

FI255 - Tópicos de Óptica e Fotônica II

Óptica Não-Linear

14ª. aula

Prof. Cid B. de Araújo
UNICAMP - 22 de junho de 2018

General theoretical approach

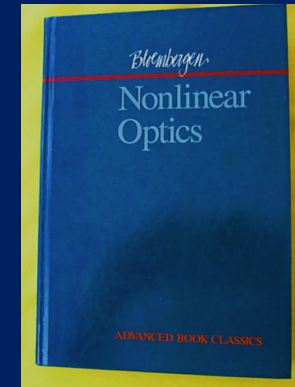
PHYSICAL REVIEW

VOLUME 127, NUMBER 6

SEPTEMBER 15, 1962

Interactions between Light Waves in a Nonlinear Dielectric*

J. A. ARMSTRONG, N. BLOEMBERGEN, J. DUCUING,† AND P. S. PERSHAN
Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts



When there is inversion symmetry:

$$\chi^{(j)} \equiv 0 \\ j = \text{even}$$

$$P_L + P_{NL} = \epsilon_0 \sum_{N=0}^{\infty} \chi^{(2N+1)} E^{(2N+1)}$$

$$n_N \propto \text{Re } \chi^{(2N+1)}$$

Nonlinear refractive index

$$\alpha_N \propto \text{Im } \chi^{(2N+1)}$$

Nonlinear absorption coefficient

linear + nonlinear

$$n = n_0 + n_2 I + n_4 I^2 + n_6 I^3 + \dots$$

$$\alpha = \alpha_0 + \alpha_2 I + \alpha_4 I^2 + \alpha_6 I^3 + \dots$$

High-order nonlinearities

- Generation of higher harmonics
- Multiphoton excitation processes
 - Multi-wave mixing

Studies of multiphoton ionization: 60's; Many studies in the 70's, 80's.

Multiphoton dissociation and ionization processes applied to isotope separation (70's and 80's).

More recently:
Solitons, filamentation, extreme events, generation of VUV and soft X-rays – 400th harmonic, proposals for attosecond X-ray pulse generation

Eq. de Maxwell regime espacial

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \vec{P}_{NL}}{\partial t^2}$$

$$\vec{P}_L = \epsilon_0 \epsilon_L \vec{E}(\vec{r}, t) \quad \vec{P}_{NL} = \epsilon_0 \epsilon_{NL} \vec{E}(\vec{r}, t) \quad \Delta n = n_{2E} |\vec{E}|^2$$

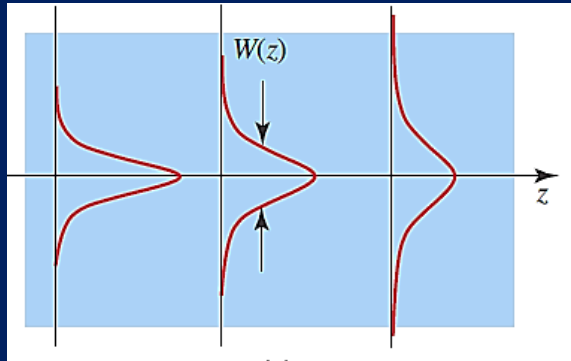
$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 [(n_0 + \Delta n)^2 \vec{E}]}{\partial t^2} = 0$$

$$\vec{E}(\vec{r}, t) = \vec{A}(\vec{r}) \exp[i(kz - \omega t)]$$

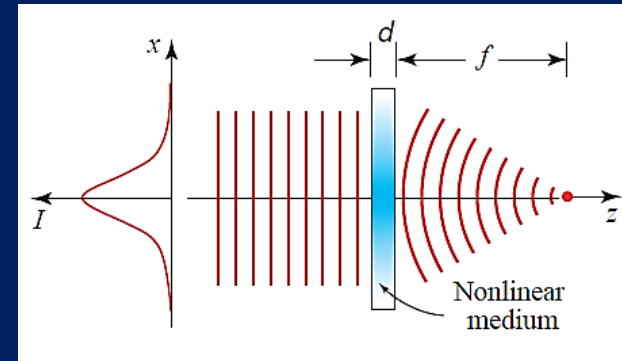
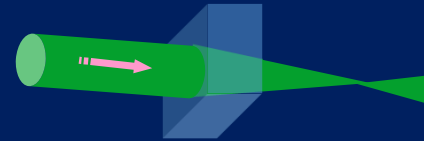
SVEA - slowing varying envelope approximation

$$i \frac{\partial \vec{A}}{\partial z} = - \frac{1}{2k} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \vec{A} - \frac{n_{2E}}{n_0} k |\vec{A}|^2 \vec{A}$$

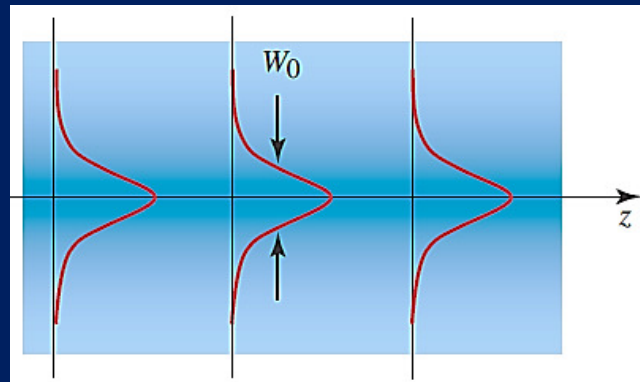
Diffraction



Self-focusing



Bright Spatial Soliton



What is “soliton” ?

A soliton is a wave with a unique shape that travels undisturbed without changing. Solitons require a balance between nonlinearity and dispersion.

Optical solitons are often thought of in the time domain, especially in fibers, where pulse compression due to the optical nonlinearity can overcome the tendency of dispersion to spread pulses out as they travel down a fiber.

This happens only for discrete values of the pulse energy and, for positive nonlinearities, the dispersion must be anomalous (negative).

Solitons are also called solitary waves and were first discovered in water waves traveling down a canal.

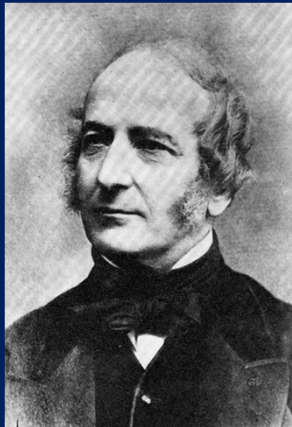
Solitons are unique in that they can interact with other solitons and emerge from the collision unchanged, except for a phase shift.

Solitons

Wave with a unique shape that travels undisturbed without changing

John Scott Russell (1808-1882)

1834: "... large solitary elevation, without change of form or diminution of speed."



Russell, Report on Waves.
Report of the fourteenth meeting of the British Association
for the Advancement of Science, York, September 1844.

How did Russell observe
a soliton?

Bright engineer: invented an improved
steam-driven road carriage in 1833.
The ``Union Canal Society'' of
Edinburgh asked him to set up a
navigation system with steam boats



Solitons



Coast (BBC2 24/04/13)

Many Faces of Solitons

Quantum Field Theory

- Quantum solitons
- Monopoles
- Instantons

General Relativity

- Bartnik-McKinnon solitons (black holes)

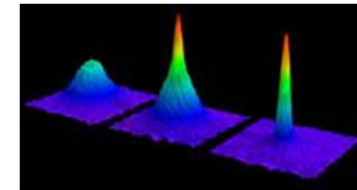
Biochemistry

- Davydov solitons (protein energy transport)

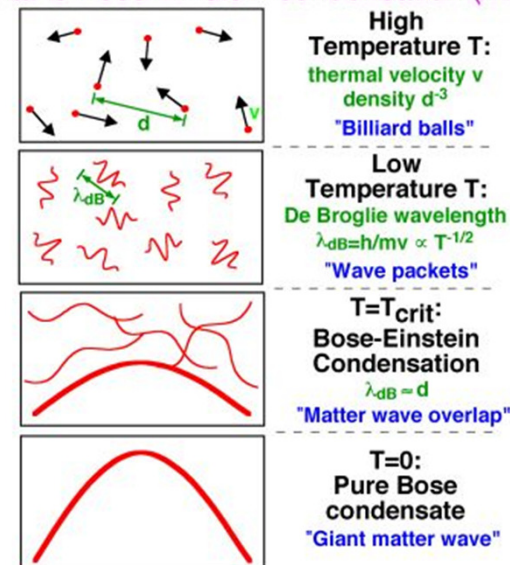
BEC

Cold atoms

- Coherent matter waves
- Dilute alkali gases



What is Bose-Einstein condensation (BEC)?



Optical solitons

solutions of Maxwell's equation with NL polarization terms

Wave with special shape such that propagates without change their shape.

When one soliton interacts with another soliton they do not change their shape but, the phases of the electric fields change.

Temporal and spatial solitons

Nonlinear Schrödinger Equation: (1+1)D

Scalar
theory

$$E(x, z, t) = A_m a(x, z) e^{i(k_0 n z - \omega t)}$$

$$\frac{1}{2k_0 n_0} \frac{\partial^2 a}{\partial x^2} + i \frac{\partial a}{\partial z} + \frac{k_0 n_0 n_2}{2\eta_0} |a|^2 a = 0$$

Diffration x self-focusing

$$\left| \frac{\partial^2 a(x, z)}{\partial z^2} \right| \ll \left| k_0 \cdot \frac{\partial a(x, z)}{\partial z} \right|$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$a(x, z) = \text{sech}(x / X_0) \exp(i z / 2L_d)$$



Soliton solution

$$L_d = X_0^2 k_0 n_0$$

$$X_0 = \frac{1}{k_0 n_0} \sqrt{\frac{2\eta_0}{|n_2| |A_m|}}$$

Zakharov, Shabat, Sov. Phys. JETP 34 (1972) 62

(1+1)D spatial solitons in CS_2

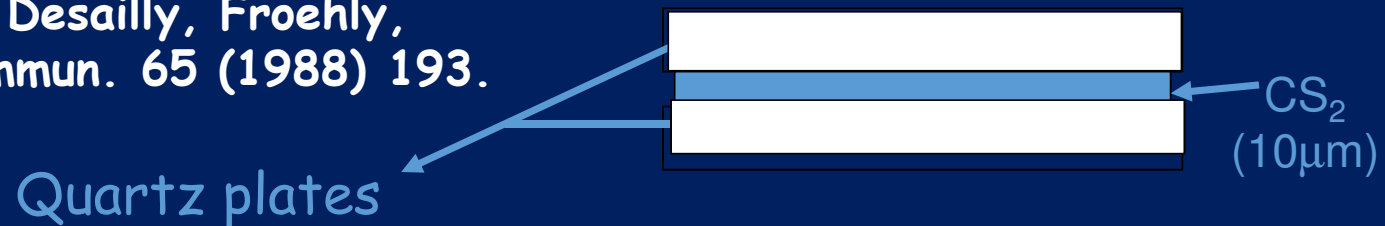
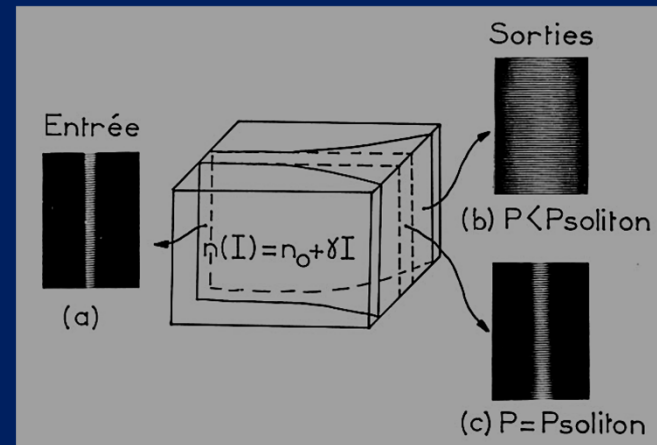
$$n_2 > 0$$

Beam focused by cylindrical lenses

Barthélémy, Maneuf, Froehly,
Opt. Commun. 55 (1985) 201.

