \mu_0 \epsilon (-i\omega)^2 \vec{E}(\vec{r}) = 0$$
$$(\nabla^2 + k^2) \vec{E}(\vec{r}) = 0$$

$$\vec{E}(\vec{r}) = \vec{\varepsilon}(\vec{r}) \exp(ikz)$$

$$\left\{ \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + k^2 \right\} \vec{\varepsilon}(\vec{r}) \exp(ikz) = 0$$

Slowly varying envelope approximation

SVEA

$$\frac{\partial^2 \varepsilon(\vec{r})}{\partial z^2} \ll k \frac{\partial \varepsilon(\vec{r})}{\partial z}$$

Variação lenta na escala do comprimento de onda

$$\nabla_{\perp}^2 \varepsilon(\vec{r}) + 2ik \frac{\partial \varepsilon(\vec{r})}{\partial z} = 0$$

Se o feixe de luz tem simetria cilíndrica

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \varepsilon(\vec{r})}{\partial \rho} \right) + 2ik \frac{\partial \varepsilon(\vec{r})}{\partial z} = 0$$

$$\rho = (x^2 + y^2)^{1/2}$$

$$\varepsilon(\rho, z) = A \exp \left\{ i \left[p(z) + \frac{k\rho^2}{2q(z)} \right] \right\}$$

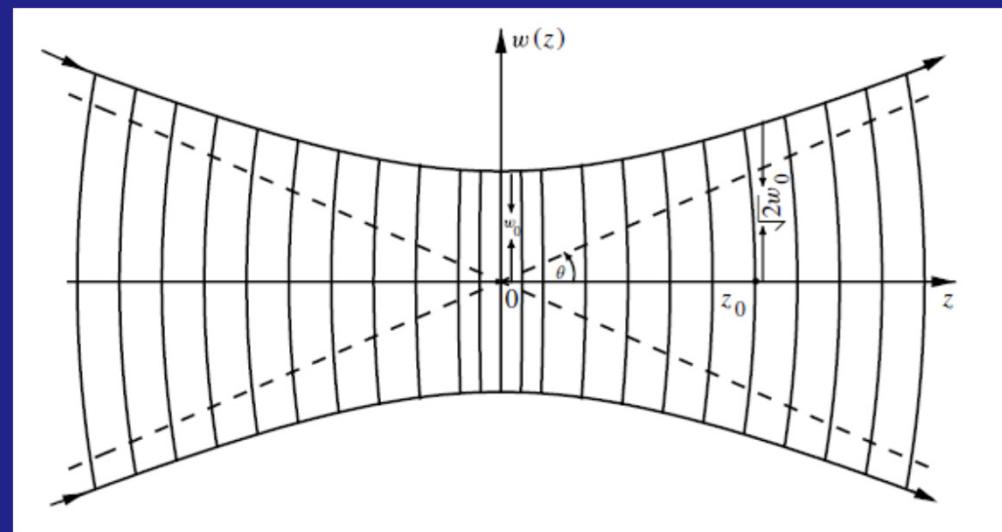
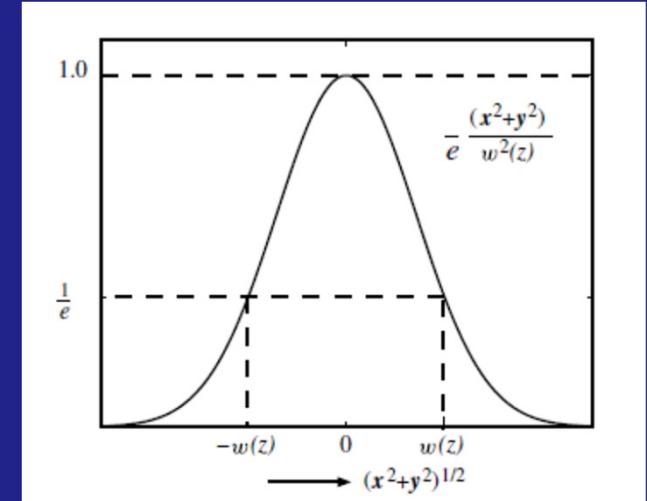
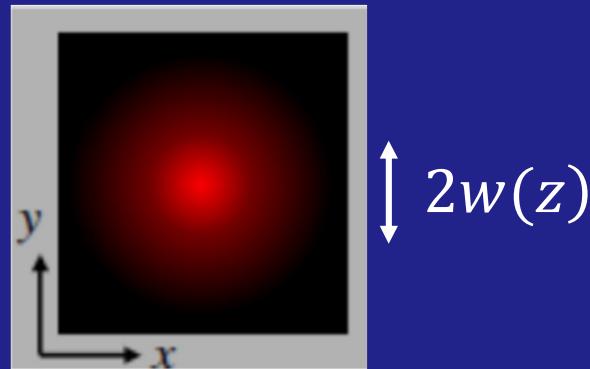
$$E(\rho) = A \frac{w_0}{w(z)} \exp \left[-\frac{\rho^2}{w^2(z)} \right]$$

$$w^2(z) = w_0^2 (1 + z^2/z_0^2) \quad w_0^2 = \frac{2z_0}{k}$$

$$E(\rho) = A \frac{w_0}{w(z)} \exp \left[-\frac{\rho^2}{w^2(z)} \right]$$

$$w(z=0) = w_0$$

$$w(z=z_0) = \sqrt{2} w_0$$



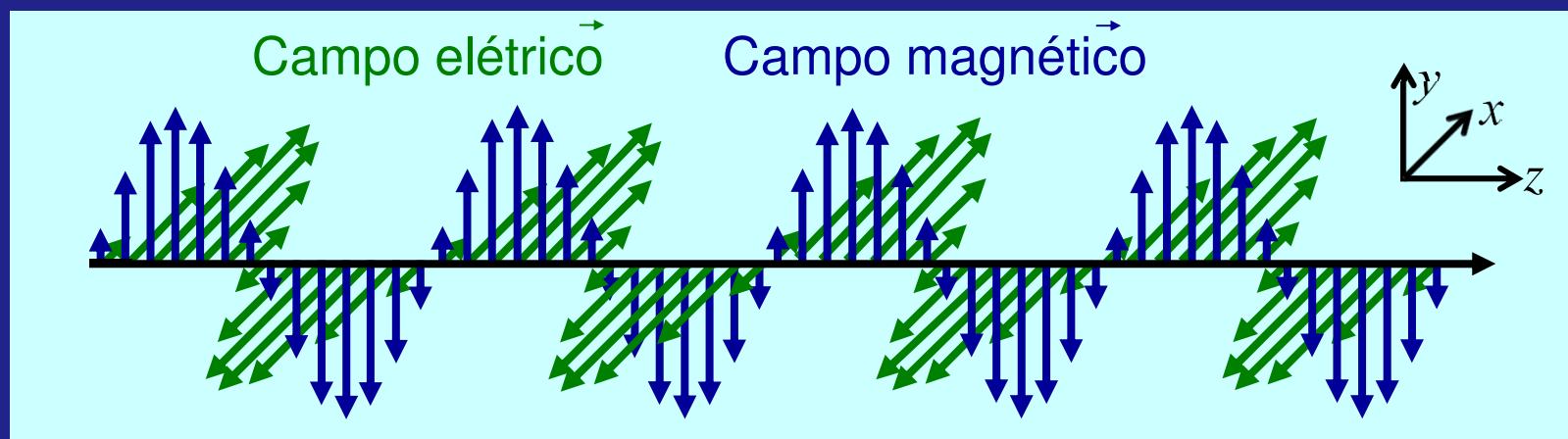
$$\theta = \lim_{z \rightarrow \infty} \frac{dw(z)}{dz} = \frac{w_0}{z_0}$$

$$\frac{w_0}{z_0} = \frac{\lambda}{\pi w_0 n_0}$$

Onda eletromagnética

Os campos elétrico e magnético estão em fase

No instante t



Os campos elétrico e magnético, e o vetor de onda são perpendiculares:

$$\vec{E} \times \vec{B} \propto \vec{k}$$

Densidade de energia na onda EM

Densidade de energia no campo elétrico: $U_E = \frac{1}{2} \epsilon E^2$

Densidade de energia no campo magnético: $U_B = \frac{1}{2} \frac{1}{\mu} B^2$

Usando $B = E/c$ e $c = \frac{1}{\sqrt{\epsilon\mu}}$, obtemos:

$$U_B = \frac{1}{2\mu} B^2 = \frac{1}{2\mu} (E^2 \epsilon\mu) = \frac{1}{2} \epsilon E^2 = U_E$$

Portanto:

$$U_B + U_E = \epsilon E^2$$

Densidade total de energia

As densidades de energia elétrica e magnética são iguais.