Planar waveguides

Maneuf, Desailly, Froehly,
Opt. Commun. 65 (1988) 193.



Picosecond lasers - 532 nm

Carbon disulfide: CS₂

VOLUME 74, NUMBER 15

PHYSICAL REVIEW LETTERS

10 APRIL 1995

Fifth Order Optical Response of Liquid CS₂ Observed by Ultrafast Nonresonant Six-Wave Mixing

Keisuke Tominaga and Keitaro Yoshihara

- n_2 : Couris *et al.*, *Chem. Phys. Lett.* **369**, 318 (2003).
 n_4 : Kong *et al.*, *J. Phys. B: At. Mol. Phys.* **42**, 065401 (2009).

$$n_2 = +3.1 \times 10^{-19} \text{ m}^2/\text{W} \quad n_4 = -2.0 \times 10^{-35} \text{ m}^4/\text{W}^2$$



Focusing nonlinearity



Defocusing nonlinearity

(2+1)D Soliton $E(x, y, z, t) = A a(x, y, z) \exp[i(k_0 n z - \omega t)]$

NLSE
$$\frac{1}{2k_0 n_0} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) a + i \frac{\partial a}{\partial z} + \frac{k_0 n_0 n_2}{2\eta_0} |a|^2 a = 0$$

(2+1)D soliton is unstable in a pure Kerr medium
Catastrophic self-focusing

Chiao, Garmire, Townes PRL (1964)

High-order nonlinearity

Modified NLS equation

$\chi^{(5)}$

$$\frac{1}{2k_0 n_0} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) a + i \frac{\partial a}{\partial z} + \frac{k_0 n_0 n_2}{2\eta_0} |a|^2 a + \frac{k_0 n_0^2 n_4}{4\eta_0^2} |a|^4 a - \frac{\alpha_3}{2} \left(\frac{n_0}{2\eta_0} \right)^2 |a|^4 a = 0$$

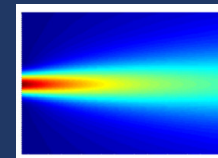
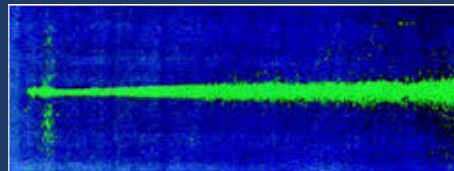
These terms may stabilize the (2+1)D spatial soliton

Este problema era conhecido desde a década de 60 mas a propagação estável de **Sólitons espaciais** em um meio homogêneo, isotrópico com não linearidade eletrônica não havia sido demonstrada

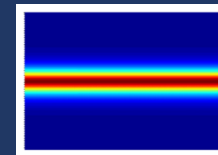
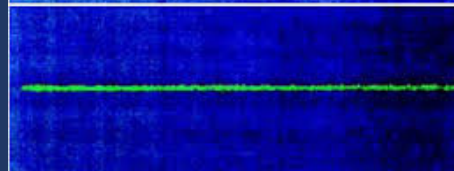
Experiência

NLSE

Propagação linear



Sóliton espacial



Robust Two-Dimensional Spatial Solitons in Liquid Carbon Disulfide

Edilson L. Falcão-Filho* and Cid B. de Araújo

Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife, Pernambuco, Brazil

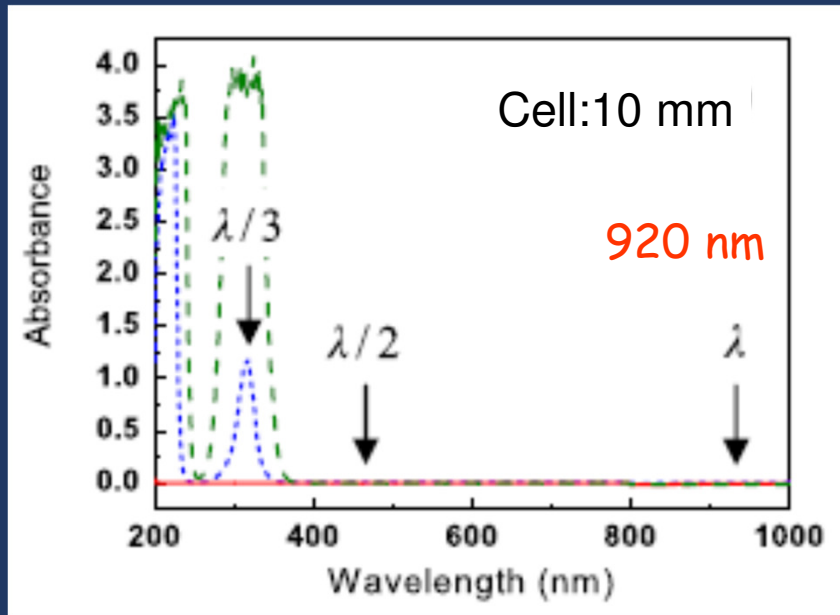
Georges Boudebs, Hervé Leblond, and Vladimir Skarka

LUNAM Université, Université d'Angers, Laboratoire de Photonique d'Angers, EA 4464, 49045 Angers, France

First demonstration of (2+1)D soliton propagating in a liquid

The laser wavelength was selected to be in the transparency region
but three-photon absorption is possible

Very important: contributions of third and fifth order of opposite signals



Absorption spectra of
 CS_2 diluted in ethanol

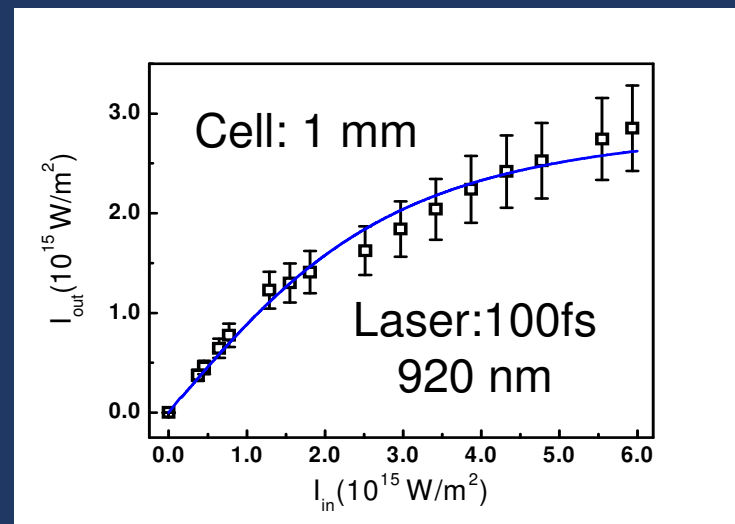
$$n_2 > 0 \quad n_4 < 0$$

Three-photon absorption
of pure CS_2

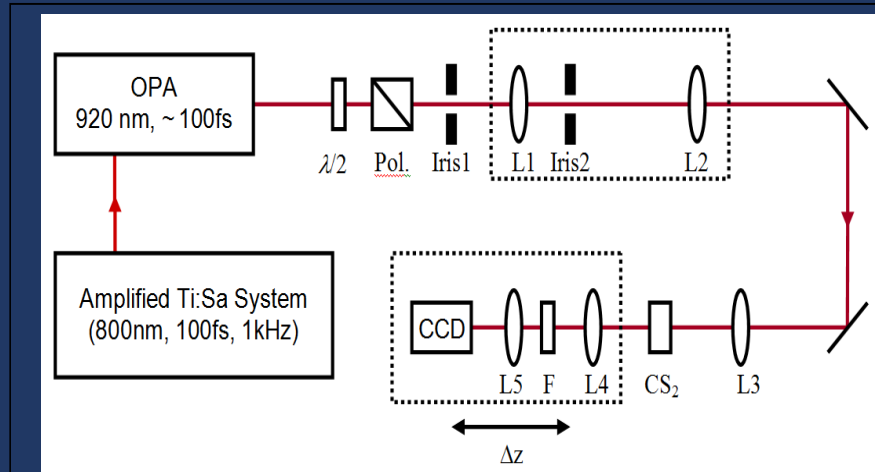
Optical limiting experiment

$$I_L = (1 - R)^2 \frac{I_0}{\sqrt{1 + 2I_0^2(1 - R)^2 \alpha_3 L}}$$

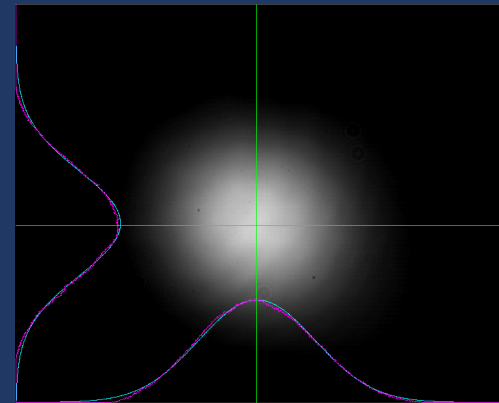
$$\alpha_3 = 5.8 \times 10^{-29} \text{ m}^3/\text{W}^2$$



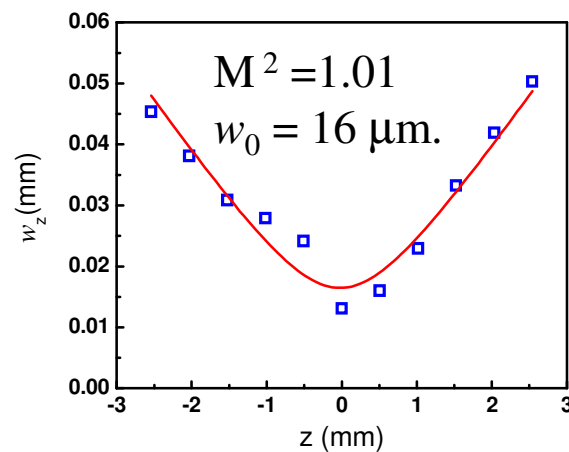
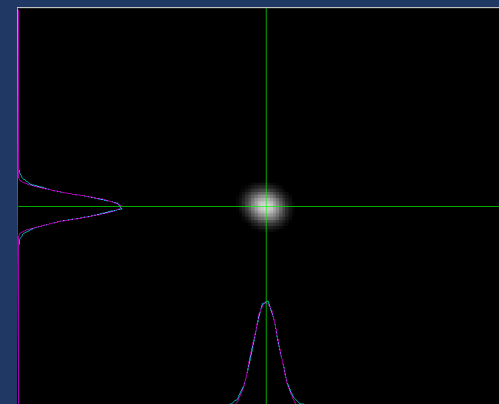
setup and beam characteristics



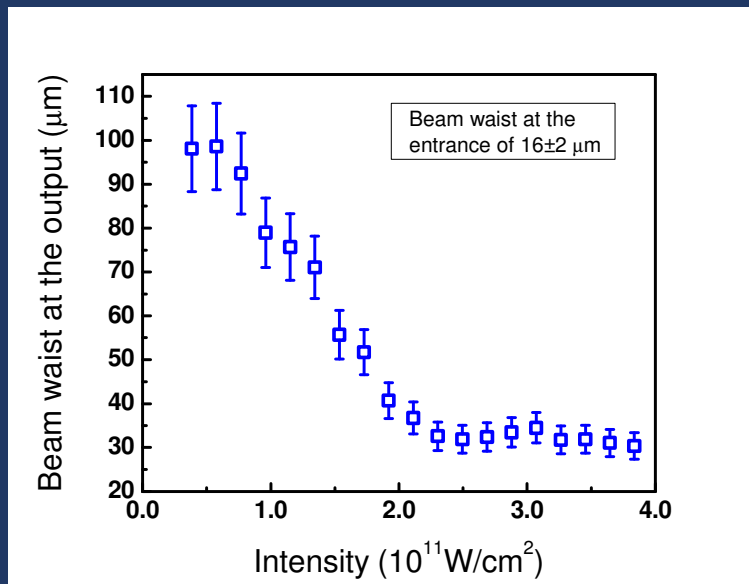
Entrance of L3: $w = 2.0$ mm



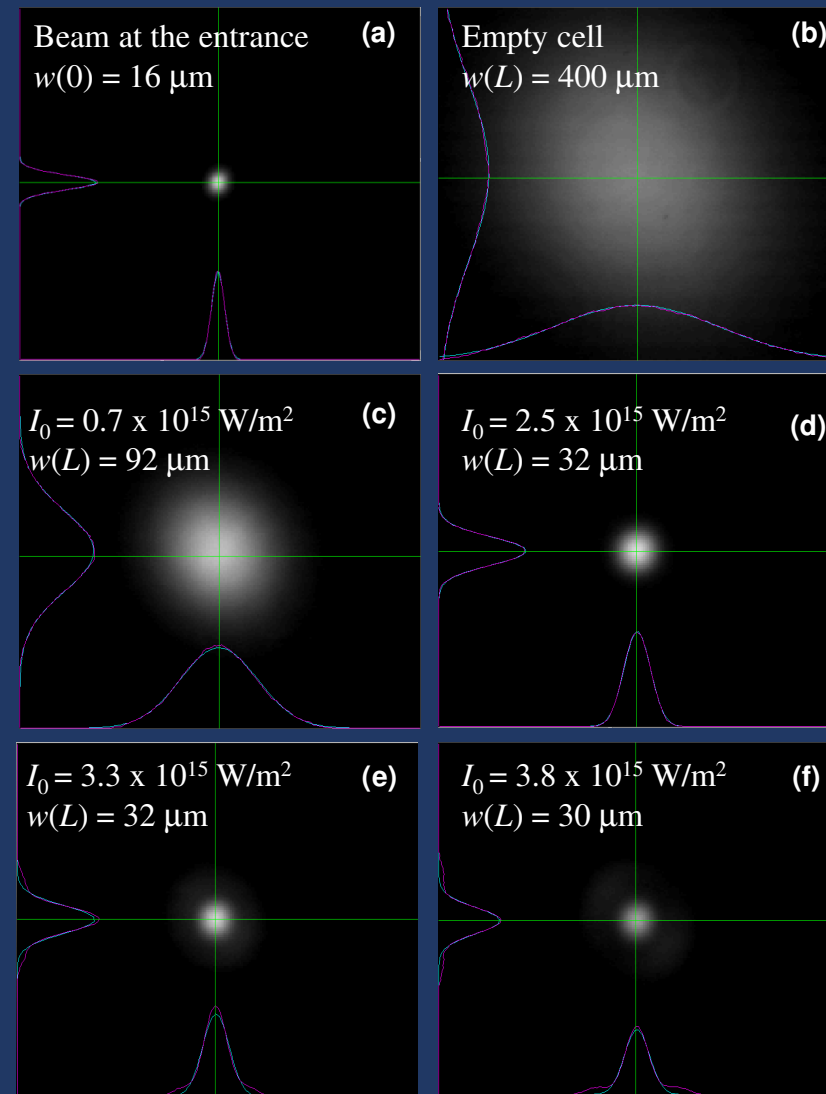
Focus: $w_0 = 16$ μm .



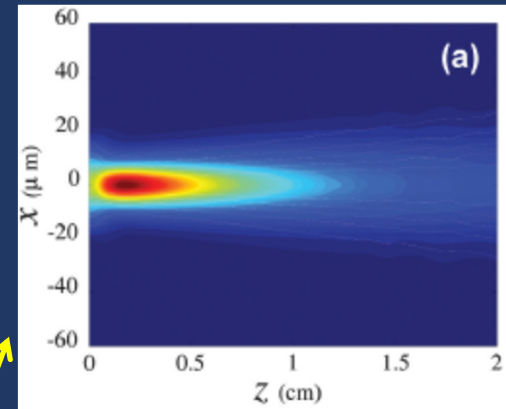
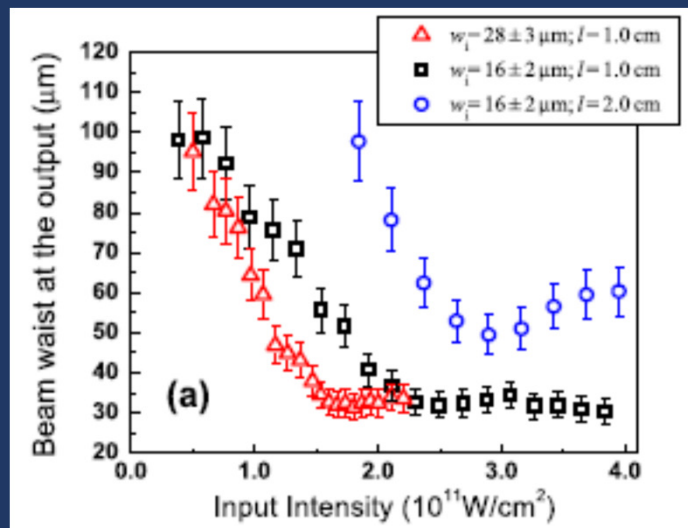
1.0 cm cell Front face at the focus



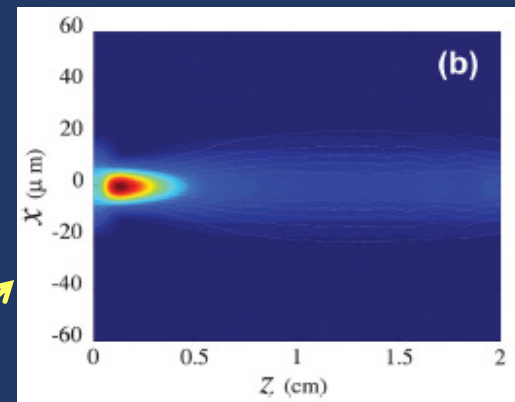
1 cm \rightarrow $10z_0$



Various beams' waist and length of cells



$0.8 \times 10^{11} \text{ W/cm}^2$



$1.6 \times 10^{11} \text{ W/cm}^2$

NLS equation

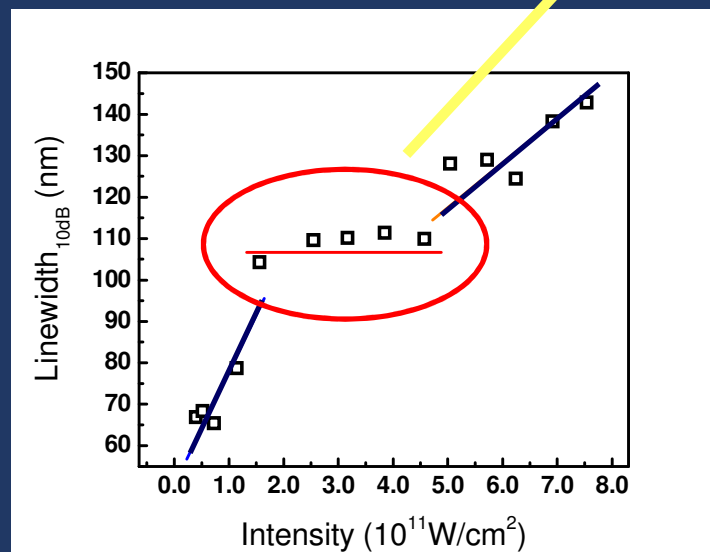
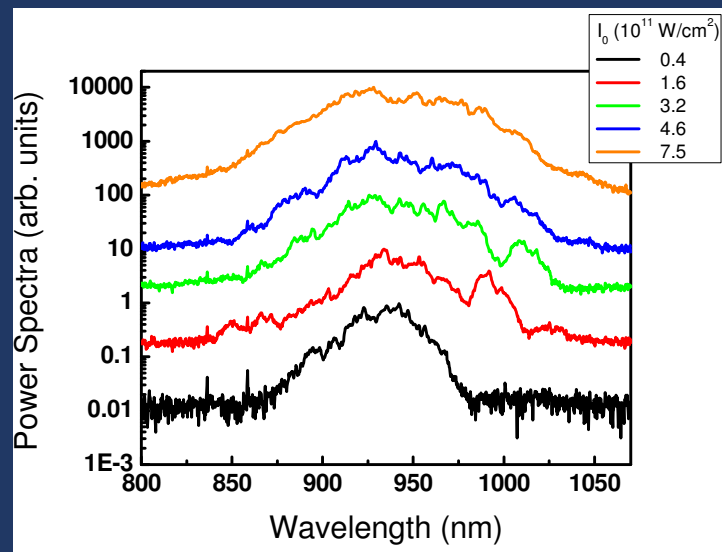
Spectral broadening

$$\phi(t) = \omega_0 t - \frac{2\pi}{\lambda_0} \cdot n(I) L$$

$$\omega(t) = \frac{d\phi(t)}{dt}$$

$$I(t) = I_0 \exp\left(-\frac{t^2}{\tau^2}\right)$$

Intensity clamping



Summary – bright spatial solitons

- (2+1)D spatial soliton propagation over more than $10z_0$ in CS_2 due to simultaneous contribution of the third- and the fifth-order susceptibilities.
- Intensity clamping effect which corroborates the soliton stability
- Computer simulations with the NLSE. Results in agreement with the experimental data.