Por que desprezamos o campo magnético para descrever a interação da luz com a matéria?

A força sobre uma carga, q , é:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad \vec{v} \text{ é a velocidade da carga}$$

A razão entre as forças:

$$\frac{|F_{magnética}|}{|F_{elétrica}|} = \frac{qvB \operatorname{sen} \theta}{qE} \leq \frac{qvB}{qE} = \frac{v}{c}$$

como $v \ll c$, podemos desprezar a força magnética

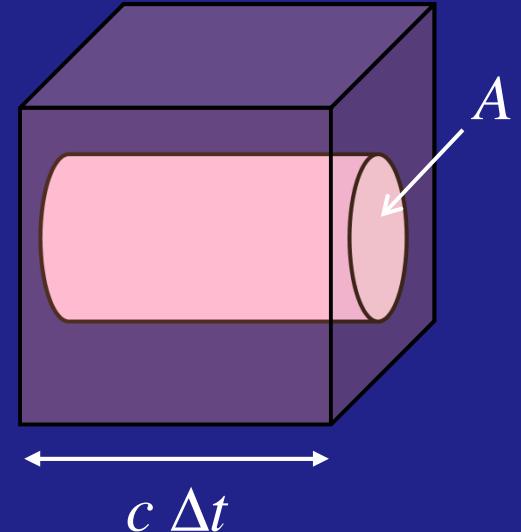
Vetor de Poynting : $\vec{S} = c^2 \epsilon \vec{E} \times \vec{B}$

U = densidade de energia

Potência por unidade de área em um feixe.

Energia passando através da área A no tempo Δt :

$$= U V = U A c \Delta t$$



portanto

$$= UV / (A \Delta t) = UA c \Delta t / (A \Delta t) = U c = c \epsilon E^2$$

$$= c^2 \epsilon EB$$

Na direção

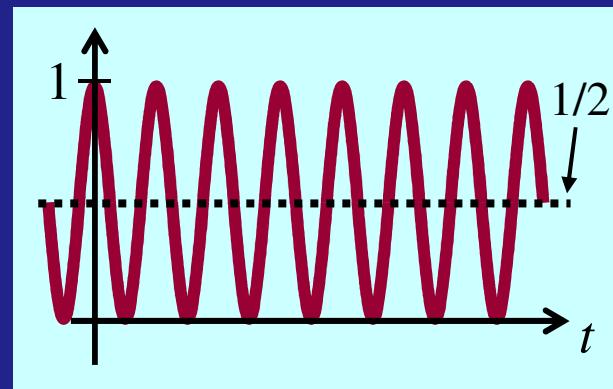
$$\vec{E} \times \vec{B} \propto \vec{k}$$

Intensidade = potência média por unidade de área

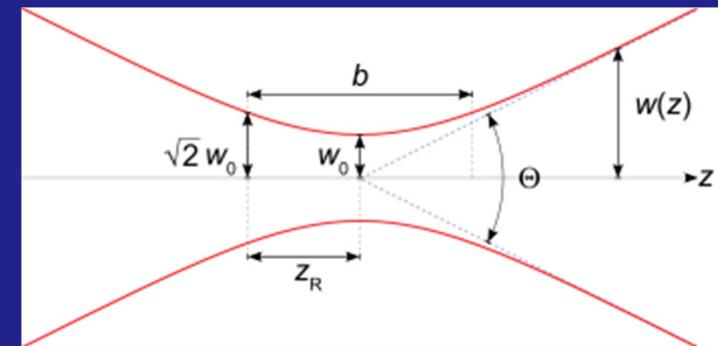
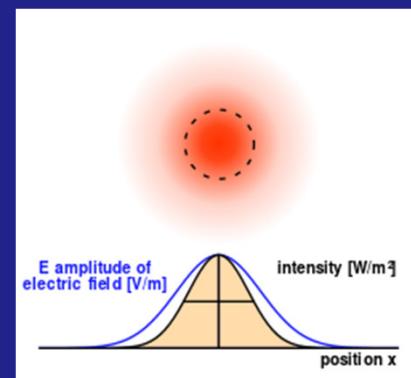
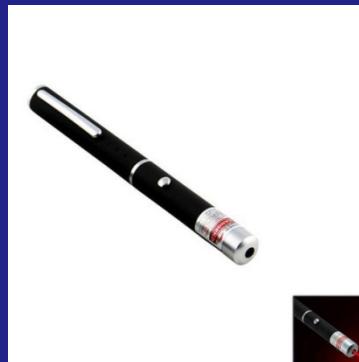
$$I(\vec{r}, t) = \langle |\vec{S}(\vec{r}, t)| \rangle = \frac{1}{T} \int_{t-T/2}^{t+T/2} |\vec{S}(\vec{r}, t')| dt'$$

$$\langle |\vec{S}(\vec{r}, t)| \rangle = c^2 |\vec{E}_0 \times \vec{B}_0| \langle \cos^2(\vec{k} \cdot \vec{r} - \omega t - \theta) \rangle$$

$$\cos^2(\omega t - a)$$



$$I(\vec{r}, t) = \frac{1}{2} c^2 \epsilon |\vec{E}_0 \times \vec{B}_0| = \frac{1}{2} c \epsilon |\vec{E}_0|^2$$



$$P = 5 \text{ mW}; w_0 = 100 \times 10^{-6} \text{ m}$$

$$I = \frac{P}{\text{área}} = \frac{P}{\pi w_0^2}$$

$$I = 2n_0 \left(\frac{\epsilon_0}{\mu_0} \right)^{0.5} |E|^2$$

|E| = 55 \text{ V/cm}

Laser pulsado
300 kW

$$|E| = 4.4 \times 10^4 \text{ V/cm}$$



Intensidade da soma de duas ondas EM

Se ambas forem proporcionais a: $\exp(i(\vec{k} \cdot \vec{r} - \omega t))$

$$I = \frac{1}{2} c \epsilon \vec{E}_0 \cdot \vec{E}_0^* = \frac{1}{2} c \epsilon [E_{0x} E_{0x}^* + E_{0y} E_{0y}^* + E_{0z} E_{0z}^*]$$

Polarizações perpendiculares (x e y):

$$I = \frac{1}{2} c \epsilon [E_{0x} E_{0x}^* + E_{0y} E_{0y}^*] = I_x + I_y$$

Mesmas polarizações - por exemplo: $E_{0x} = E_1 + E_2$

$$I = \frac{1}{2} c \epsilon [E_1 E_1^* + 2 \operatorname{Re}\{E_1 E_2^*\} + E_2 E_2^*]$$

$$I = I_1 + c \epsilon \operatorname{Re}\{E_1 E_2^*\} + I_2$$

Interferência óptica – só ocorre para feixes que têm mesma polarização.

Intensidade da soma de duas ondas

	Mesmas polarizações	Polarizações perpendiculares
Soma coerente	$I = I_1 + I_2 + c\varepsilon \text{Re}\{E_1 E_2^*\}$	
Frequências iguais		
Soma incoerente	$I = I_1 + I_2$	
Frequências diferentes		$I = I_1 + I_2$

Interferência só ocorre quando as ondas têm mesma côr e mesma polarização.