(2+1)D soliton is stable in a homogeneous medium if:

$$\text{Re } \chi^{(3)} > 0 \qquad \text{Re } \chi^{(5)} < 0$$

Phys. Rev. Lett. 110 (2013) 013901

The contribution of $\chi^{(5)}$ prevents the catastrophic self-focusing

Is it possible to observe a stable (2+1)D soliton in a system with:

$$\text{Re } \chi^{(3)} = 0 \quad , \quad \text{Re } \chi^{(5)} > 0 \quad , \quad \text{Re } \chi^{(7)} < 0 \quad ?$$

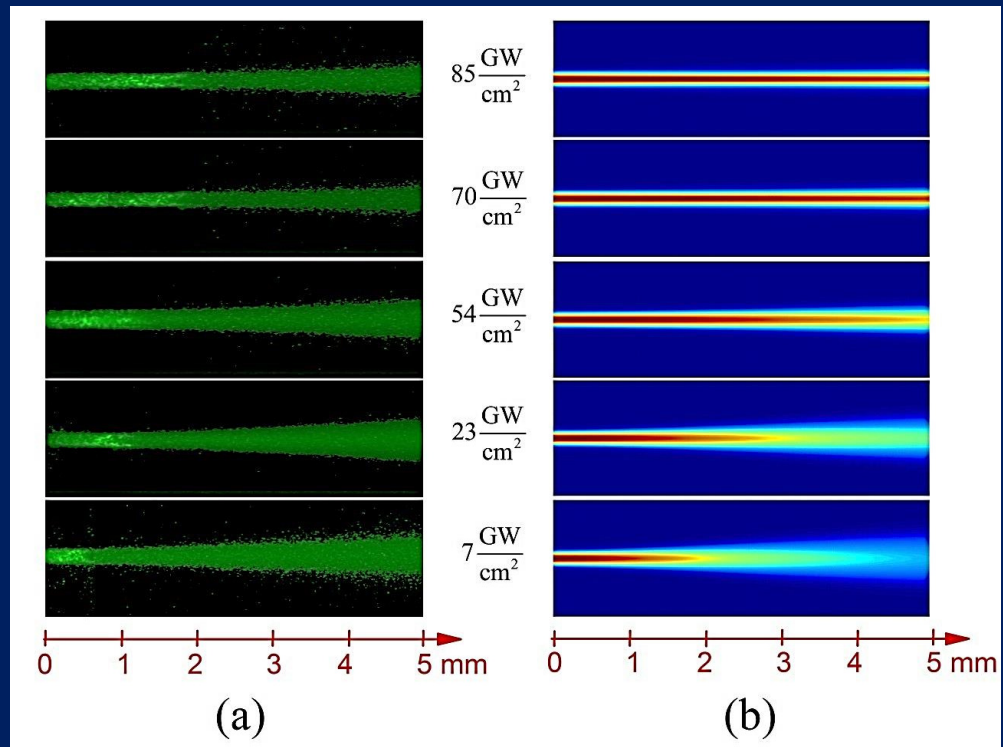
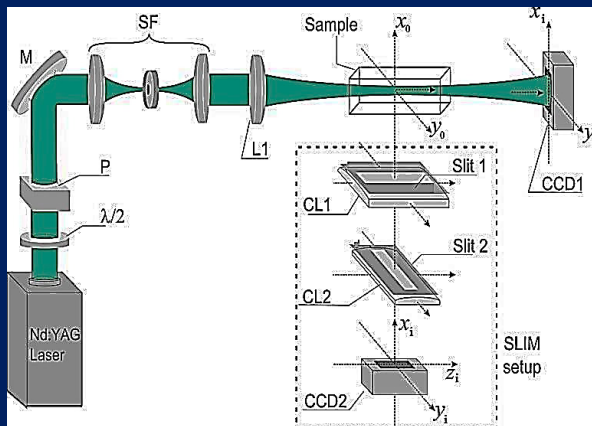
$$2ik \frac{\partial E}{\partial z} + \Delta E = -\frac{\omega^2}{c^2} \left[3\chi_{\text{eff}}^{(3)} |E|^2 E + 10\chi_{\text{eff}}^{(5)} |E|^4 E + 35\chi_{\text{eff}}^{(7)} |E|^6 E \right]$$

First observation of 2D Spatial-Solitons in a quintic-septimal medium

Silver NPs
in acetone

$$\chi^{(2N+1)}$$

$$n_2 = 0; \quad n_4 > 0; \quad n_6 < 0$$



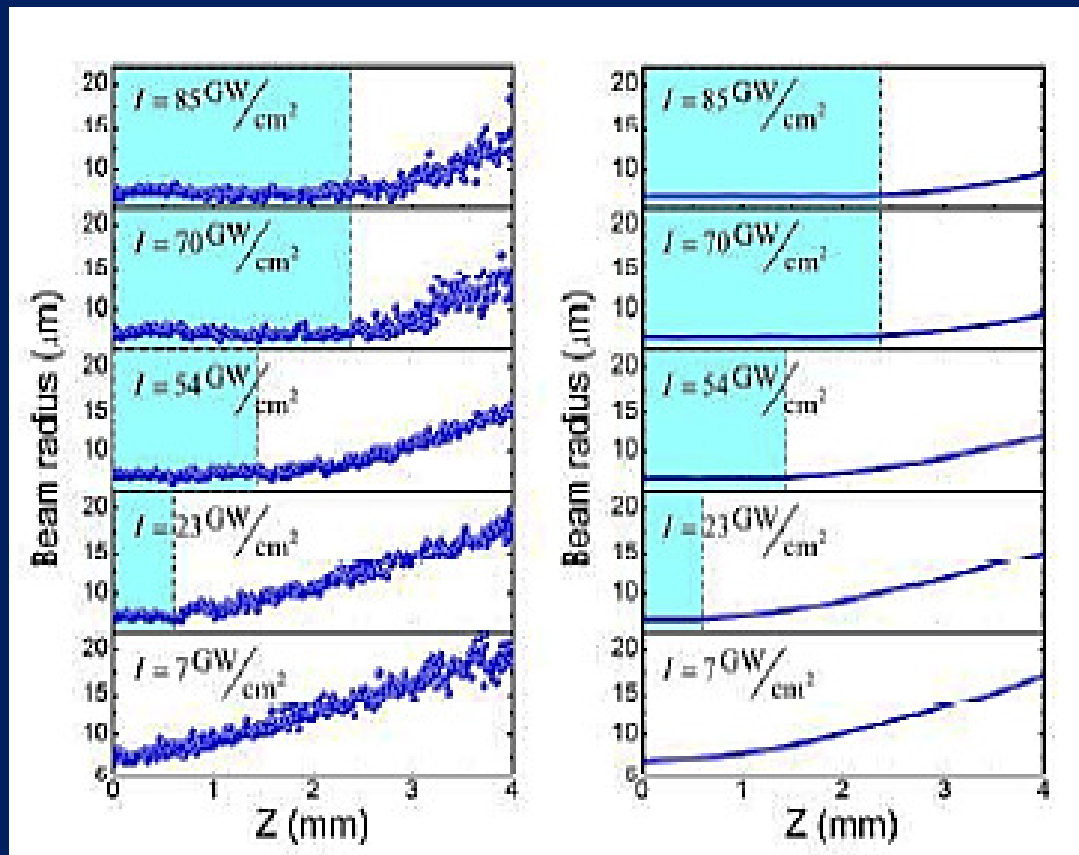
experiment

theory

Reyna, Jorge, de Araújo,
de Araújo et al. ,

Phys. Rev. A 90 (2014) 063835
J. Lumin. 169 (2016) 492-496

$$2ik \frac{\partial E}{\partial z} + \Delta E = -\frac{\omega^2}{c^2} \left[3\chi_{\text{eff}}^{(3)} |E|^2 E + 10\chi_{\text{eff}}^{(5)} |E|^4 E + 35\chi_{\text{eff}}^{(7)} |E|^6 E \right]$$



experiment

theory

Optical vortices

- Beams with phase singularity
- Zero field in the center of the vortex
- Helical wavefront
- Phase

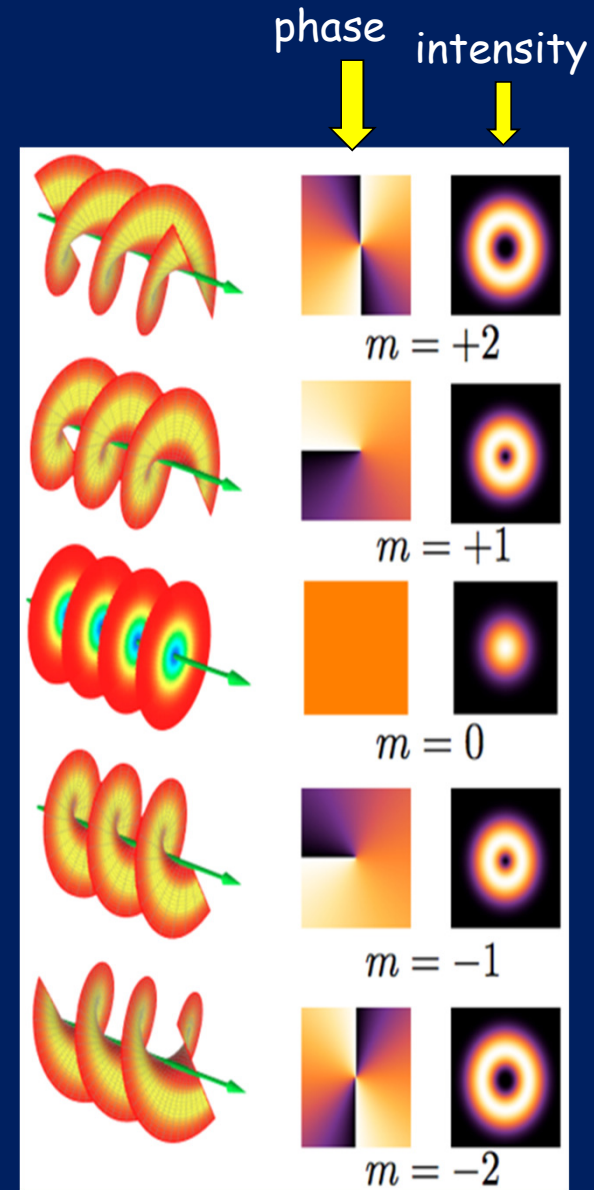


$$\phi(t, z, \theta) = kz + \omega t + m\theta$$

$m = 0$ Plane wave

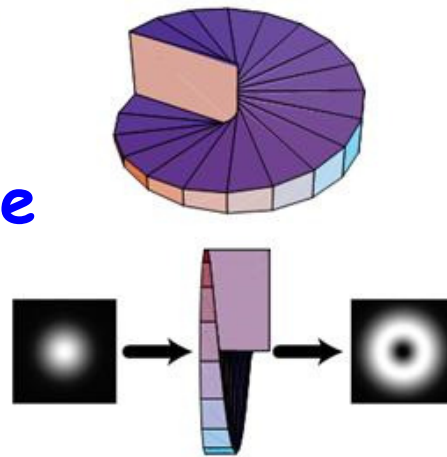
$m \neq 0$ Wave with topological phase

m is the "topological charge"



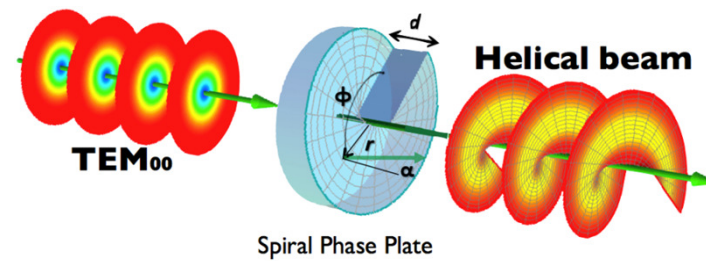
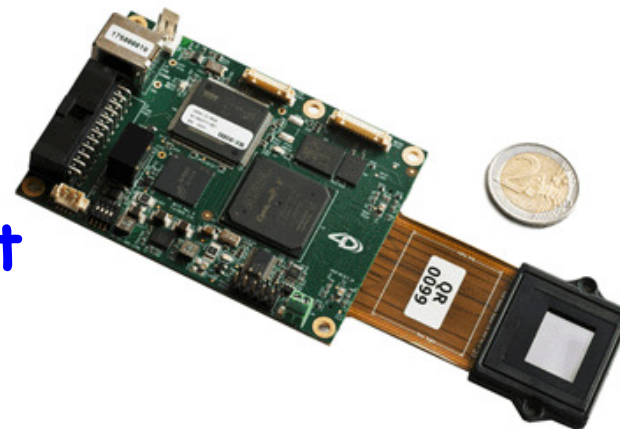
Optical Vortex beam carries an orbital angular momentum of $m\hbar$ per photon

Vortex
phase plate

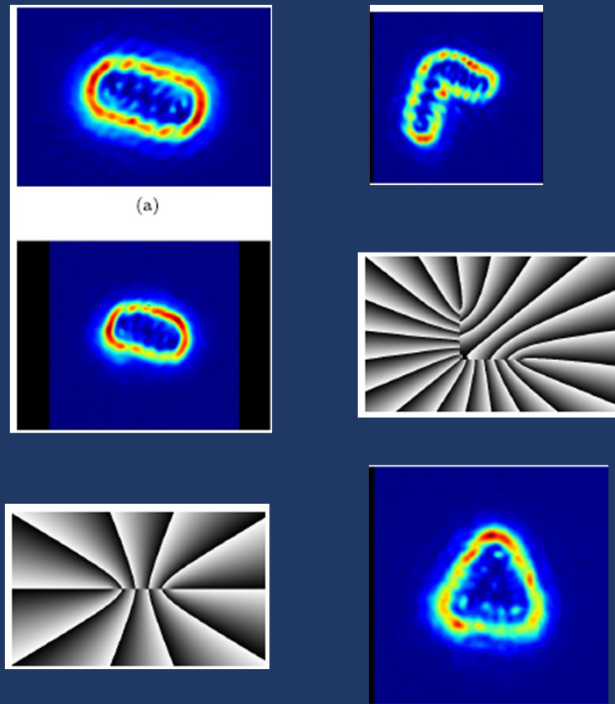


Images
from the
internet

Spatial light
modulator

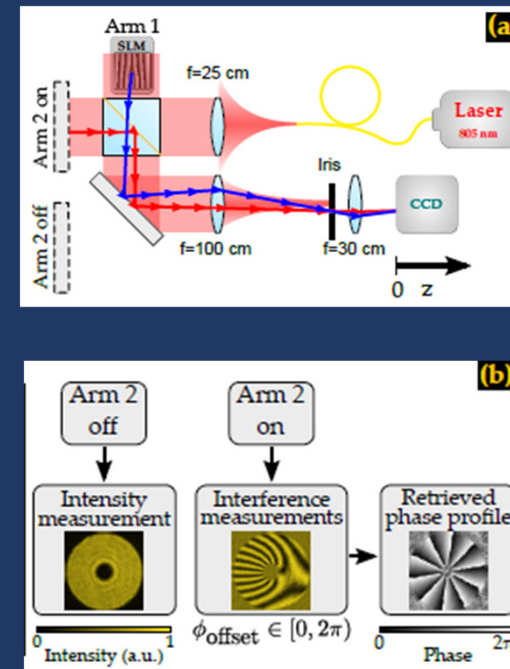


Shaping optical beams with topological charges



Amaral, Falcão-Filho, de Araújo
Opt. Lett. 38 (2013) 1579

Characterization of topological charge and orbital angular momentum of shaped optical vortices



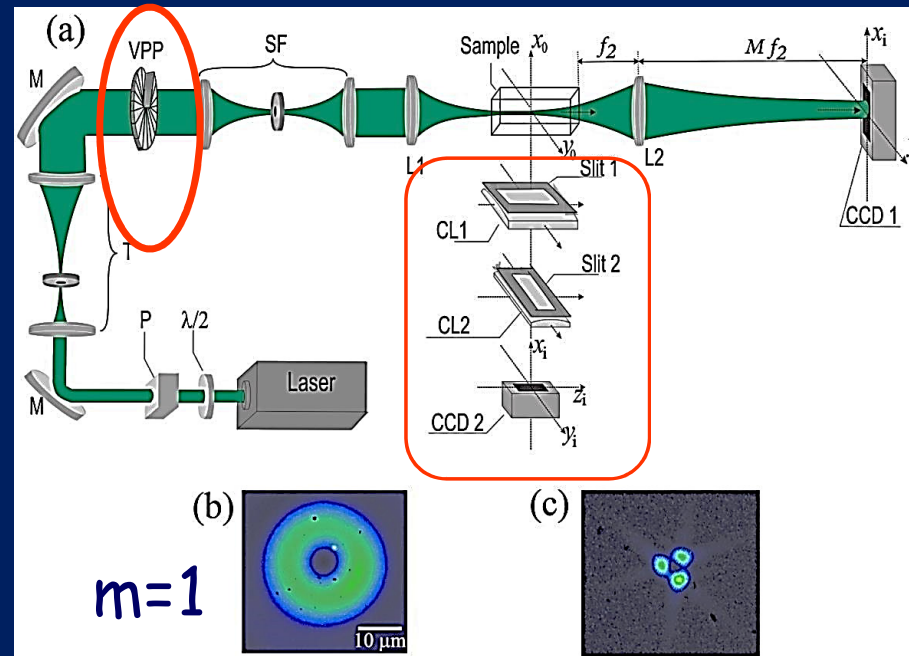
Amaral, Falcão-Filho, de Araújo
Opt. Express 22 (2013) 30315

PHYSICAL REVIEW A 93, 013840 (2016)

Robust self-trapping of vortex beams in a saturable optical medium

Albert S. Reyna,^{1,*} Georges Boudebs,² Boris A. Malomed,^{1,†} and Cid B. de Araújo¹

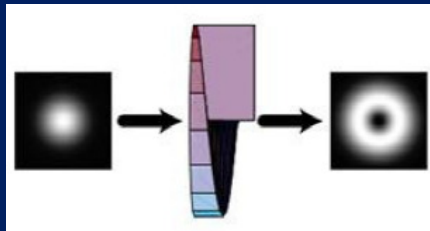
532 nm
80 ps
10 Hz



CS₂

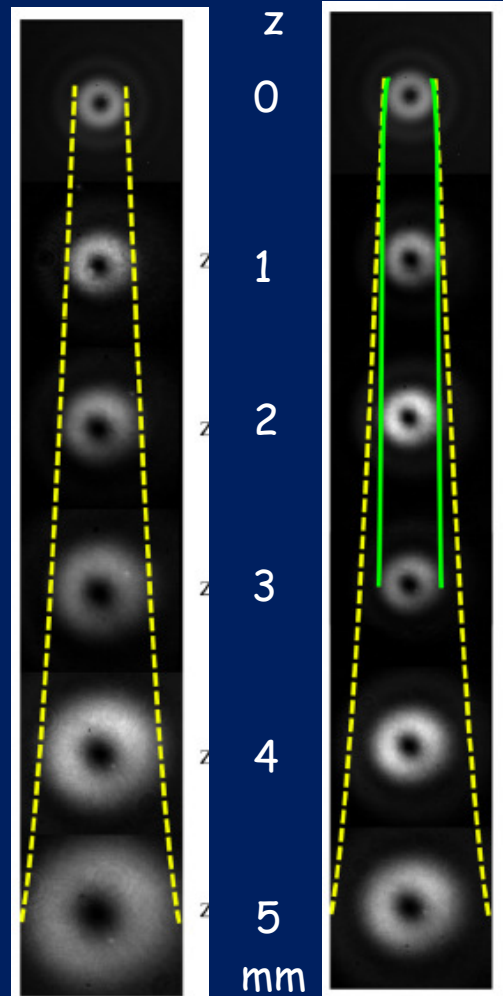
$m=1$

VPP- vortex phase plate



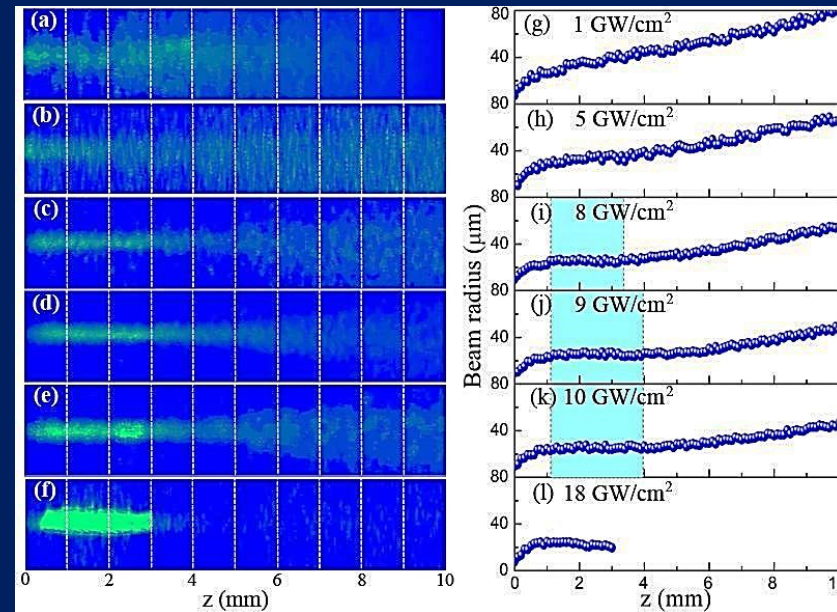
Optical Vortex beam carries an orbital angular momentum of $m\hbar$ per photon

Optical Vortex Solitons in CS_2



1.0 GW/ cm²

9.0 GW/ cm²



$$i \frac{\partial E}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp}^2 E - \left[\frac{k a I^2}{1 + b^2 I^2} + i \frac{\gamma I^2}{2} \right] E$$

First observation of an OVS in a self-focusing medium with local nonlinearity

Optical Vortex Soliton - OVS

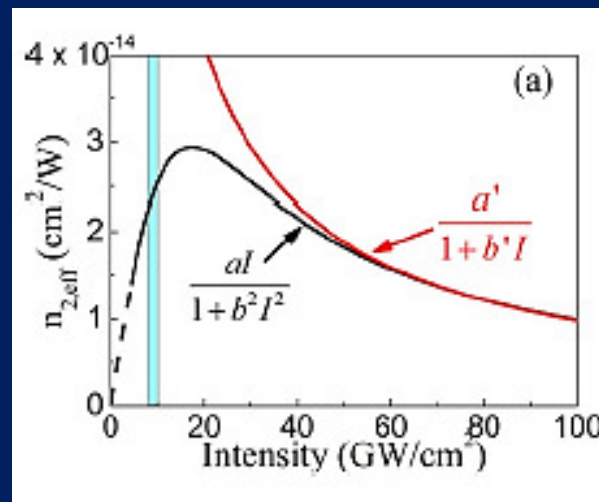
Stable propagation in a self-defocusing medium

Unstable propagation in self-focusing

Conditions on the NLSE to observe stable OVS in a self-focusing medium?