Óptica Não Linear

Interação luz-matéria sob condições que violam o princípio de superposição linear

Polarização óptica = momento de dipolo por unidade de volume

$$\vec{P} = \epsilon_0 \overleftrightarrow{\chi^{(1)}} \otimes \vec{E} + \epsilon_0 \overleftrightarrow{\chi^{(2)}} \otimes \vec{E} \vec{E} + \epsilon_0 \overleftrightarrow{\chi^{(3)}} \otimes \vec{E} \vec{E} \vec{E} + \dots$$

$\overleftrightarrow{\chi^{(n)}}$ ≡ 0 , n = par (meios centro-simétricos)

\vec{P} induz mudança na velocidade da luz no meio e novos comprimentos de onda podem ser gerados

Significado de cada parcela

Vetor polarização

$$\overrightarrow{P^{(1)}} = \epsilon_0 \overleftrightarrow{\chi^{(1)}} \otimes \overrightarrow{E}$$

Componentes da polarização

$$P_i^{(1)} = \epsilon_0 \chi_{ij}^{(1)} E_j$$

$$\overrightarrow{P^{(2)}} = \epsilon_0 \overleftrightarrow{\chi^{(2)}} \otimes \vec{E} \vec{E}$$

$$P_i^{(2)} = \epsilon_0 \chi_{ijk}^{(2)} E_j E_k$$

$$\overrightarrow{P^{(3)}} = \epsilon_0 \overleftrightarrow{\chi^{(3)}} \otimes \vec{E} \vec{E} \vec{E}$$

$$P_i^{(3)} = \epsilon_0 \chi_{ijkl}^{(3)} E_j E_k E_l$$

analogamente para os demais têrmos ...

Susceptibility tensors \Leftrightarrow structural symmetry

$$\chi_{ij}^{(1)}(\omega, \omega)$$

9 elements
Isotropic media: scalar

$$\chi_{ijk}^{(2)}(\omega; \omega_1, \omega_2)$$

27 elements
All elements are identically null
in systems with inversion symmetry

$$\chi_{ijkl}^{(3)}(\omega; \omega_1, \omega_2, \omega_3)$$

81 elements
Isotropic media: 21 nonzero
elements (only 3 are independent)

Higher-order susceptibilities: more elements

The early days...

1958 - Schawlow and Townes; optical maser (laser) proposed

1960 - Maiman, ruby laser demonstrated

1961 - Franken, Hill, Peters, and Weinreich, second-harmonic generation (SHG) observed

1961- Kaiser and Garret, two –photon absorption in Eu:CaF₂

1962 - McClung, Hellwarth, Woodbury, and Ng, stimulated Raman scattering (SRS)

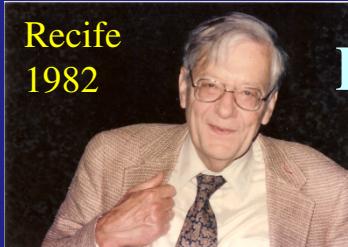
1962 - Armstrong, Bloembergen, Ducing, and Pershan; systematic formulation of NLO

1964 -Bloembergen, Shen, quantum formalism of NLO

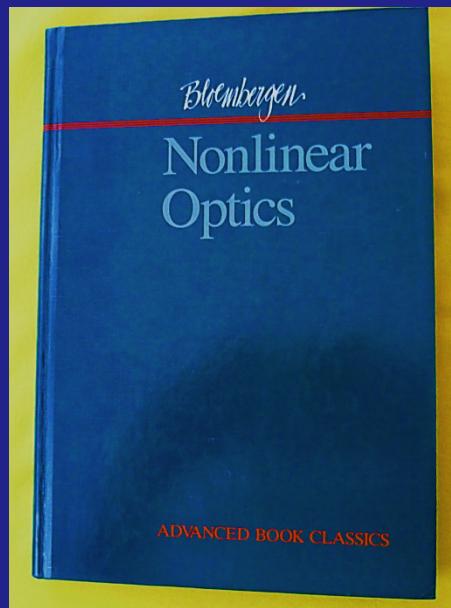
1964 - Chiao, Townes, and Stoicheff, stimulated Brillouin scattering (SBS)

1964 - Chiao, Garmire, and Townes, self-trapping of light

Interactions between Light Waves in a Nonlinear Dielectric*

J. A. ARMSTRONG, N. BLOEMBERGEN, J. DUCUING,[†] AND P. S. PERSHAN*Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts*

Physics Nobel Prize
1981



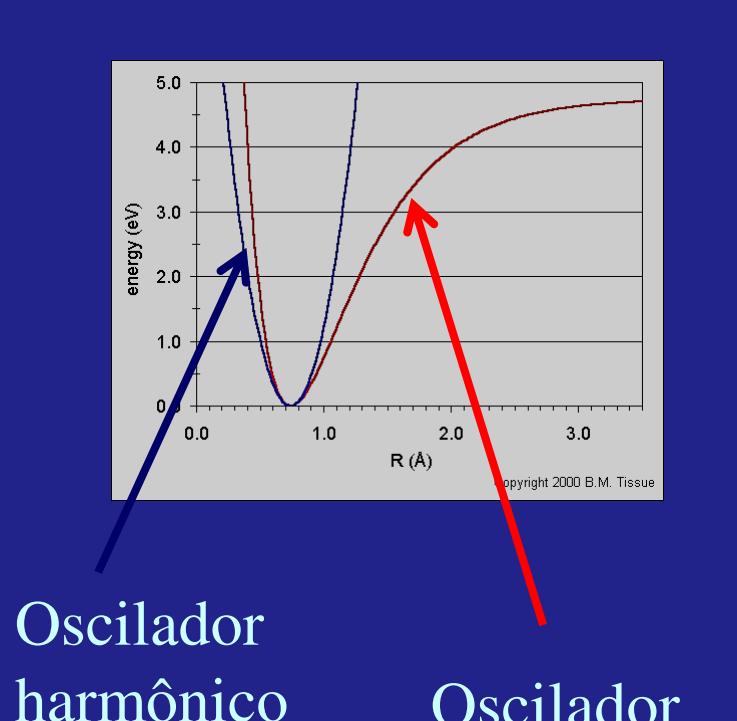
Conservation laws in nonlinear optics[‡]

N. Bloembergen

TABLE I. Historical dates of linear and nonlinear optical laws.

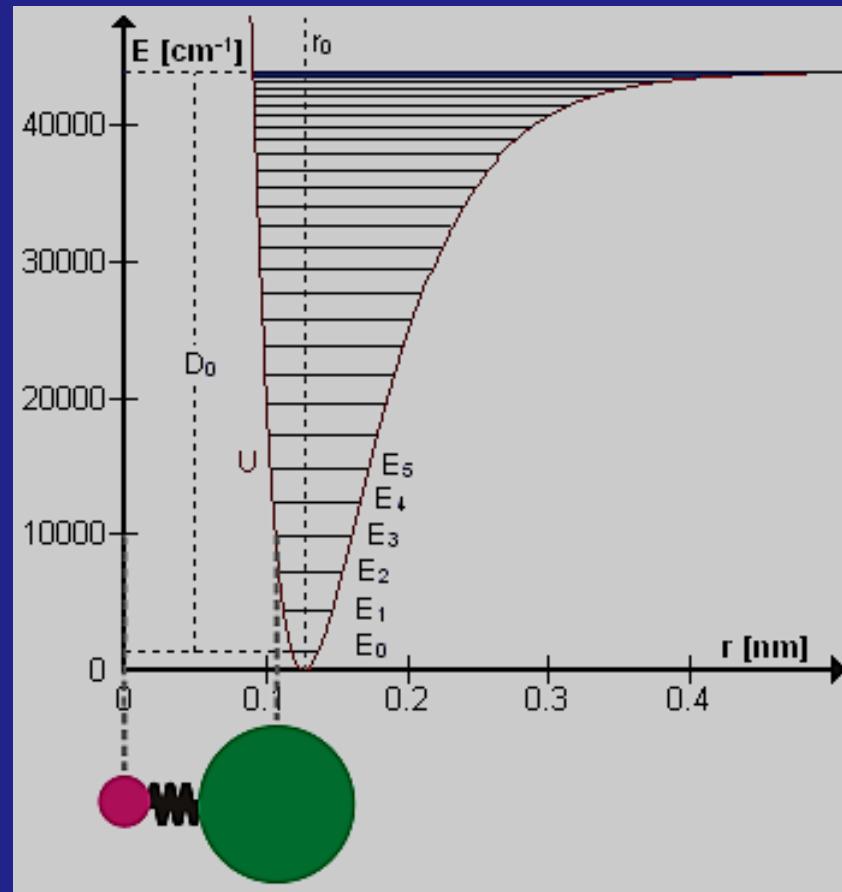
	Linear	Nonlinear
Law of reflection	1st century (Hero of Alexandria)	1962 (Bloembergen and Pershan)
Law of refraction	1621 (Snell)	1962 (Bloembergen and Pershan)
Intensity of reflected and refracted light	1823 (Fresnel)	1962 (Bloembergen and Pershan)
Conical refraction Theory	1833 (Hamilton)	1969 (Bloembergen and Shih)
Experiment	1833 (Lloyd)	1977 (Schell and Bloembergen)