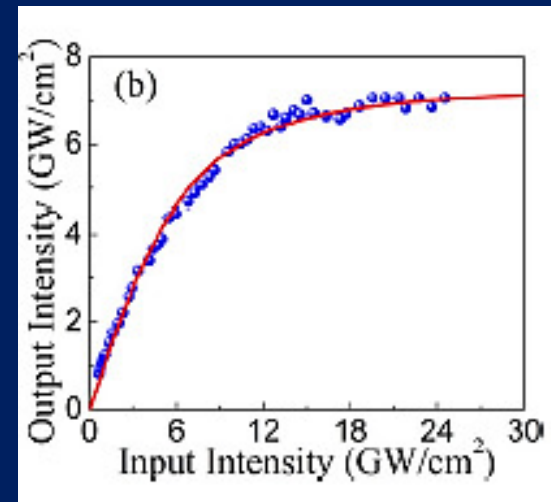
Saturable NL refractive index and NL absorption

CS_2
532 nm
psec



Effective NL refractive index

Besse, Leblond, Boudebs
Phys. Rev. A 89 (2014) 043840

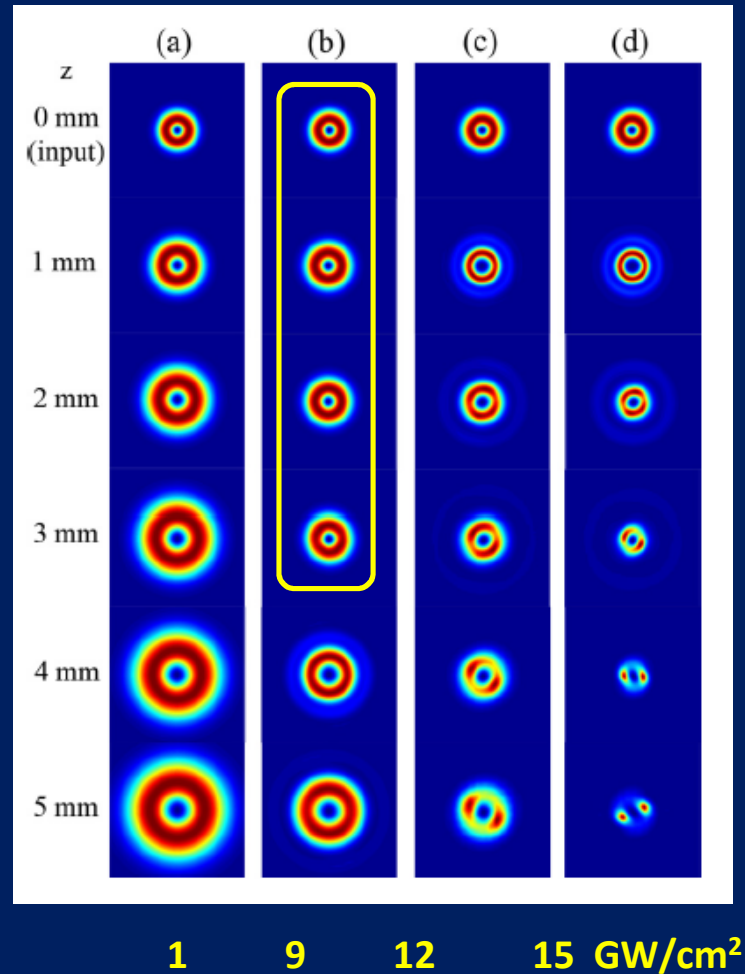


NL transmittance

This experiment

Numerically simulated
images
showing the evolution
of the transverse
vortex-beam profiles

$$i \frac{\partial E}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp}^2 E - \left[\frac{kaI^2}{1+b^2I^2} + i \frac{\gamma I^2}{2} \right] E$$

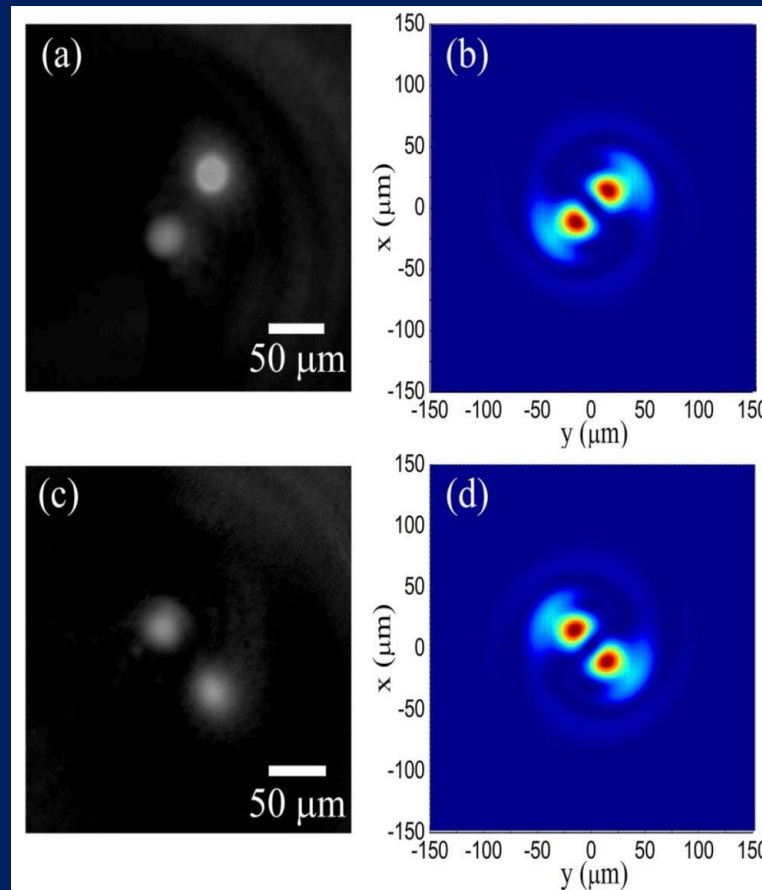


Azimuthal symmetry breaking

$$I = 18 \text{ GW/cm}^2$$

$$I = 15 \text{ GW/cm}^2$$

$$m = +1$$



$$m = -1$$

Considering 3PA
Stable OVS

$$8 \text{ GW/cm}^2 \leq I < 10 \text{ GW/cm}^2$$

splitting

$$13 \text{ GW/cm}^2 \leq I$$

No 3PA

$$7.4 \text{ GW/cm}^2 \leq I \leq 7.6 \text{ GW/cm}^2$$

$$8 \text{ GW/cm}^2 \leq I$$

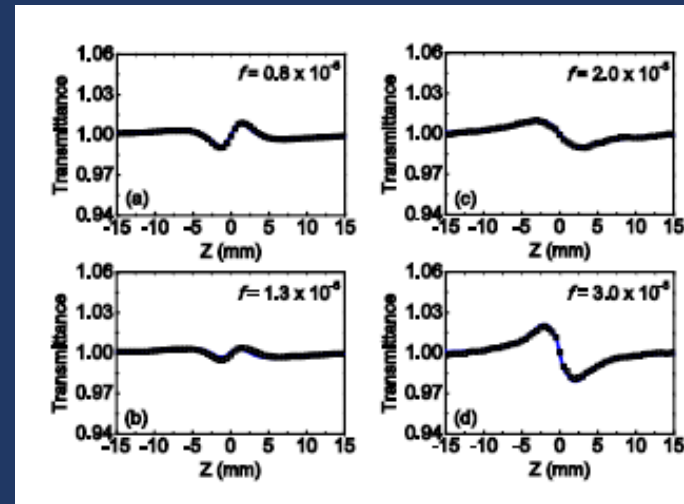
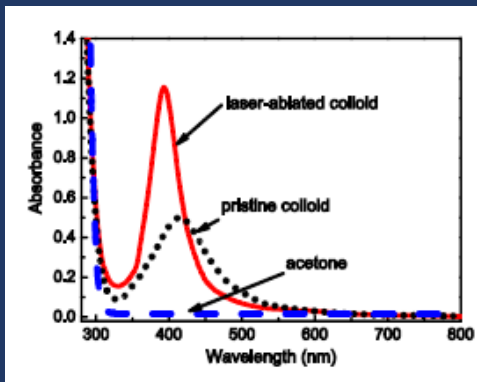
FILM



topological charge $m = +1$ final.mp4

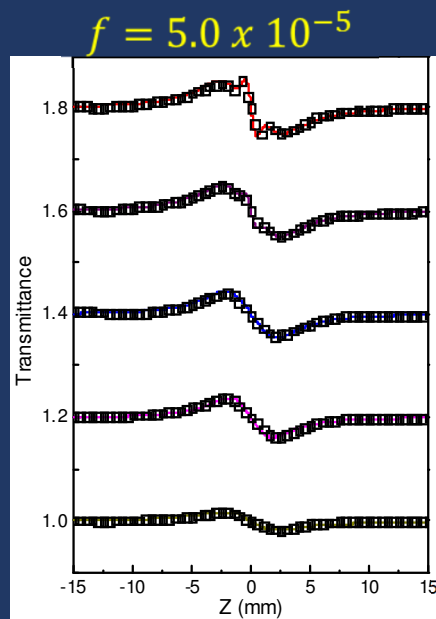
Observation of fifth-order refraction in a colloid with suppressed third-order refraction

Silver NPs in acetone



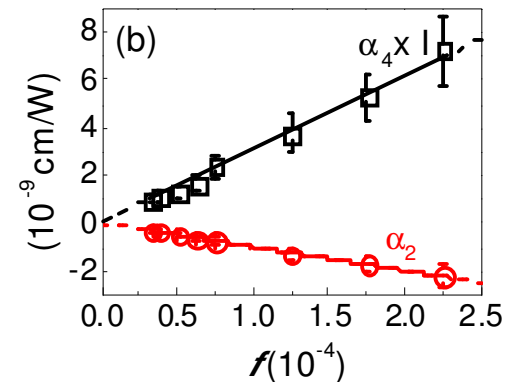
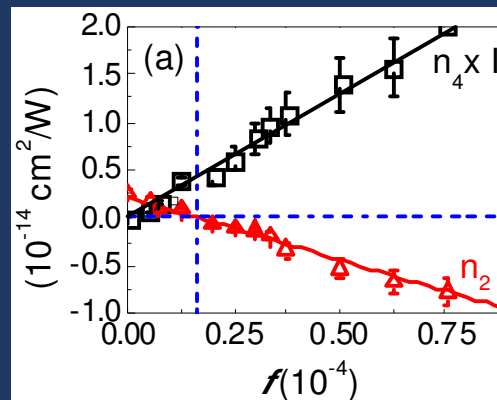
Z-scan

532 nm
Single pulses
5 GW/cm²



9
8
6
4
2

GW/cm²



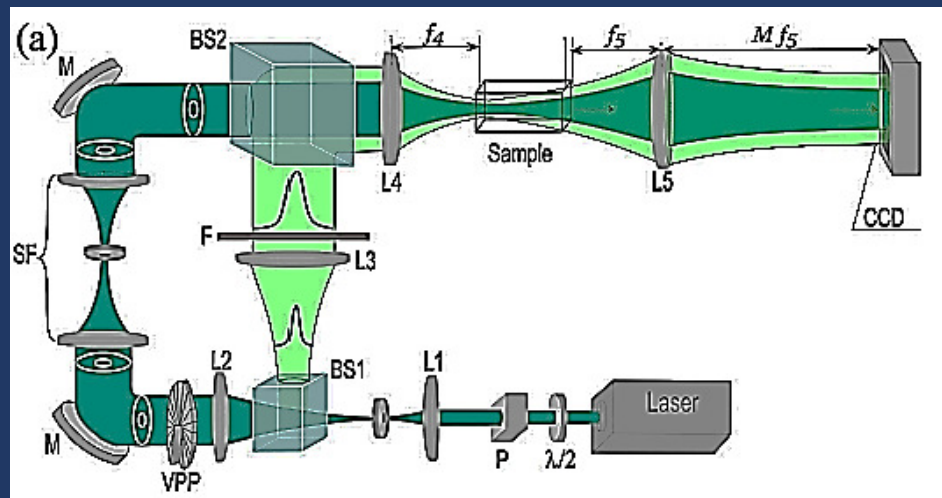
9 GW/cm²

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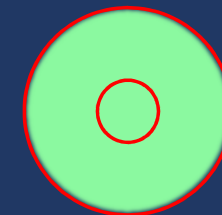
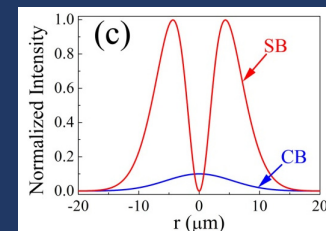
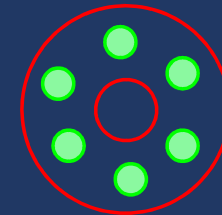
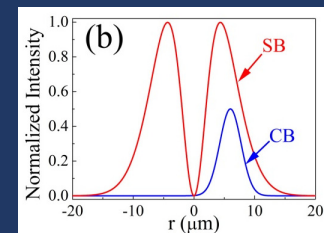
Taming the emerging beams after the split of optical vortex solitons in a saturable medium