Oscilador anarmônico devido à grande amplitude do campo elétrico



Oscilador
harmônico

Oscilador
anarmônico



$$U(x) = \frac{1}{2}m\omega_0^2x^2 + \frac{1}{3}mKx^3 + \frac{1}{4}mK'x^4 + \dots$$

Força restauradora

$$\text{Considerando só até 2a. ordem: } F(x) = -\frac{dU}{dx} = -m\omega_0^2x - mKx^2$$

$$m \frac{d^2x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_0^2 x + mKx^2 = -e\varepsilon(t)$$

$$\varepsilon(t) = \varepsilon_0 \cos \omega t = \frac{1}{2} \varepsilon_0 (e^{i\omega t} + e^{-i\omega t})$$

Solução tentativa

$$x(t) = \frac{1}{2} (X_1 e^{i\omega t} + X_2 e^{i2\omega t} + c.c.) \quad X_1 \gg X_2$$

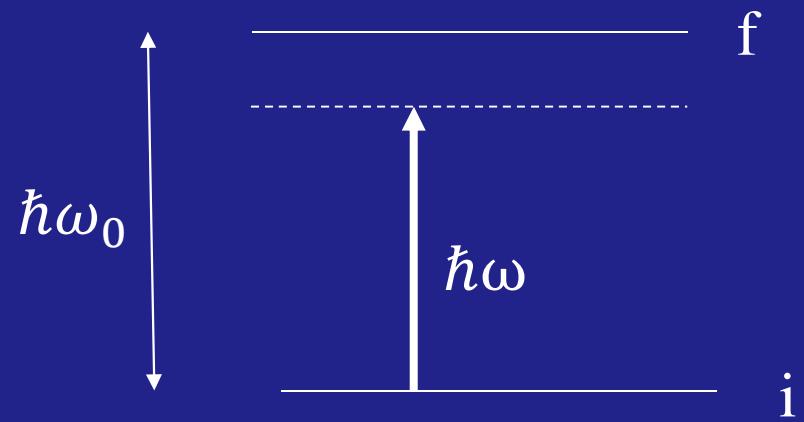
Polarização
induzida

$$X_1 = - \frac{e\varepsilon_0}{m} \frac{1}{(\omega_0^2 - \omega^2) + i\gamma\omega}$$

$$\begin{aligned} P(\omega, t) &= -Nex(\omega, t) = -\frac{Ne}{2} (X_1 e^{i\omega t} + c.c.) \\ &= \epsilon_0 \chi^{(1)}(\omega) \varepsilon(t) \end{aligned}$$

$$\boxed{\chi^{(1)}(\omega; \omega) = \frac{Ne^2}{m\epsilon_0[(\omega_0^2 - \omega^2) + i\gamma\omega]}}$$

$$\chi^{(1)}(\omega; \omega) = \frac{Ne^2}{m\epsilon_0[(\omega_0^2 - \omega^2) + i\gamma\omega]}$$

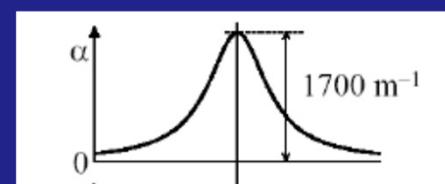
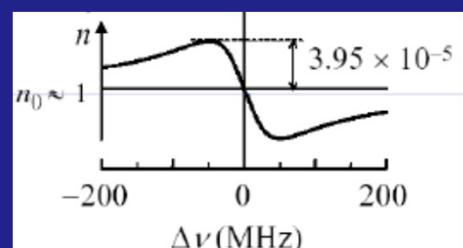


$n_0 \propto \operatorname{Re} \chi^{(1)}(\omega; \omega)$

Índice de refração

$\alpha_0 \propto \operatorname{Im} \chi^{(1)}(\omega; \omega)$

Coeficiente de absorção

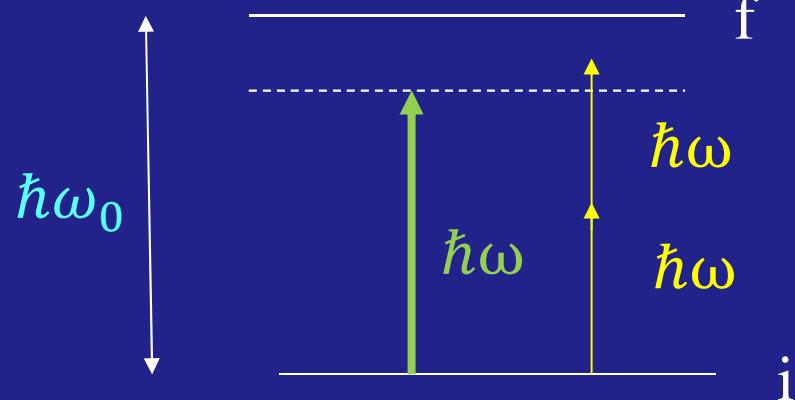


2^a.aula – sódio gasoso

$$\begin{aligned}
 P(2\omega, t) &= -Nex(2\omega, t) = -\frac{Ne}{2} (X_2 e^{i2\omega t} + c.c.) \\
 &= \epsilon_0 \chi^{(2)} \varepsilon(t)^2
 \end{aligned}$$

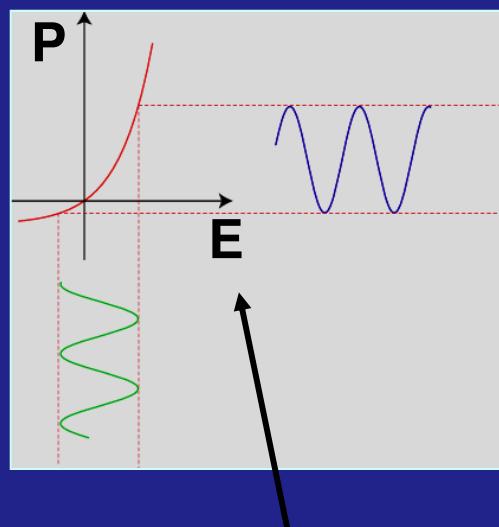
$$\chi^{(2)}(2\omega, \omega, \omega) = \frac{Ne^3 K}{\epsilon_0 m^2 [(\omega_0^2 - \omega^2) + i\gamma\omega]^2 (\omega_0^2 - 4\omega^2 + i2\omega\gamma)}$$

ressonâncias

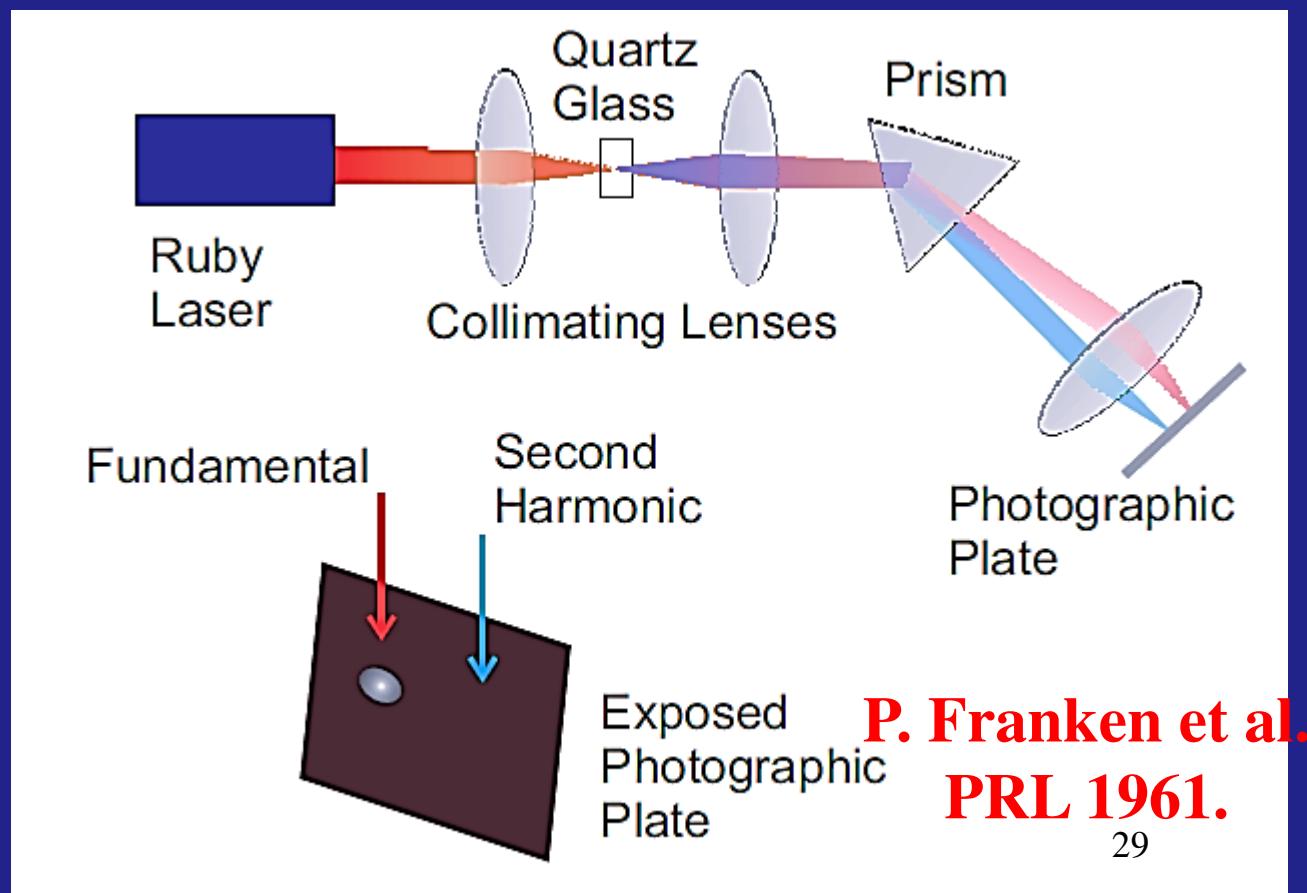


$\chi^{(2)}$ - Second Harmonic Generation

$$P^{(2)} = \epsilon_0 \chi^{(2)} (E \cos \omega t)^2 = \frac{1}{2} \epsilon_0 \chi^{(2)} E^2 (1 + \cos 2\omega t)$$



Electric field of
intense light



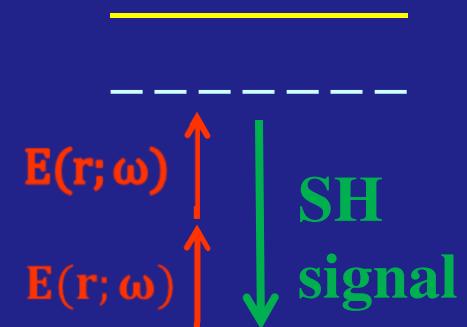
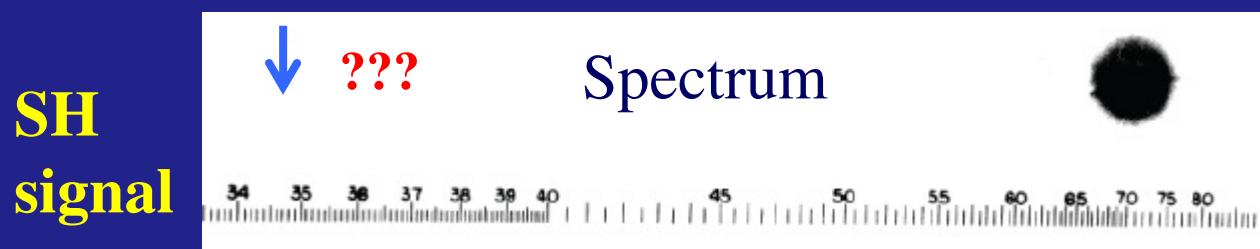
P. Franken et al.
PRL 1961.

GENERATION OF OPTICAL HARMONICS*

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich

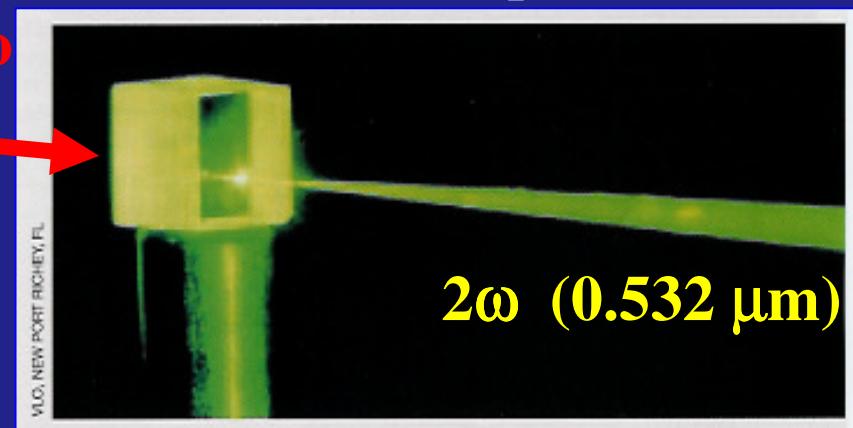
The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan

(Received July 21, 1961)



Cristal transparente

Infra-vermelho

 ω (1.064 μm)

Phase matching:
 $k(2\omega) = 2k(\omega)$



$$\varepsilon(t) = \varepsilon_1 \cos \omega_1 t + \varepsilon_2 \cos \omega_2 t$$

$$P = -Ne \quad x = P^{(1)} + P^{(2)} + P^{(3)} \dots$$

Polarização de primeira ordem

$$P^{(1)}(\omega_1) \text{ and } P^{(1)}(\omega_2)$$

Polarização de segunda ordem

$$P^{(2)}(2\omega_1), \quad P^{(2)}(2\omega_2), \quad P^{(2)}(\omega_1 \pm \omega_2), \quad P^{(2)}(0)$$