Albert S. Reyna* and Cid B. de Araújo

Possible applications of OVS in all-optical devices

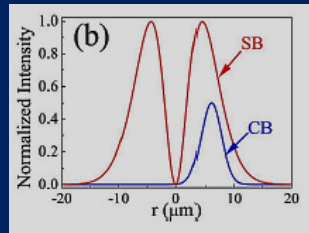


532 nm, 80 ps, 10Hz

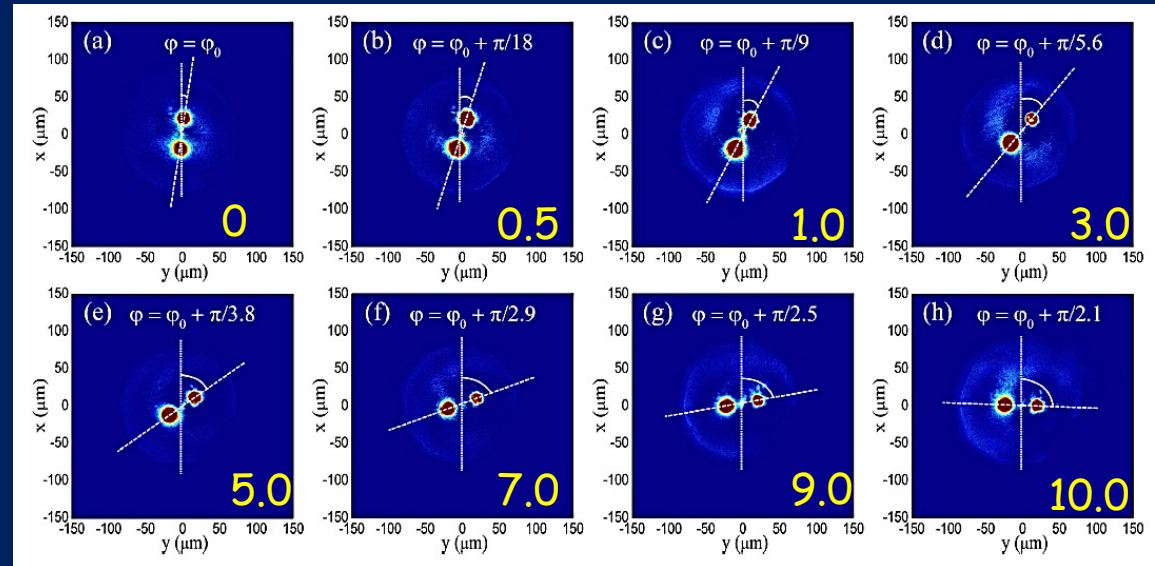


cell length: 10 mm

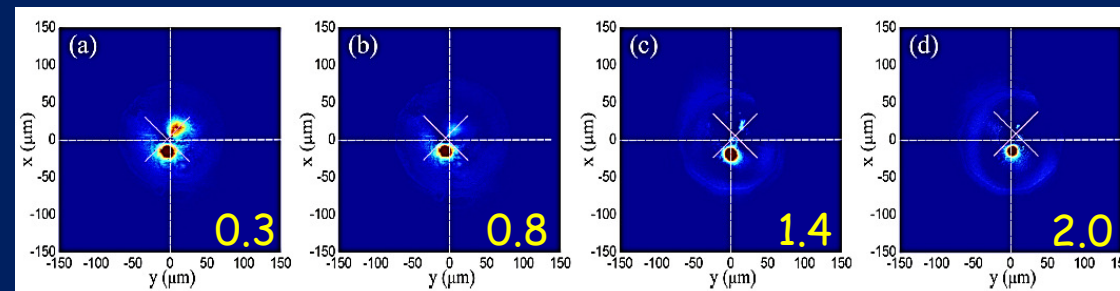
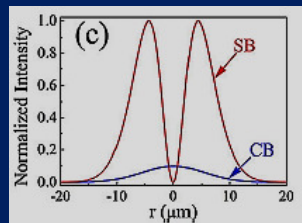
VB intensity
18 GW/cm²



Control beam
GW/cm²



Control beam less intense than the signal beam



Control of energy transfer

Two coupled NL equation considering saturable nonlinearity and 3PA

Vortex beam

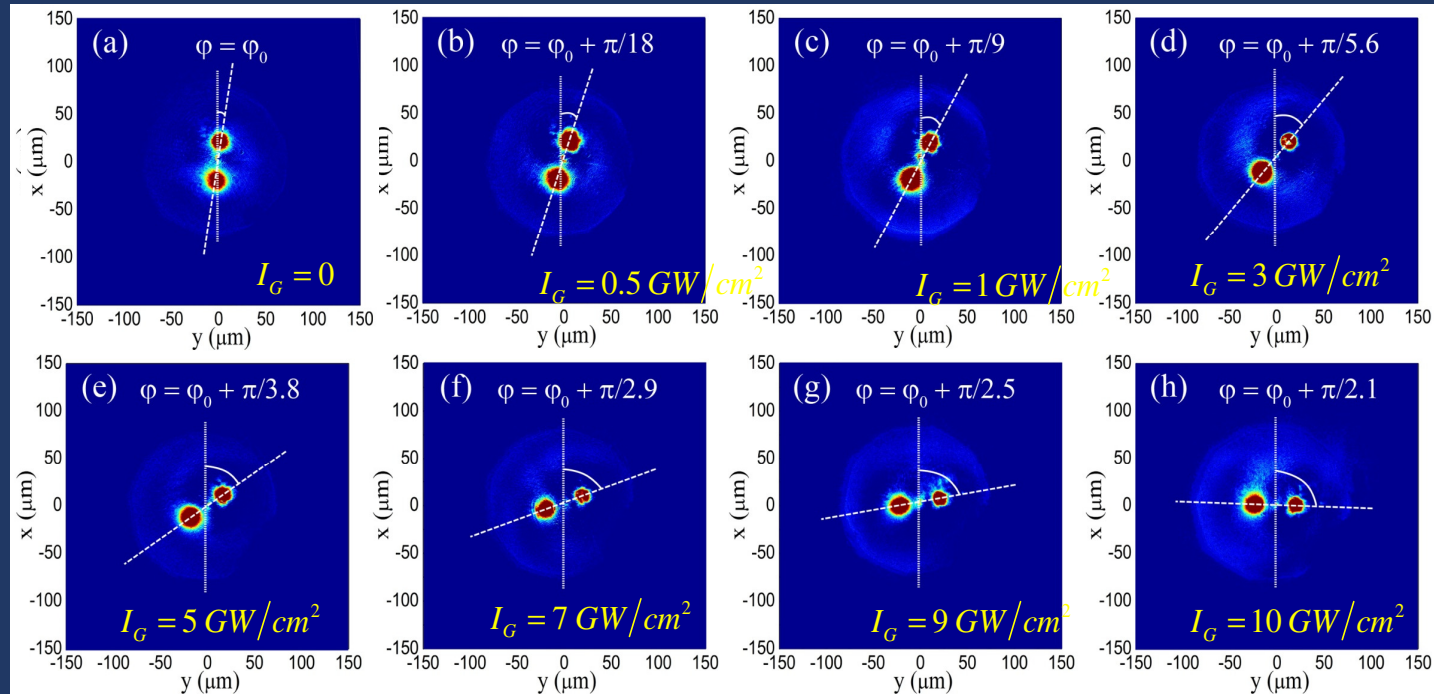
$$\longrightarrow i \frac{\partial E_S}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp}^2 E_S - \left[\frac{ka(I_S^2 + I_S I_C)}{1 + b^2(I_S + I_C)^2} + i \frac{\gamma}{2} (I_S^2 + 3I_S I_C) \right] E_S$$

Gaussian beam

$$\longrightarrow i \frac{\partial E_C}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp}^2 E_C - \left[\frac{ka(I_C^2 + I_S I_C)}{1 + b^2(I_S + I_C)^2} + i \frac{\gamma}{2} (I_C^2 + 3I_S I_C) \right] E_C$$



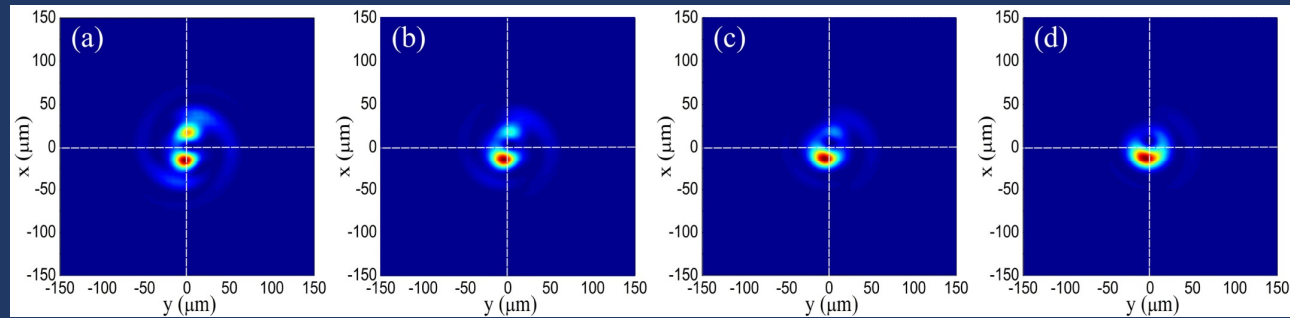
$$I_v = 18 \text{ GW/cm}^2$$



Two coupled NL equation considering saturable nonlinearity and 3PA

Vortex beam $\rightarrow i \frac{\partial E_s}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp}^2 E_s - \left[\frac{ka(I_s^2 + I_s I_C)}{1+b^2(I_s + I_C)^2} + i \frac{\gamma}{2} (I_s^2 + 3I_s I_C) \right] E_s$

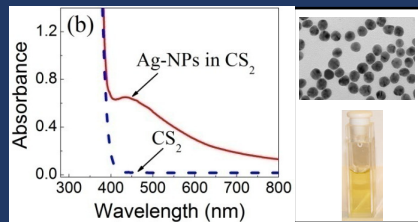
Gaussian beam $\rightarrow i \frac{\partial E_C}{\partial z} = -\frac{1}{2n_0 k} \nabla_{\perp}^2 E_C - \left[\frac{ka(I_C^2 + I_s I_C)}{1+b^2(I_s + I_C)^2} + i \frac{\gamma}{2} (I_C^2 + 3I_s I_C) \right] E_C$