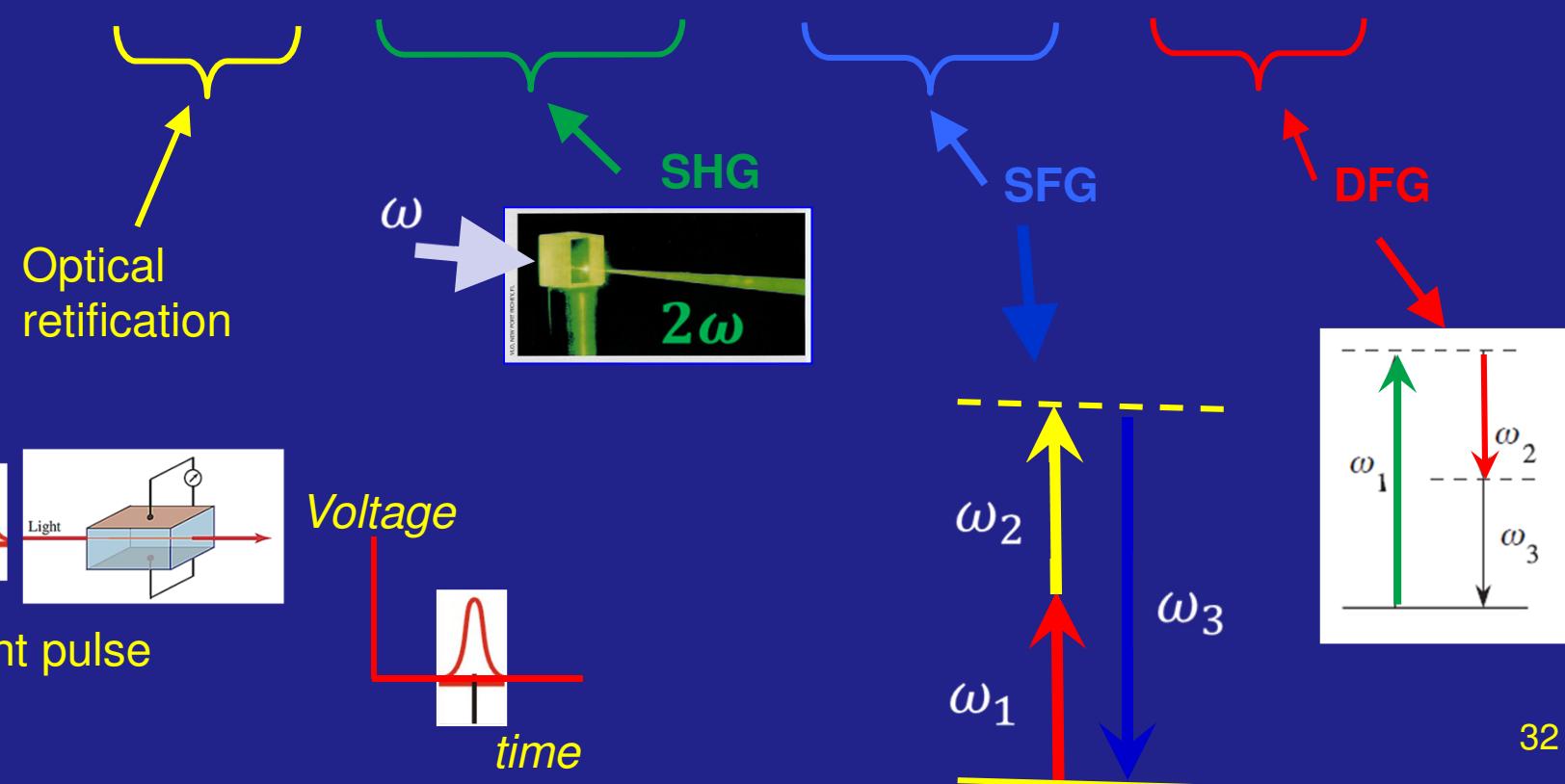
Polarização de terceira ordem... etc

$\chi^{(2)}$

Second order polarization

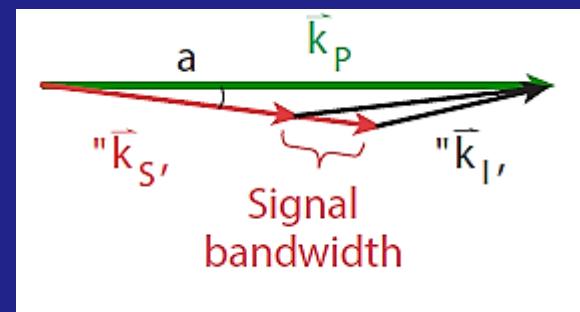
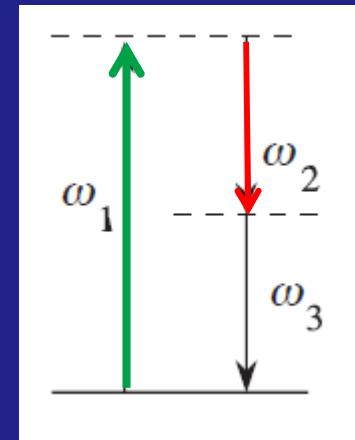
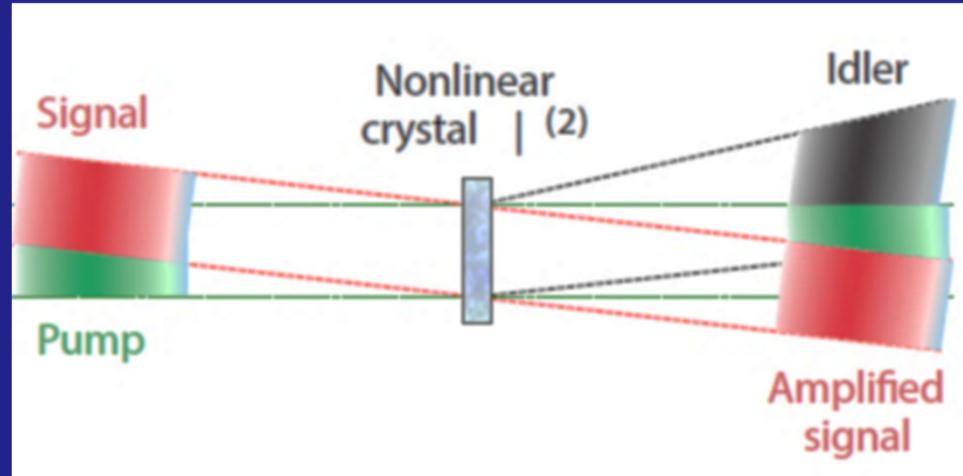
Non centro-symmetric materials

$$P^{(2)} = \epsilon_0 \chi^{(2)} (E_1 \cos \omega_1 t + E_2 \cos \omega_2 t)^2 =$$
$$P_0^{(2)} + P_{2\omega_1}^{(2)} + P_{2\omega_2}^{(2)} + P_{\omega_1+\omega_2}^{(2)} + P_{\omega_1-\omega_2}^{(2)}$$



$\chi^{(2)}$: Amplificador óptico paramétrico

Pode também amplificar pulsos de banda larga



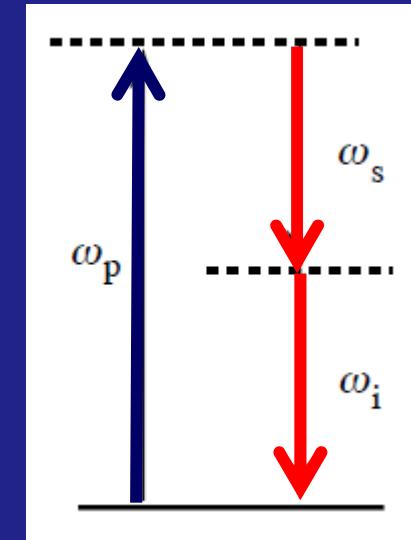
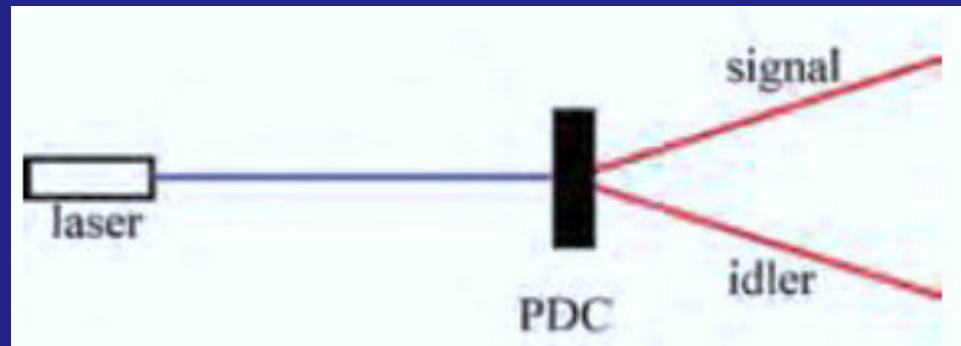
100 fs

Pump: 800 nm

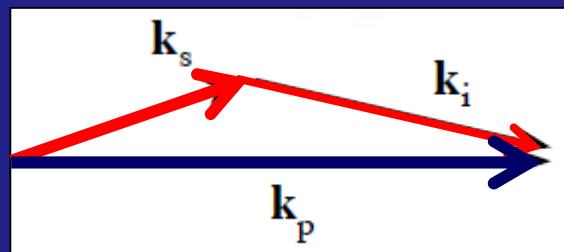
Signal: 400 nm - 2 μm



$\chi^{(2)}$: Parametric down-conversion



Phase - matching



Signal and idler photons are entangled in polarization, time, energy, position, transverse momentum, angular position and orbital angular momentum

Entanglement is important for fundamental tests of QM and quantum technologies.

$$P^{(3)}(t) = \epsilon_0 \chi^{(3)} E^3(t)$$

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + E_3 e^{-i\omega_3 t}$$

$$\begin{array}{cccccc} \omega_1, & \omega_2, & \omega_3, & 3\omega_1, & 3\omega_2, & 3\omega_3, (\omega_1 - \omega_2 + \omega_3,), \\ (-\omega_1 + \omega_2 + \omega_3,), (2\omega_1 \pm \omega_2,), (2\omega_1 \pm \omega_3,), (2\omega_2 \pm \omega_1,), \\ (2\omega_2 \pm \omega_3,), & (2\omega_3 \pm \omega_1,), & (2\omega_3 \pm \omega_2,) \end{array}$$

$$P(3\omega_1) = \epsilon_0 \chi^{(3)} E_1^3$$

$$P(\omega_1 + \omega_2 + \omega_3) = 6\epsilon_0 \chi^{(3)} E_1 E_2 E_3$$

$$P(\omega_1 + 2\omega_3) = 3\epsilon_0 \chi^{(3)} E_1 E_3^2$$

$$P^{(3)}(t) = \sum_n P^{(3)}(\omega_n) e^{-i\omega_n t}$$

$\chi^{(3)}$: third order polarization

$$P^{(3)}(r; \omega) e^{-i\omega t} = \epsilon_0 [\chi^{(3)}(r; \omega, \omega, -\omega, \omega) E(r; \omega) E^*(r; -\omega) E(r; \omega)] e^{-i\omega t}$$

NL refractive index

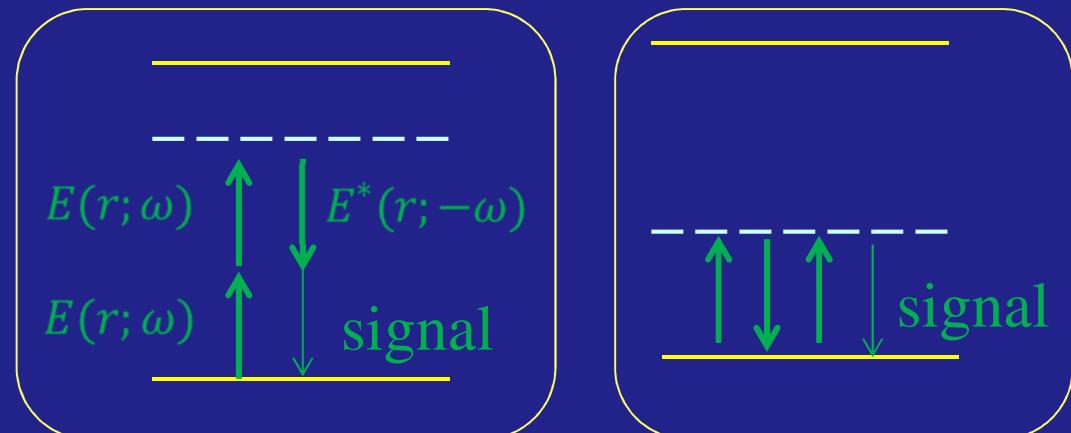
$$n_2 \propto \text{Re } \chi^{(3)}$$

NL absorption coefficient

$$\alpha_2 \propto \text{Im } \chi^{(3)}$$

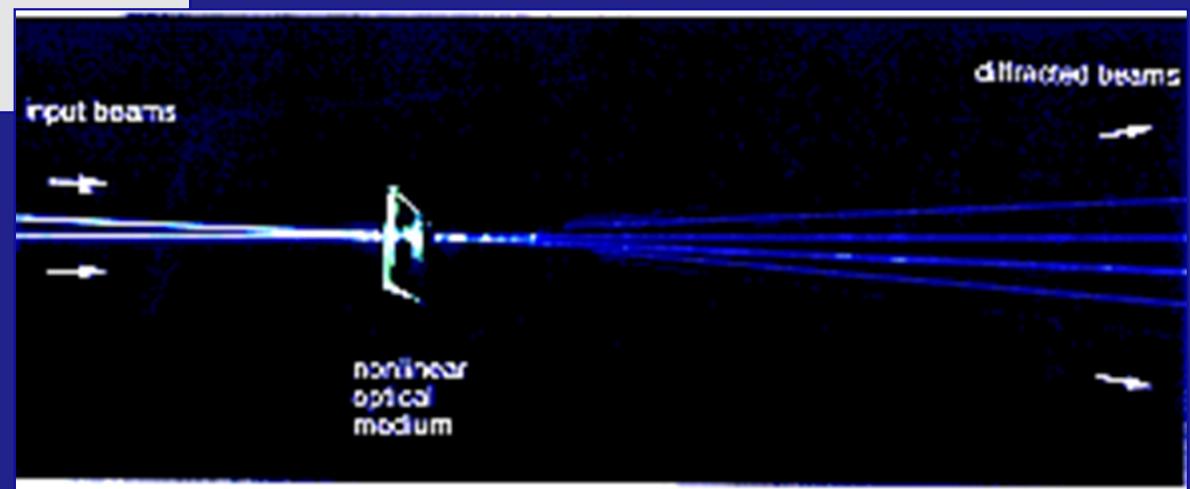
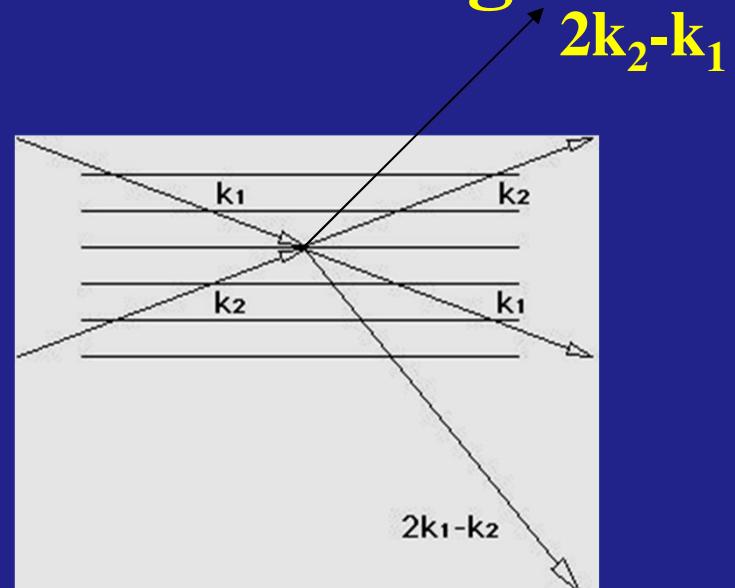
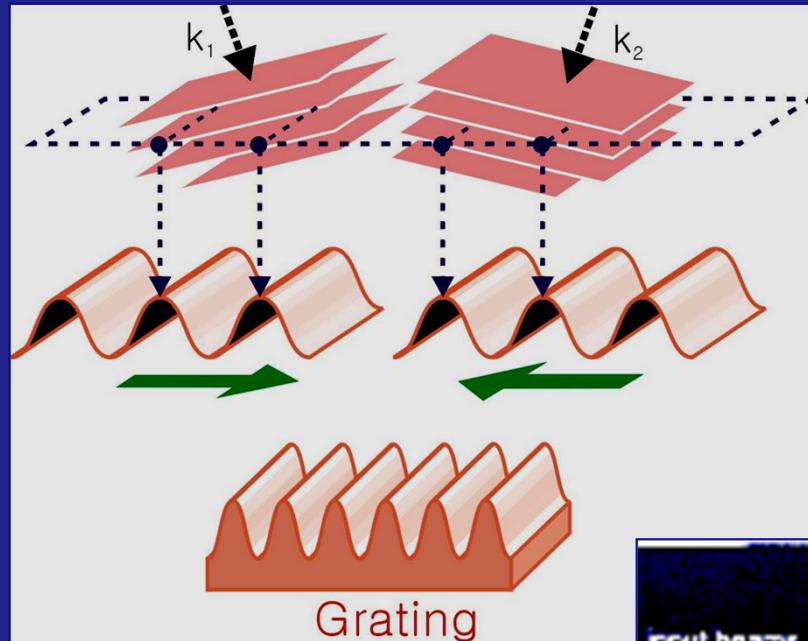
Degenerate wave-mixing