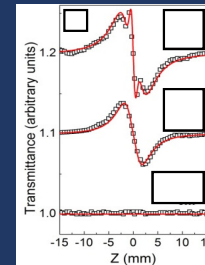
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Demonstration of optically induced waveguide using OVS

Metal colloid



Z-scan of AgNP in CS_2

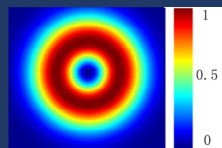


$$n^{(NL)} < 0$$

Self-defocusing medium

$$n^{(TOTAL)} = n_0 + n^{(NL)} < n_0$$

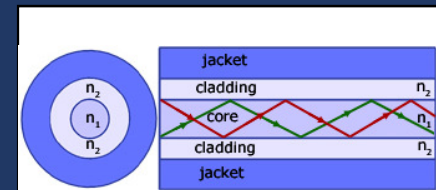
Vortex beam profile ($l = +1$)



Intensity

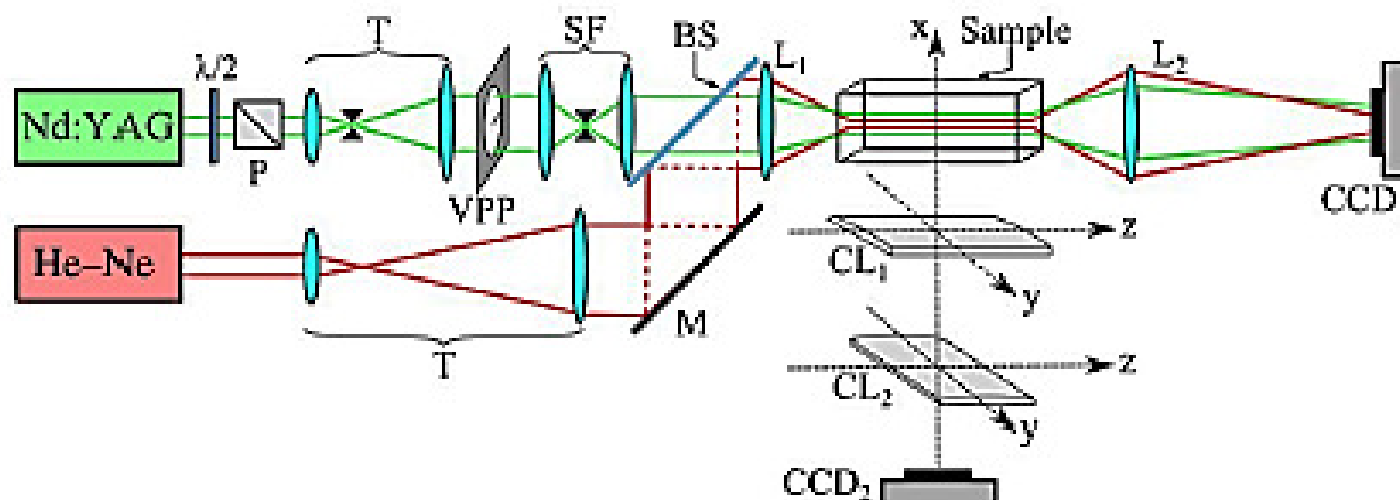


Optical fiber

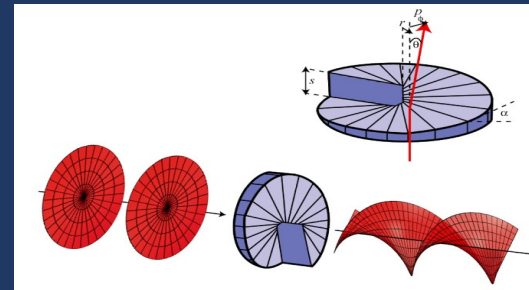
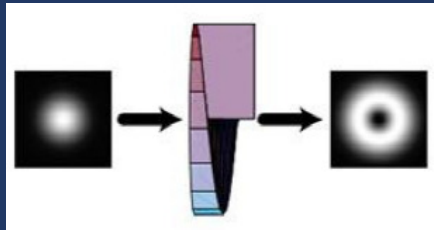


Guiding and confinement of light induced by optical vortex solitons in a cubic–quintic medium

ALBERT S. REYNA* AND CID B. DE ARAÚJO



VPP- vortex phase plate

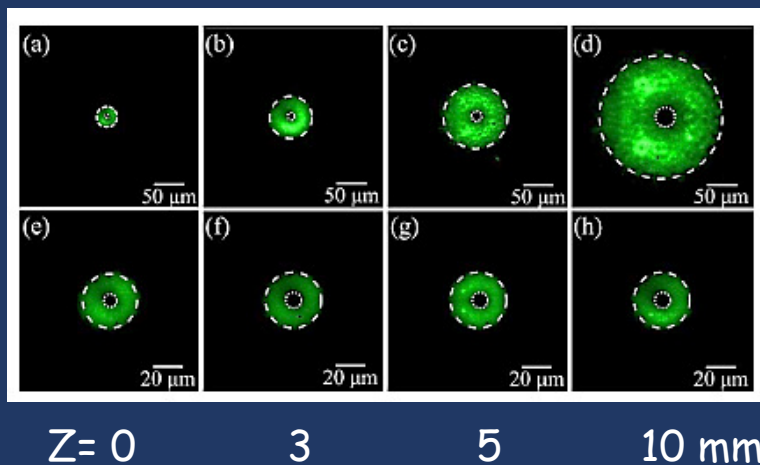


Optical Vortex beam carries an orbital angular momentum of $m\hbar$ per photon

Ag NPs (9.0 nm) suspended in acetone $f = 3 \times 10^{-5}$

$$\chi_{\text{eff}}^{(3)} = -(8.3 + i2.7) \times 10^{-21} \text{ m}^2/\text{V}^2$$

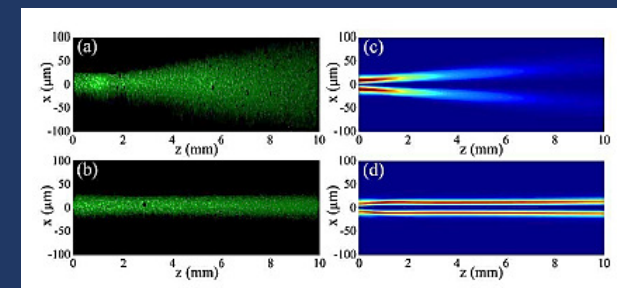
$$\chi_{\text{eff}}^{(5)} = (2.8 + i0.2) \times 10^{-35} \text{ m}^4/\text{V}^4$$



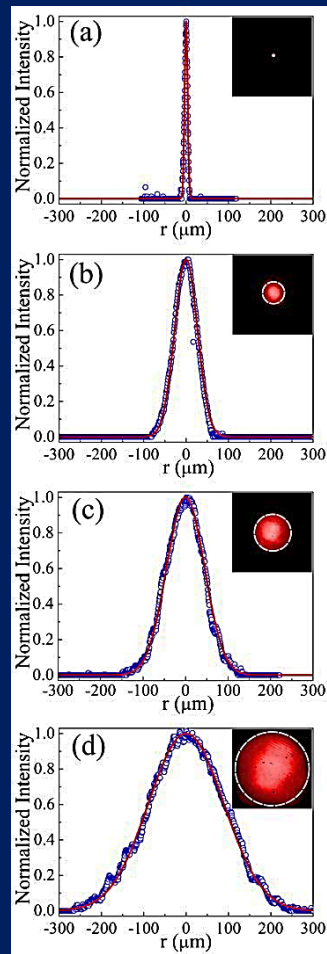
0.1

3.0

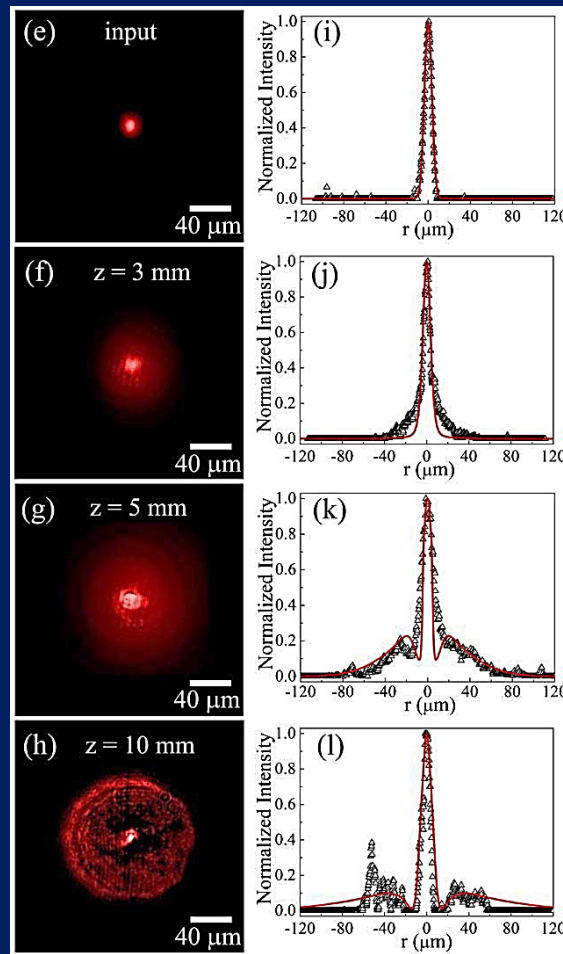
GW/cm²



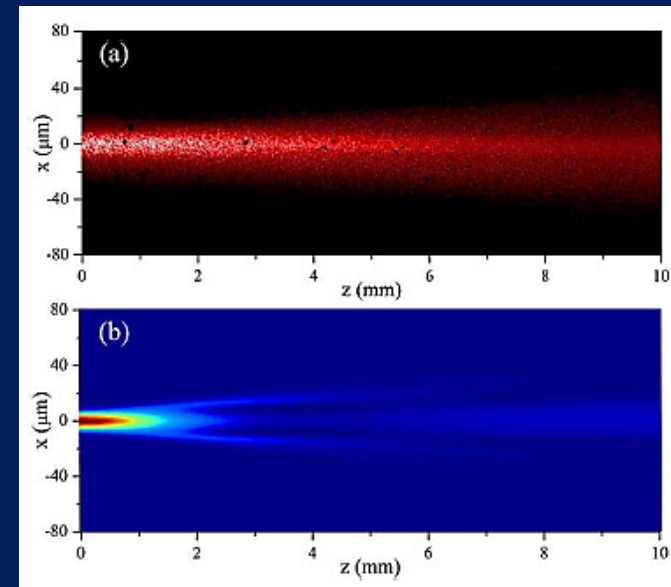
10 mm - 25 Z_R



HeNe



Guided HeNe



$$I_{Ovs} = 3.0 \text{ GW/cm}^2$$

$$I_{HeNe} = 0.1 \text{ GW/cm}^2$$

Exploitation of high-order electronic nonlinearities allows:

- observation of $(2+1)D$ bright spatial solitons in a homogeneous NL medium with electronic nonlinearity.
- observation of stable propagation of optical vortex solitons (OVS) in a medium with saturable refractive index and presenting NL absorption.
- controlling the instability of an OVS by using a control beam with smaller intensity than the OVS.
- guiding and confinement of an OVS in a self-defocusing cubic-quintic medium