Transparent medium



$\chi^{(3)}$

Degenerate four-wave-mixing



$$P^{(3)} = \epsilon_0 \chi^{(3)} E^3$$

Boyd, Escola Swieca (Belo Horizonte, 2004)

$\chi^{(3)}$: self - focusing

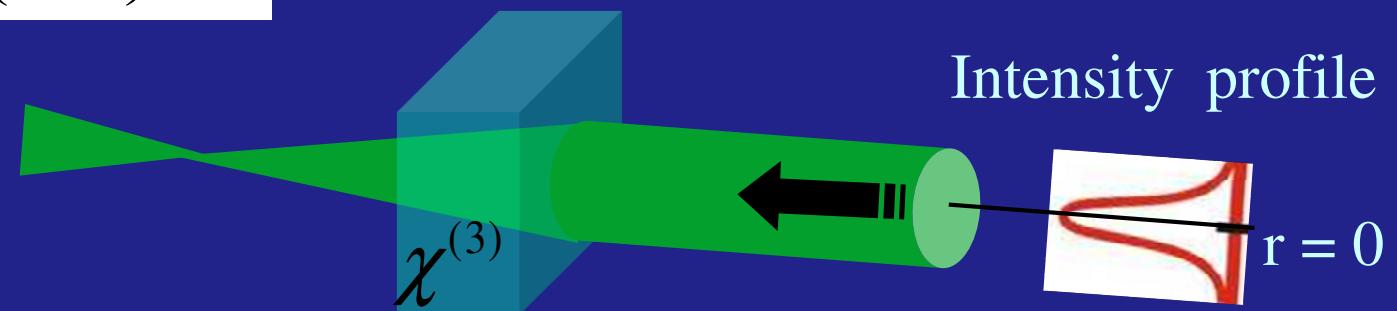
$$n_2 \propto \operatorname{Re} \chi^{(3)}$$

A laser beam with Gaussian intensity profile will induce a Gaussian refractive index profile inside the NL sample.

$$E(\omega) \exp\left(-\frac{r^2}{\Delta^2}\right) e^{-i\omega t}$$

$$n_2 > 0$$

$$I(r) = I_0 \exp\left\{-r^2/w^2\right\}$$



$$n = n_0 + n_2 I_0 \exp\{-r^2/w^2\}$$

Sample behaves as
a convergent lens

$$n_2 \propto \operatorname{Re} \chi(3)$$

Mechanism	$n_2(\text{cm}^2/\text{W})$	$\chi^{(3)}_{1111}(\text{esu})$	Response time (sec)
Electronic Polarization	10^{-16} - 10^{-13}	10^{-14} - 10^{-11}	10^{-15}
Molecular Orientation	10^{-14}	10^{-12}	10^{-12}
Electrostriction	10^{-14}	10^{-12}	10^{-9}
Saturated Absorption	10^{-10}	10^{-8}	10^{-8}
Thermal effects	10^{-6}	10^{-4}	10^{-3}

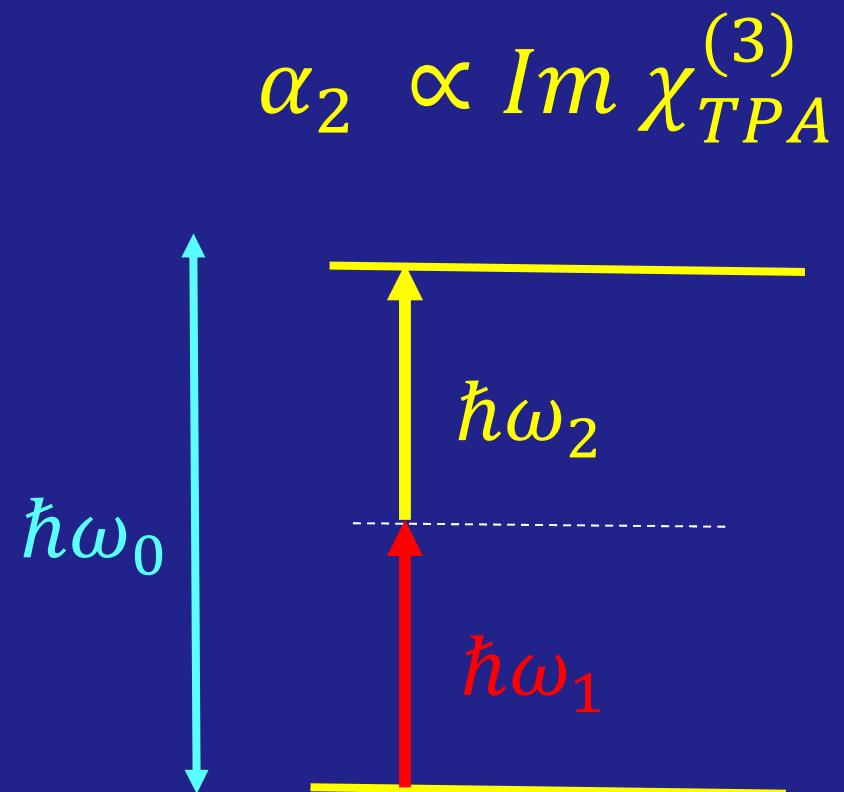
$$\Delta n(r=0) = n_2 I_0$$

$$I_0 = 1 \text{ GW/cm}^2$$

$$\Delta n = 10^{-7} - 10^{-4}$$

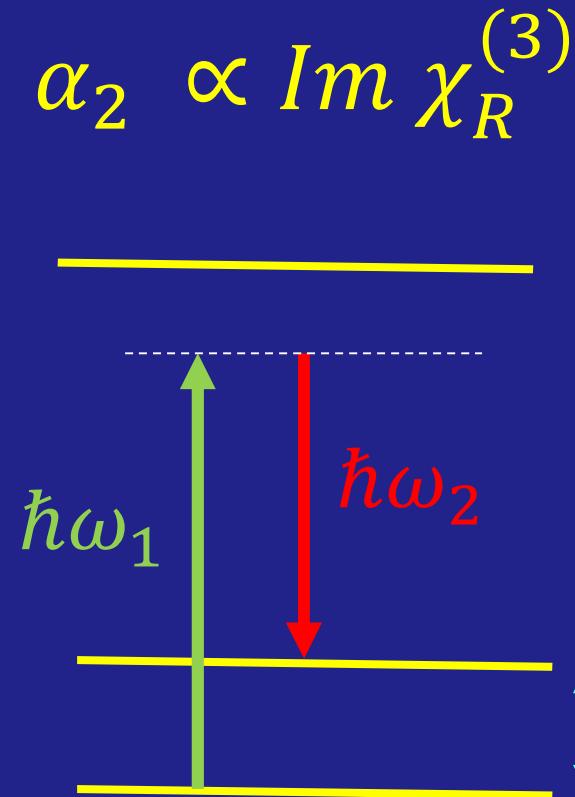
Electronic polarization

Two-photon absorption



$$\hbar\omega_1 + \hbar\omega_2 = \hbar\omega_0$$

Raman process



$$\hbar\omega_1 - \hbar\omega_2 = \hbar\omega_0$$